# Guided Work Series Number 3 ( Numerical Series)

**Exercise 3.3.1** (Convergence of Various Series) Study the convergence of the following series $\sum u_n$:

**1.** $u_n = \dfrac{n}{n^3+1}$ **2.** $u_n = \dfrac{\sqrt{n}}{n^2+\sqrt{n}}$ **3.** $u_n = n\sin(1/n)$

**4.** $u_n = \dfrac{1}{\sqrt{n}}\ln\left(1+\dfrac{1}{\sqrt{n}}\right)$ **5.** $u_n = \dfrac{(-1)^n+n}{n^2+1}$ **6.** $u_n = \dfrac{1}{n!}$

**7.** $u_n = \dfrac{3^n+n^4}{5^n-2^n}$ **8.** $u_n = \dfrac{n+1}{2^n+8}$ **9.** $u_n = \dfrac{1}{\ln(n^2+1)}$

**Exercise 3.3.2** (Power Series and Radius of Convergence) Study the convergence of the following series $\sum u_n$:

**1.** $u_n = \left(\dfrac{1}{2}\right)^{\sqrt{n}}$ **2.** $u_n = a^n n!,\ a \in \mathbb{R}_+$ **3.** $u_n = ne^{-\sqrt{n}}$

**4.** $u_n = \dfrac{\ln(n^2+3)\sqrt{2^n+1}}{4^n}.$ **5.** $u_n = \dfrac{\ln n}{\ln(e^n-1)}$ **6.** $u_n = \left(\dfrac{1}{n}\right)^{1+\frac{1}{n}}$

**7.** $u_n = \dfrac{(n!)^3}{(3n)!}.$

**Exercise 3.3.3** (Function Series and Uniform Convergence) Study the uniform convergence of the function series:

$$\sum_{n=1}^{\infty} \frac{\sin(nx)}{n^2}$$

on $\mathbb{R}$.

**Exercise 3.3.4** (Alternating Series and Error Estimation) Consider the alternating series:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$$

(a) Show that the series converges.
(b) Estimate the error when approximating the sum by the first 10 terms.
(c) How many terms are needed to ensure the error is less than $10^{-4}$?

**Exercise 3.3.5** (Fourier Series) Find the Fourier series of the function:

$$f(x) = \begin{cases} 0 & \text{if } -\pi \le x < 0 \\ x & \text{if } 0 \le x < \pi \end{cases}$$

with period $2\pi$.