Exercise 9 : Problems set 3

9 groups of students are learning 5 courses in a semester.

The course C1 is taken by groups 1,2,3

The course C2 is taken by groups 6,7

The course C3 is taken by groups 1,2,7,9

The course C4 is taken by groups 4,6,8

The course C5 is taken by groups 2,3,4,5

We want to schedule the exams such that no group will have more than one exam in one day, and the length of the exam period will be as short as possible.

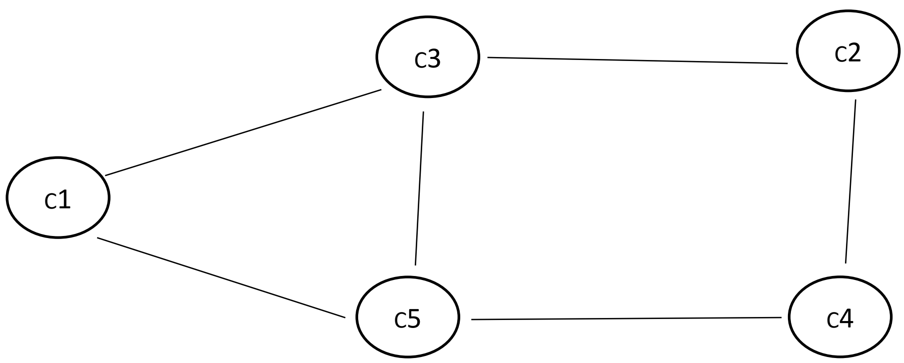
Solution

Build a conflict graph where vertices are courses and an edge joins two courses when they share at least one student group.

Course group sets:

- C1 = {1,2,3}
- C2 = {6,7}
- C3 = {1,2,7,9}
- C4 = {4,6,8}
- C5 = {2,3,4,5}

Edges (shared groups): (C1–C3), (C1–C5), (C2–C3), (C2–C4), (C3–C5), (C4–C5).



The subgraph C1–C3–C5 forms a triangle, so at least 3 colors (days) are required. (m= 3)

Max degree d= 3 $m \leq X(G) \leq d+1$ ➔ $3 \leq X(G) \leq 4$

We apply Powell-Welsh algorithm to calculate le color number, and we find 3 colors.

The chromatic number now is equal 3 **X(G) = 3**

Every colo is a day exam

Schedule 1

- Color A (Day 1): C1, C2
- Color B (Day 2): C3, C4
- Color C (Day 3): C5

Check: C1 and C2 do not conflict;

C3 and C4 do not conflict.

C5 conflicts with C1, C3, C4 so it sits alone on Day 3.

verification (per-day group lists)

- Day 1 (C1, C2) → groups {1,2,3,6,7} — no duplicate group inside the day.
- Day 2 (C3, C4) → groups {1,2,7,9,4,6,8} — no duplicate group inside the day.
- Day 3 (C5) → groups {2,3,4,5}.

All groups appear at most once per day, so the schedule is valid and minimal (3 days).

We can find other solutions, by varying the order of the vertices.

Schedule 2

Color 1 : C3, C4

Color 2 : C2, C5

Color 3: C1

Schedule 3

Color 1 : C1, C4

Color 2 : C2, C5

Color 3: C3