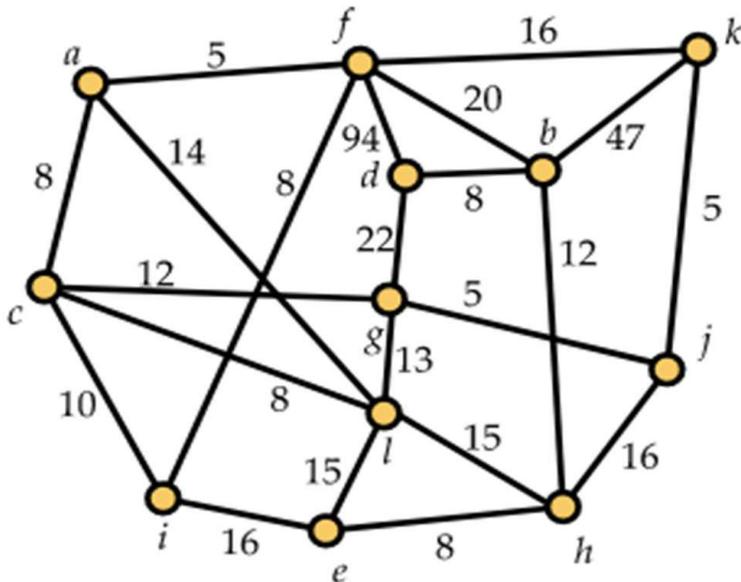


Graph theory (L2)

Problems Set 4 (Trees and pathfinding)**Exercise 1 :**

- 1-Use Kruskal's algorithm to find a minimum weight spanning tree (MST) for this graph
- 2-Use Know, Prim's algorithm to find a minimum weight spanning

**Exercise 2 :**

Consider the problem of computing a maximum spanning tree, namely the spanning tree that maximizes the sum of edge costs. Do Prim and Kruskal's algorithm work for this problem.

Exercise 3 :

Let k and n be strictly positive integers. Find the number of edges and the sum of the degrees of the vertices in a forest with k trees and n vertices.

Exercise 4 :

Let G be a rooted tree of height 4. Each internal node has exactly 3 children.

1. How many vertices are in the tree?
2. How many leaves does it have?

Exercise 5 :

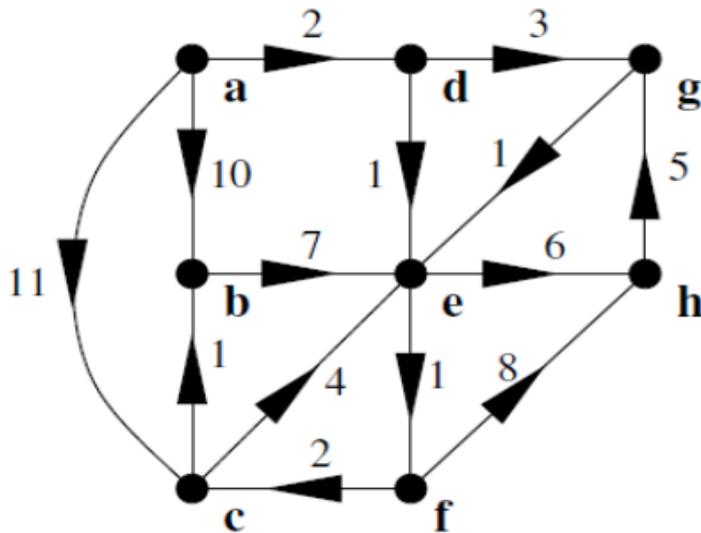
A tree has 12 vertices. The longest path has length 8.(diameter)

1. How many edges does the tree have?
2. Give a possible shape for such a tree.

Exercise 6 :

We consider the directed and weighted graph as follows:

- 1-Use **Dijkstra's** algorithm to find a shortest path tree starting from vertex a.
- 2-The length of the arc **ge** is actually **-8**. Redo the previous question. What do you observe?
3. Use the **Bellman algorithm** this time to find a shortest path tree starting from a.
- 4- The length of the arc **ge** know is **-12**. What do you observe? Which algorithm we can use ?



Exercise 7 :

We consider the directed and weighted graph as follows:

$$V=\{1,2,3,4,5,6,7\},$$

$$E=\{(1,2 ; 3),(1,3 ; 2),(2,3,3),(2,5,2),(2,4 ; 2),(3,6 ; 1),(4,3 ; 0),(4,5 ; 2),(5,7 ; 2),(6,4 ; 2),(6,5 ; 3),(6,7 ; 6)\}$$

Note : $(1,2 ; 3)$ means : the weight of the arc $(1,2)$ equal 3

- 1-Use the Bellman-Ford (**Improved Bellman**) algorithm to find a shortest path tree starting from vertex **1**.
- 2- The length of the arc $(6,7)$ know is **-6**. Use the Bellman-Ford (Improved Bellman) algorithm to **detect a negative circuit**.