Chapter 1: DC circuits and basic laws - 3 weeks

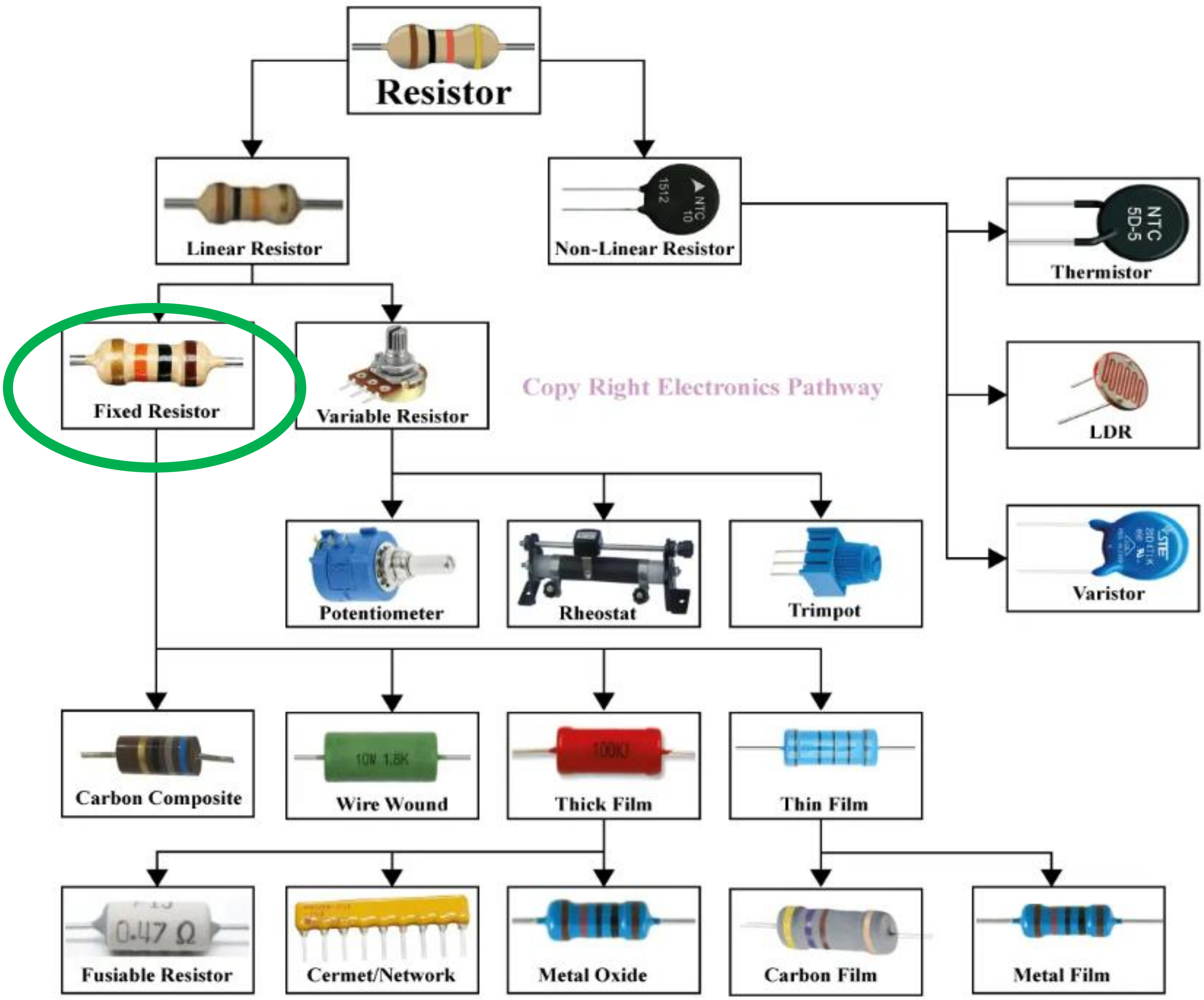
1.1. Definitions:

a. A two-terminal component (Dipole):

A two-terminal component (electrical dipole) is an electrical conductor, an electrical component, an electronic component, or any electrical circuit with two terminals. **Lamps**, **switches**, **generators**, **batteries**, **diodes**, **LEDs**, **resistors**, **capacitors** and **inductors** are dipoles. The electrical state of a two-terminal network is characterized by the potential difference (voltage) across its terminals and the current flowing through it.

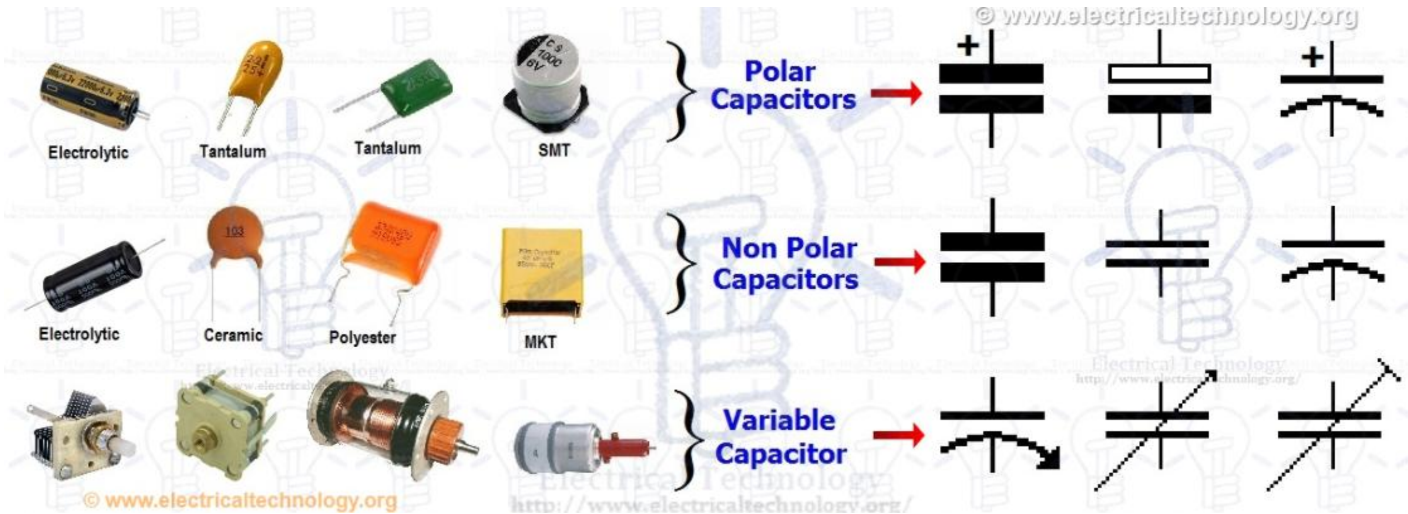
$$V = f(I)$$

Resistors: The relationship is given by **Ohm's law for linear resistors**: $V = R \cdot I$. Where V is the voltage, I the current and R the resistance.

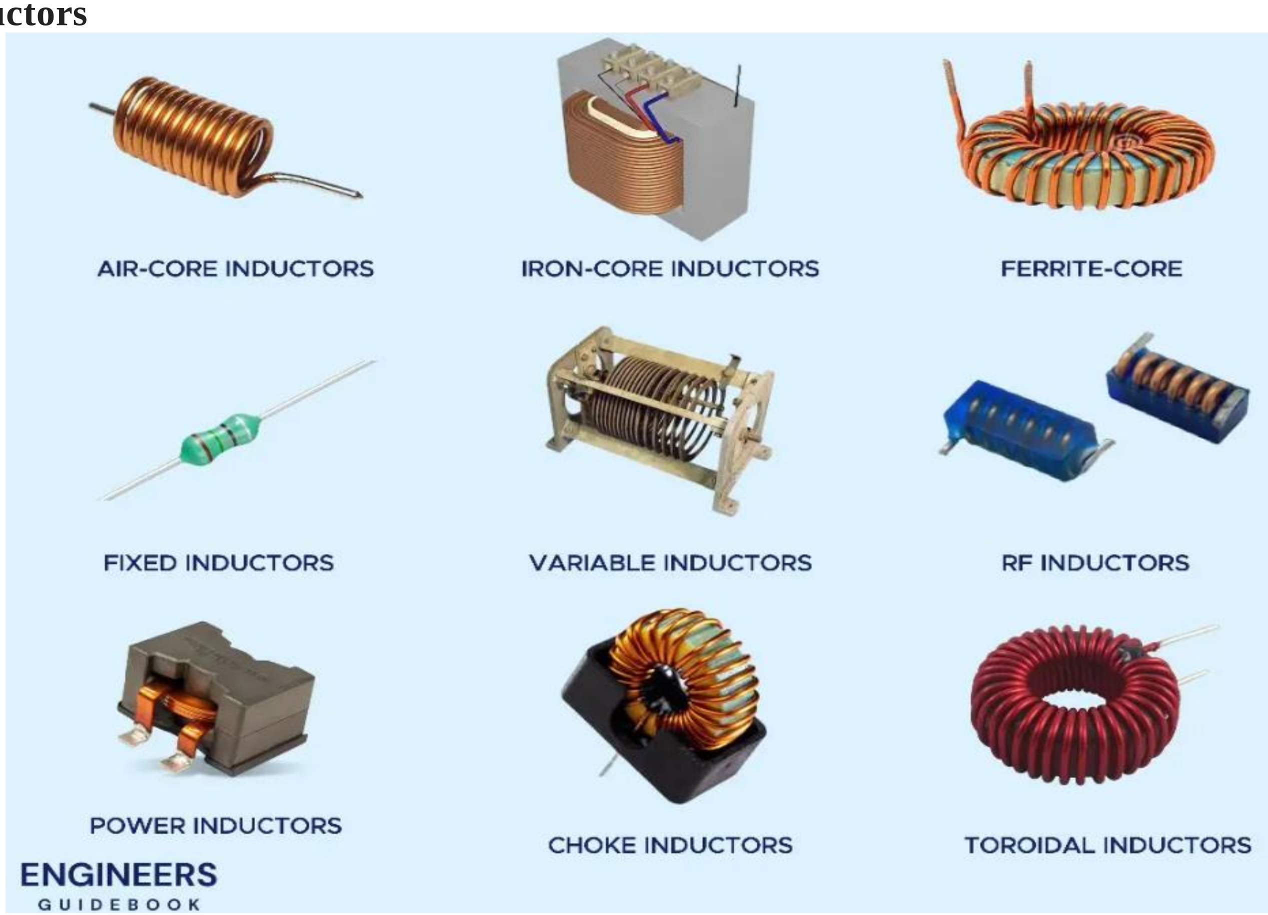


Capacitors

- Time domain I-V relationship: $i(t) = C\frac{dv(t)}{dt}$, where i(t) and v(t) are the instantaneous values of current and voltage respectively and C is the capacitance.
- Frequency domain I-V relationship: $V(\omega) = Z_C I(\omega)$, where Z_C is the impedance of the capacitor $Z_C = \frac{1}{j\omega C}$.



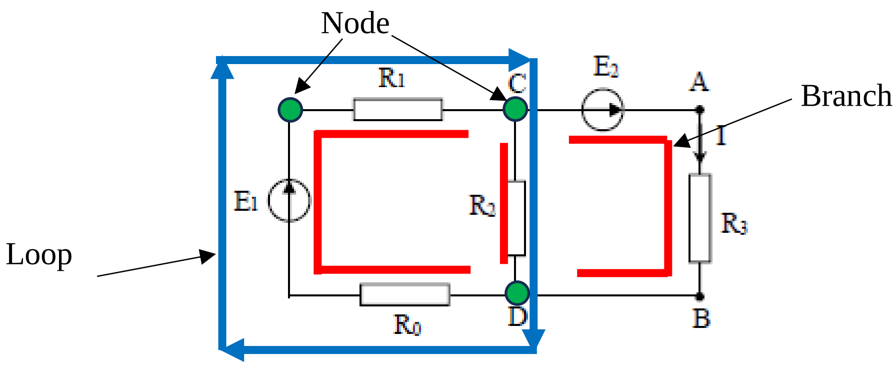
Inductors



- Time domain I-V relationship: $v(t) = L\frac{di(t)}{dt}$, where i(t) and v(t) are the instantaneous values of current and voltage respectively and L is the inductance.
- Frequency domain I-V relationship: $V(\omega) = Z_L I(\omega)$, where Z_L is the impedance of the inductor $Z_L = j\omega L$.

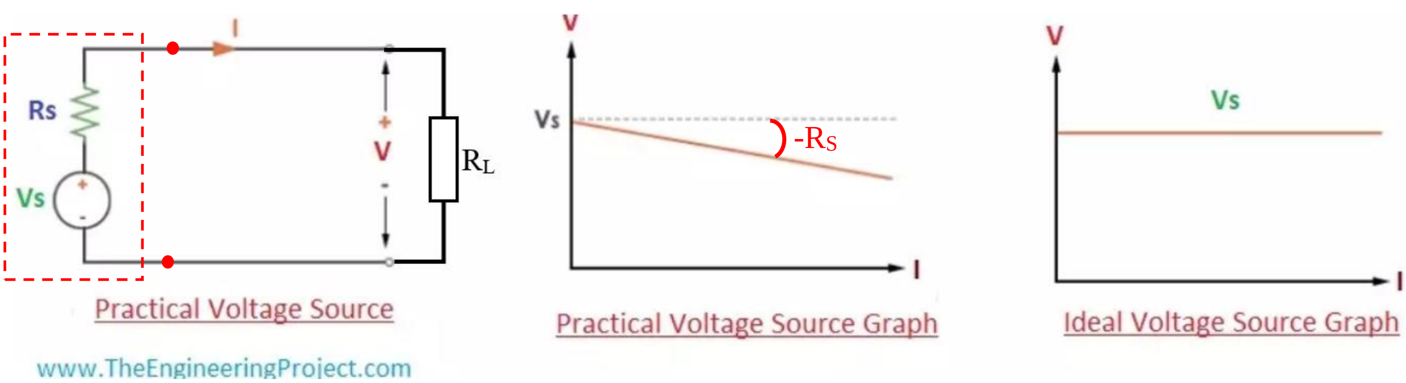
b. Branch, node and loop:

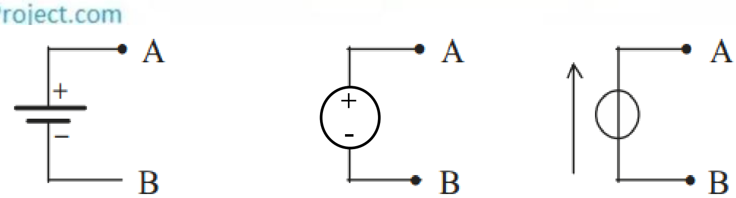
- a node is a connection point where two or more elements meet,
- a branch is the path or component(s) between two nodes,
- and a loop is any closed path in the network formed by connecting branches, starting and ending at the same node.



c. Independent voltage source

An ideal independent voltage source provides a perfectly constant voltage regardless of the current drawn, having zero internal resistance, while a real (or practical) voltage source has internal resistance R_S, causing its terminal voltage to drop as current increases. $\boldsymbol{V = R_L I = V_S - R_S I}$

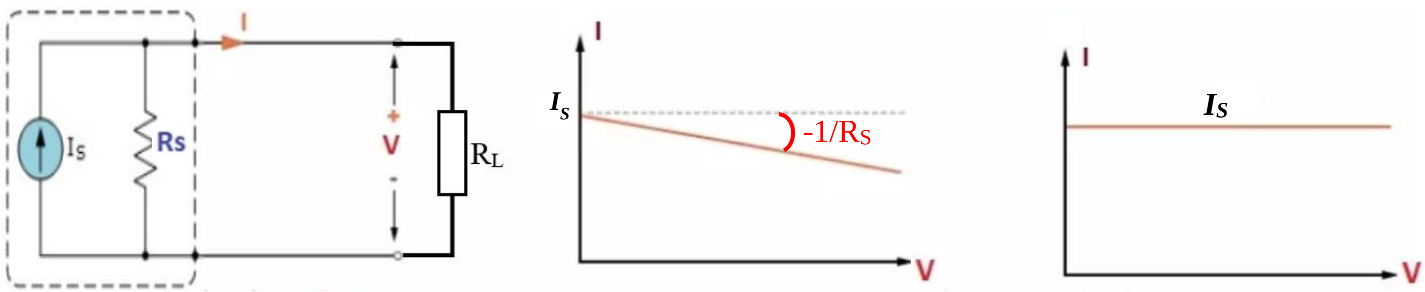




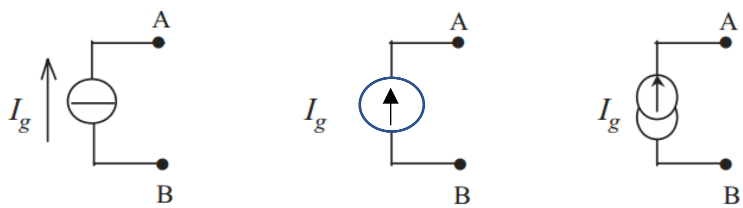
d. Independent current source

An ideal independent current source is a two terminal device which supplies constant current regardless of the load resistance or impedance.

$$I = \frac{V}{R_L} = I_S - \frac{1}{R_S} I$$



Practical current source | Practical current source graph | Ideal current source graph



e. Current-Voltage relationship in time domain and frequency domain (impedances)

Element	DC regime	Time domain (t)	Frequency domain (ω = 2π*f*)
R	$U = R\, I$	$u(t) = R\, i(t)$	$U(\omega) = Z_R\, I(\omega), \;\; Z_R = R$
L	$U = 0$	$u(t) = L\frac{di(t)}{dt}$	$U(\omega) = Z_L\, I(\omega), \;\; Z_L = j\omega L$
C	$I = 0$	$i(t) = C\frac{dv(t)}{dt}$	$U(\omega) = Z_C\, I(\omega), \;\; Z_C = \frac{1}{j\omega C}$

1.2. Foundational laws

1.2.1. Ohm's Law

Ohm's Law applies to both DC and AC circuits.

For **DC** circuits, it uses resistance: **V = R x I**.

For **AC** circuits, it uses impedance, which includes resistance (R) and reactance (X) from capacitors and inductors: **V = I x Z**.

Where: $Z = \begin{cases} R & for\ resistace \\ j\omega L & for\ inductor \\ \frac{1}{j\omega C} & for\ capacitor \end{cases}$

1.2.2. Kirchhoff's Current Law (KCL)

Kirchhoff's *current law* states that *the sum of the currents entering a junction is equal to the sum of the currents leaving the junction*: $\sum I_{IN} = \sum I_{OUT}$

1.2.3. Kirchhoff's Voltage Law (KVL)

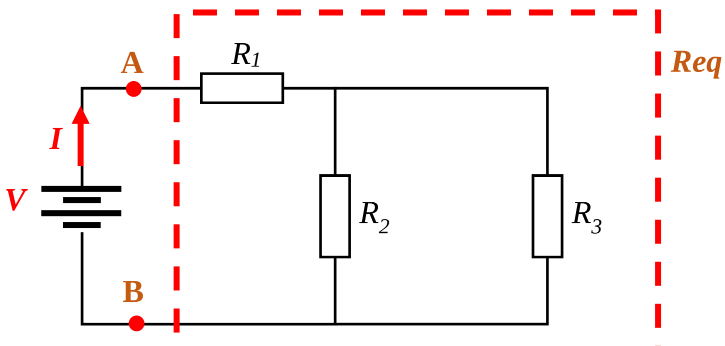
Kirchhoff's voltage law states that the voltage applied to a closed circuit equals the sum of the voltage drops in that circuit.

Another way of stating KVL is that **the algebraic sum of all the voltages around any closed circuit, equals zero**.

Example 1.1

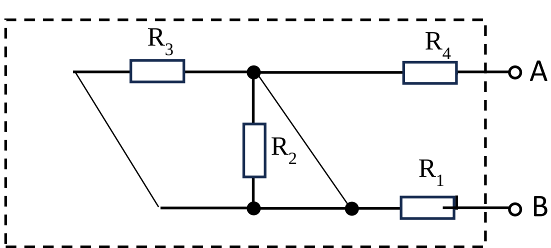
Find the equivalent resistance between points A and B using:

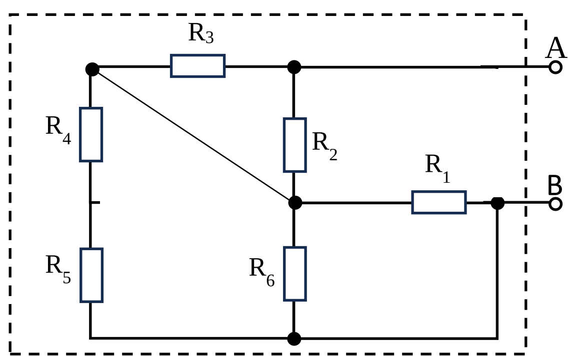
- Using series-parallel combination method.
- Ohm's and Kirchhoff's laws.



Find the equivalent resistance between points A and B using series-parallel combination method.

Example 1.2





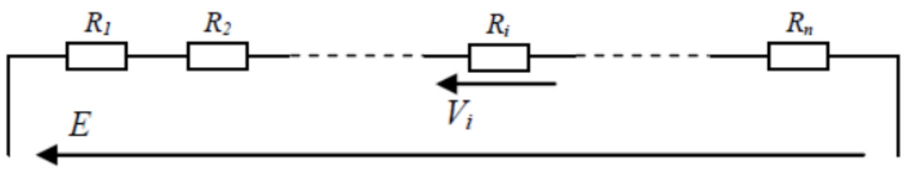
A resistor is short-circuited when its two terminals are connected together. This force both terminals to be at the same potential, effectively causing a zero-voltage drop across the resistor and making any current flow through the shorting wire instead of the resistor.

1.3. Basic theorems in electricity

1.3.1. Voltage divider rule

The total voltage is divided among components in proportion to their resistances. The current through all components in a series circuit is the same. It applies to series circuit.
Application: Used to obtain a lower, specific voltage from a higher voltage source or in sensors like thermistors to measure resistance.

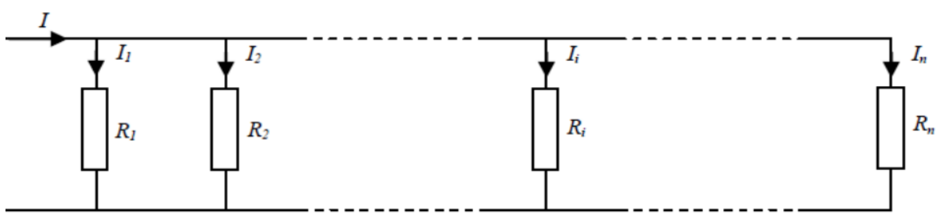
$$V_i = \frac{R_i}{\sum_{j=1}^{n} R_j} E$$



1.3.2. Current divider rule

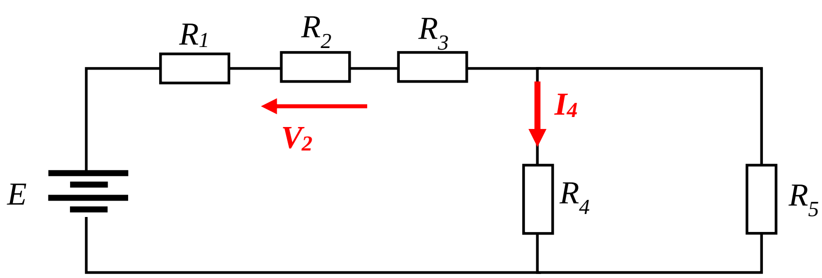
The total current divides among parallel paths, with less current flowing through branches with higher resistance and more current flowing through branches with lower resistance. It applies to parallel circuit.

$$I_i = \frac{\frac{1}{R_i}}{\sum_{j=1}^{n} \frac{1}{R_j}} I$$



Example 1.3

Find the current flowing through R_4 and the voltage across R_2 using current and voltage divider rule.



$E = 12$ V, $R_1 = 1\ k\Omega$, $R_2 = 2\ k\Omega$, $R_3 = 500\ \Omega$, $R_4 = 0.004\ M\ \Omega$, $R_4 = R_5$.

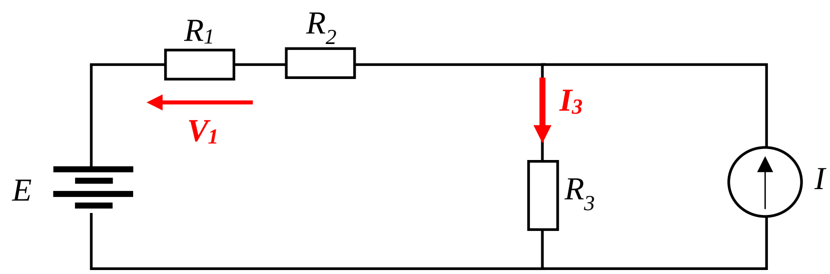
1.3.3. Superposition

The superposition theorem states that in a linear circuit with **multiple independent sources**, the total current or voltage across any element is the **algebraic sum** of the currents or voltages produced by **each source acting alone**, with other sources deactivated.

- To deactivate a voltage source, it is replaced by a short circuit (ideal) or by its internal resistor (practical);
- To deactivate a current source, it is replaced by an open circuit (ideal) or by its internal resistor (practical).

Example 1.4

Find V_1 and I_3 using superposition theorem.



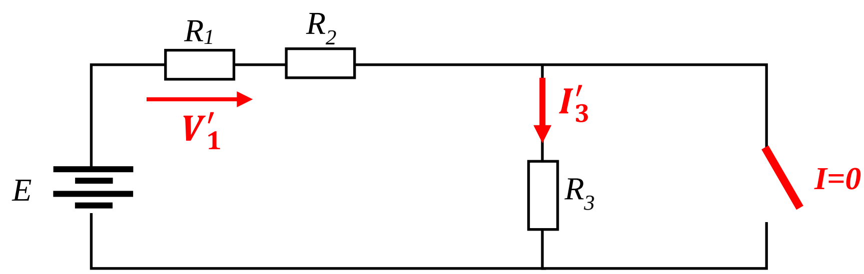
E = 12 V, I = 10 mA, $R_1 = 1\ k\Omega$, $R_2 = 2\ k\Omega$, $R_3 = 500\ \Omega$, $R_4 = 0.004\ M\ \Omega$, $R_4 = R_5$.

To determine the signs of each term in the algebraic sum for the superposition theorem, Compare with the reference direction:

- If the calculated current or voltage flows in the same direction as your reference, add it with a positive sign "+" to the total.
- If the calculated current or voltage flows in the opposite direction to your reference, add it with a negative sign "-".

$$V_1 = -V_1' + V_1'' = -V_1(E \neq 0, I = 0) + V_1(E = 0, I \neq 0)$$
$$I_3 = +I_3' - I_3'' = +I_3(E \neq 0, I = 0) - I_3(E = 0, I \neq 0)$$

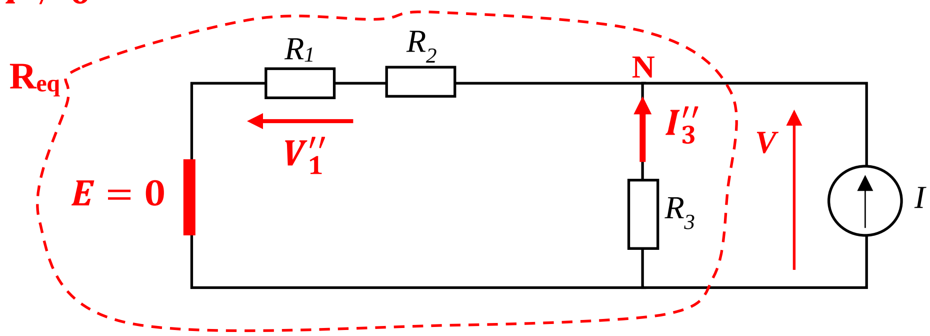
- $E \neq 0, I = 0$



The sign "-" in the voltage divider indicates that V_1' and E are in the same direction in the loop: $V_1' = -\frac{R_1}{R_1+R_2+R_3}E$.

By applying Ohm's law: $I_3' = \frac{E}{R_1+R_2+R_3}$.

- $E = 0, I \neq 0$



$$R_{eq} = (R_1 + R_2)//R_3$$
$$V = R_{eq}I$$
$$V_1'' = -\frac{R_1}{R_1 + R_2}V = -\frac{R_1}{R_1 + R_2}\big((R_1 + R_2)//R_3\big)I = -\frac{R_1R_3}{R_1 + R_2 + R_3}I$$

The sign "-" in the expression of I_3'' indicates that both I_3'' and I are both entering (this case) or both leaving the node N.

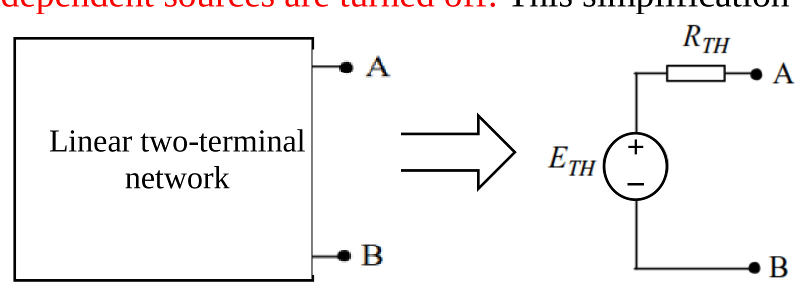
$$I_3'' = -\frac{\frac{1}{R_3}}{\frac{1}{R_1 + R_2} + \frac{1}{R_3}}I$$

$$V_1 = -V_1' + V_1'' = -\left(-\frac{R_1}{R_1 + R_2 + R_3}E\right) + \left(-\frac{R_1R_3}{R_1 + R_2 + R_3}I\right)$$
$$= \frac{R_1}{R_1+R_2+R_3}(E - R_3I) = 2 \text{ V.}$$

$$I_3 = +I_3' - I_3'' = +\frac{E}{R_1 + R_2 + R_3} - \left(-\frac{\frac{1}{R_3}}{\frac{1}{R_1 + R_2} + \frac{1}{R_3}}I\right)$$
$$= \frac{E}{R_1+R_2+R_3} + \frac{R_1+R_2}{R_1+R_2+R_3}I = 12 \text{ mA}$$

1.3.4. Thevenin's theorem

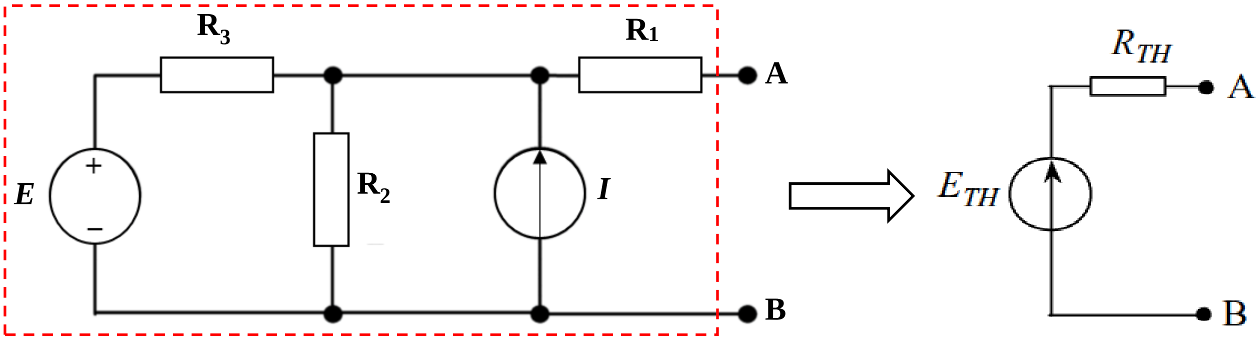
Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off. This simplification is valid for both AC and DC circuits.



To find R_{Th} (Thevenin's equivalent resistance), remove the load resistor, then deactivate all independent sources by replacing voltage sources with short circuits and current sources with open circuits.

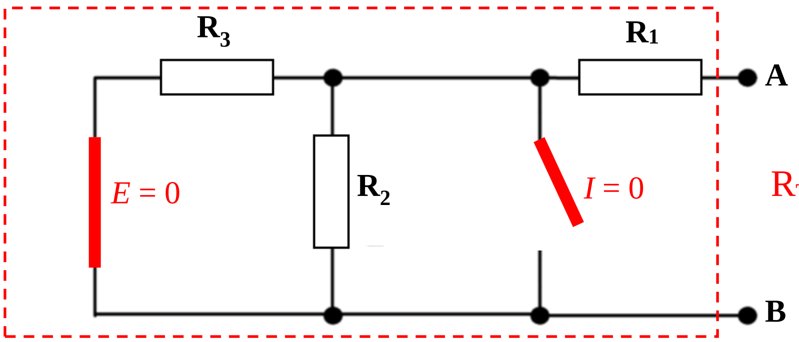
Example 1.5

Find Thevenin equivalent circuit of the following network.



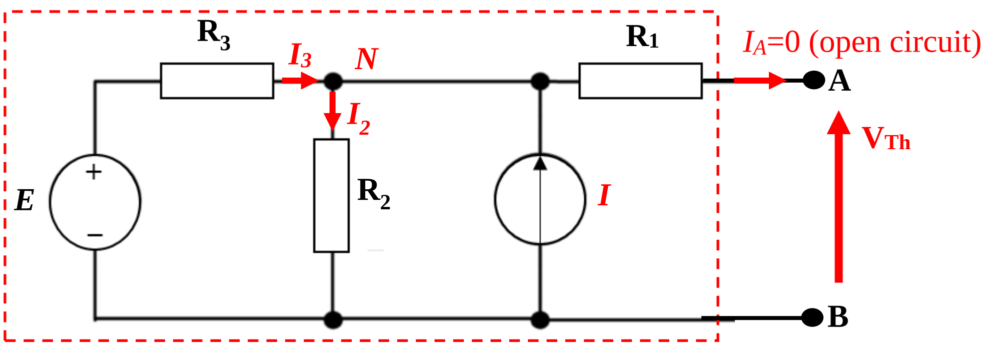
- Thevenin's equivalent resistance $R_{Th} = R_{AB}$

  E and *I* are independent sources (they provide a fixed voltage or current regardless of other circuit conditions). To find R_{Th}, the independent voltage source is short-circuited and the independent current source is open-circuited.



$R_{Th} = R_{AB} = R_1 + R_2 // R_3$

- Thevenin's voltage source V_{Th}



To determine V_{Th}, we can use KVL/KCL, superposition or other theorems.
We will avoid applying KVL to the loops containing current source, we will use KCL instead.
Apply KCL to N:

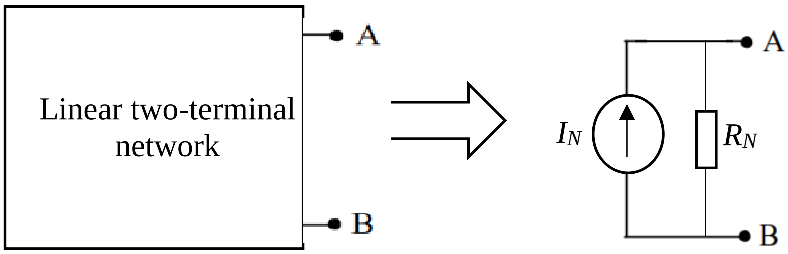
$$I_3 + I = I_2 + I_A \quad (1) \quad \text{with } I_A = 0.$$

Apply KVL the first loop (E, R_3 and R_2) and the second loop (R_2, R_1 and V_{Th}):
$E - R_3 I_3 - R_2 I_2 = 0$ (2)
$R_2 I_2 - R_1 I_A - V_{Th} = 0$ (3)
We can solve for V_{Th} the system of three linear equations:

$$V_{Th} = \frac{R_2}{R_2 + R_3}(E + R_3 I)$$

1.3.5. Norton's theorem

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.



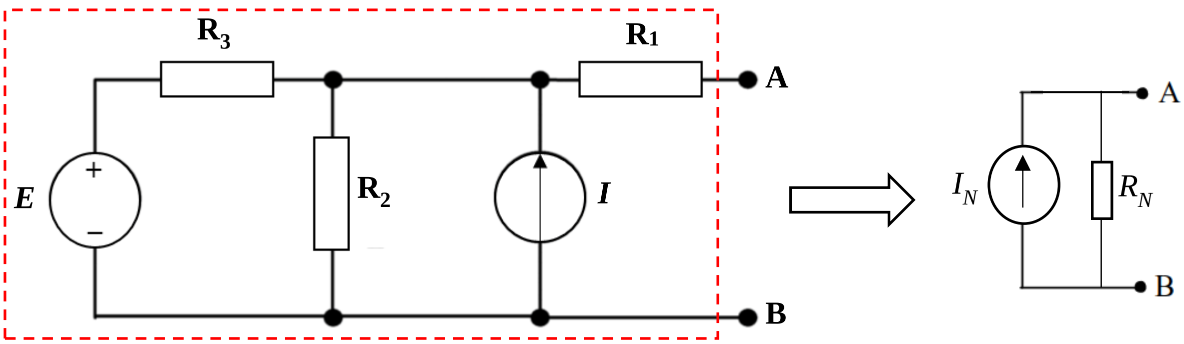
We find R_N in the same way we find R_{Th}: $R_N = R_{Th}$.
To find the Norton current we determine the short-circuit current flowing from terminal A to B.

Observe the close relationship between Norton's and Thevenin's theorems:

$R_{Th} = R_N$ and $V_{Th} = R_{Th} I_N$.

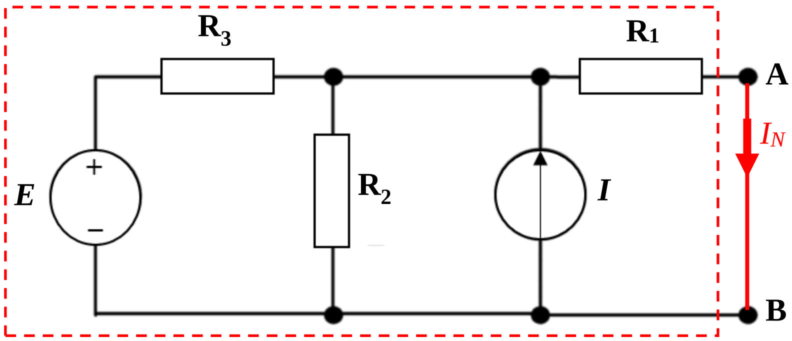
Example 1.6

Find Norton equivalent circuit of the following network.

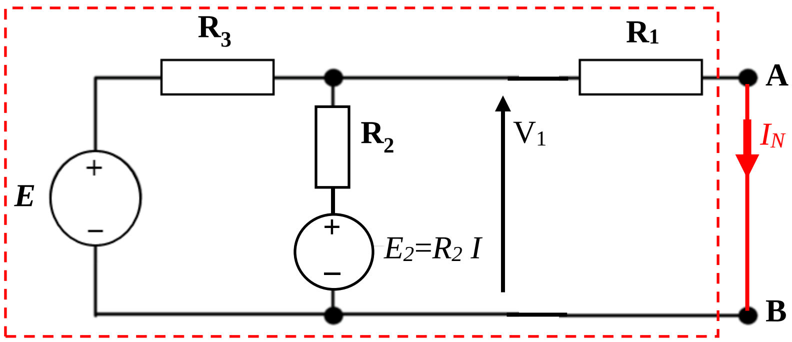


- We find R_N in the same way we find R_{Th}: $R_N = R_{Th}$.

- Norton's current source



To find I_N, we can use: KCL/KVL, superposition, Millman, current divider, …etc.
Let's convert current source *I* and R_2 to a voltage source in series with R_2.



Now we can apply Millman's theorem because we have a parallel network of three branches, each containing a voltage source and its series resistance.

$$V_1 = \frac{\frac{E}{R_3} + \frac{E_2}{R_2} + \frac{0}{R_1}}{\frac{1}{R_3} + \frac{1}{R_2} + \frac{1}{R_1}} = \frac{\frac{E}{R_3} + I}{\frac{1}{R_3} + \frac{1}{R_2} + \frac{1}{R_1}}$$

Using Ohm's law

$$I_N = \frac{V_1}{R_1} = \frac{1}{R_1} \frac{\frac{E}{R_3} + I}{\frac{1}{R_3} + \frac{1}{R_2} + \frac{1}{R_1}} = \frac{R_2(E + R_3 I)}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$