Module: Advanced Semiconductor Physics
Series TD4 of Chapter IV: p-n Junction diode and solar cell

Exercise 1: Poisson's Equation and Depletion Analysis in a Silicon p–n Junction

Consider a one-dimensional abrupt p–n junction at equilibrium, with doping concentrations $N_A = 10^{18}\ \text{cm}^{-3}$on the p-side and $N_D = 10^{16}\ \text{cm}^{-3}$on the n-side. The intrinsic carrier concentration is $n_i = 1.5 \times 10^{10}\ \text{cm}^{-3}$, the dielectric constant is $\varepsilon = 11.7 \cdot \varepsilon_0$, and the vacuum permittivity ε_0 is: $\varepsilon_0 = 8.854 \times 10^{-14}$ F/cm , the thermal voltage at $T = 300$ K is $V_T = k_B T = 0.025875$ eV. Assume complete ionization and depletion approximation. 1-**Poisson's Equation**: Solve the one-dimensional Poisson's equation $\frac{d^2\psi(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon}$ in the depletion regions: For the p-side $(-x_p < x < 0)$: $\rho(x) = -qN_A$, for the n-side $(0 < x < x_n)$: $\rho(x) = +qN_D$, apply the boundary conditions: $\psi(-x_p) = 0$, $\psi(x_n) = V_d$, and the continuity of electric field at $x = 0$: $E_p(0^-) = E_n(0^+)$ to derive expressions for: (a) Electric field $E(x) = -\frac{d\psi}{dx}$, (b) Electrostatic potential $\psi(x)$. 2- **Depletion Width**: Using the built-in potential $V_d = V_T . \ln\left(\frac{N_A N_D}{n_i^2}\right)$, derive the total depletion width $W = x_p + x_n$ as a function of V_d, N_A, and N_D.3- **Numerical Evaluation**: Calculate: (a) The built-in potential V_d in volts. (b) The total depletion width W in cm and μm.

Exercise 2: Minority Carrier Dynamics and Diffusion Current in a Silicon p–n Junction

Consider a silicon p–n junction at thermal equilibrium with bandgap energy $E_g = 1.12$ eV, effective masses $m_e^* = 1.08\, m_0$ and $m_h^* = 0.56\, m_0$, temperature $T = T_0 = 300$ K, effective density of states $N_0 = 2.508 \times 10^{19}\ cm^{-3}$, and thermal energy $k_B T = 0.025875$ eV. (1) Write the steady-state continuity equations for minority carriers: electrons in the neutral p-type region and holes in the neutral n-type region. (2) Solve these equations assuming constant diffusion coefficients and the following boundary conditions: $n_p(-x_p) = n_{p0} e^{qV/k_B T}$, $n_p(-d_p) = n_{p0}$, $p_n(x_n) = p_{n0} e^{qV/k_B T}$, and $p_n(d_n) = p_{n0}$. (3) Derive expressions for the minority carrier diffusion current densities at the edges of the depletion region: $J_n(-x_p) = qD_n \frac{dn_p}{dx} |_{-x_p}$ and $J_p(x_n) = -qD_p \frac{dp_n}{dx} |_{x_n}$. (4) Finally, calculate the total current density $J(V)$ and show that it follows the ideal diode equation $J(V) = J_0(e^{qV/k_B T} - 1)$, where J_0 is the reverse saturation current density determined by the equilibrium minority concentrations and diffusion parameters. (5) calculate J_0 value for $N_A = 10^{18}\ cm^{-3}$ and $N_D = 10^{15}\ cm^{-3}$ with $\mu_n = 1350\ \text{cm}^2/(V.s)$ and $\mu_p = 450\ \text{cm}^2/(V.s)$, $\tau_n = \tau_p = 10^{-7}$ s.

Exercise 3: Illumination-Induced Carrier Dynamics and Photovoltaic Response in a Silicon p–n Junction solar cell

A silicon p–n junction is formed with acceptor concentration $N_A = 10^{18}\ cm^{-3}$ and donor concentration $N_D = 10^{15}\ \text{cm}^{-3}$, operating at temperature $T = T_0 = 300$ K. The material has a bandgap energy $E_g = 1.12$ eV, relative permittivity $\varepsilon = \varepsilon_r \varepsilon_0 = 11.7 \cdot 8.854 \times 10^{-14}$ F/cm, and effective masses $m_e^* = 1.08\, m_0$, $m_h^* = 0.56\, m_0$. Carrier mobilities are $\mu_n = 1350\ \text{cm}^2/(V.s)$ and $\mu_p = 450\ \text{cm}^2/(V.s)$, with minority carrier lifetimes $\tau_n = \tau_p = 10^{-7}$ s. The junction is illuminated with monochromatic light of intensity $P_{op} = 0.099\ \text{W/cm}^2$ and wavelength $\lambda = 0.55\ \mu$m, assuming each photon generates one electron–hole pair and the generation rate is uniform across the depletion width. Calculate: (1) the depletion width W, (2) the photocurrent density J_{ph} in A/cm^2, and (3) the open-circuit voltage V_{oc} at 300 K.

Solutions

Exercise 1: 1- **Poisson's Equation Solution**: In the depletion approximation, solving $\frac{d^2\psi(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon}$ yields linear electric fields and quadratic potentials. For the p-side $(-x_p < x < 0)$, $\rho(x) = -qN_A$, so $\frac{d^2\psi}{dx^2} = \frac{qN_A}{\varepsilon} \Rightarrow \frac{d\psi}{dx} = \frac{qN_A}{\varepsilon}(x + x_p) \Rightarrow \quad E(x) = \left(-\frac{d\psi}{dx}\right) = -\left(\frac{qN_A}{\varepsilon}\right)(x + x_p)$ and $\psi(-x_p) = 0$ so $\psi(x) = \frac{qN_A}{2\varepsilon}(x + x_p)^2$. For the n-side $(0 < x < x_n)$, $\rho(x) = +qN_D$, so $\frac{d^2\psi}{dx^2} = -\left(\frac{qN_D}{\varepsilon}\right) \Rightarrow \frac{d\psi}{dx} = -\left(\frac{qN_D}{\varepsilon}\right)(x - x_n)$ and $E(x) = \left(-\frac{d\psi}{dx}\right) = \left(\frac{qN_D}{\varepsilon}\right)(x - x_n)$ and $\psi(x_n) = V_d$ so $\psi(x) = V_d - \left(\frac{qN_D}{2\varepsilon}\right)(x - x_n)^2$. Continuity of electric field at $x = 0$ gives $E(0^-) = -\frac{qN_Ax_p}{\varepsilon}, E(0^+) = -\frac{qN_Dx_n}{\varepsilon} \Rightarrow N_Ax_p = N_Dx_n$.. 2- **Depletion Width Expression**: The built-in potential is $V_d = V_T \ln\left(\frac{N_AN_D}{n_i^2}\right)$, and total depletion width is $W = x_p + x_n$.

Depletion widths: $x_n = \left(\frac{N_A}{N_D+N_A}\right)W, x_p = \left(\frac{N_D}{N_D+N_A}\right)W$ and $\psi(0) = V_d - \left(\frac{qN_D}{2\varepsilon}\right)x_n^2 = \left(\frac{qN_A}{2\varepsilon}\right)x_p^2$ which leads to :

$V_d = \frac{qN_Dx_n^2}{2\varepsilon} + \frac{qN_Ax_p^2}{2\varepsilon}$, Therefore : $V_d = \frac{q}{2\varepsilon}\left[N_D\left(\frac{N_A}{N_D+N_A}W\right)^2 + N_A\left(\frac{N_D}{N_D+N_A}W\right)^2\right]$

Factor out W^2: $V_d = \frac{qW^2}{2\varepsilon} \cdot \left[\frac{N_DN_A^2+N_AN_D^2}{(N_D+N_A)^2}\right] = \frac{qW^2}{2\varepsilon} \cdot \left[\frac{N_DN_A(N_D+N_A)}{(N_D+N_A)^2}\right]$. Simplify: $V_d = \frac{qW^2}{2\varepsilon} \cdot \left[\frac{N_DN_A}{N_D+N_A}\right]$

Final Step: Solving for $W : W^2 = \left(\frac{2\varepsilon V_d}{q}\right) \cdot \left(\frac{N_D+N_A}{N_DN_A}\right) \Rightarrow W = \sqrt{\left(\frac{2\varepsilon V_d}{q}\right) \cdot \left(\frac{N_D+N_A}{N_DN_A}\right)}$.

3-**Numerical Evaluation**: (a) With $V_T = 0.025875$ eV, $V_d = 0.025875 \cdot \ln\left(\frac{10^{18} \cdot 10^{16}}{(1.5\times10^{10})^2}\right) \approx 0.025875 \cdot \ln(4.44 \times 10^{13}) \approx 0.025875 \cdot 31.42 \approx 0.813$ V. (b) Using $\varepsilon = 11.7 \cdot 8.854 \times 10^{-14} = 1.0358 \times 10^{-12}$ F/cm, we find $W = \sqrt{\frac{2 \cdot 1.0358\times10^{-12} \cdot 0.812}{1.6\times10^{-19}}\left(\frac{1}{10^{18}} + \frac{1}{10^{16}}\right)} \approx 3.26 \times 10^{-5}$ cm $= 0.326\ \mu$m.

Exercise2:

(1) The steady-state continuity equations for minority carriers are $D_n\left(\frac{d^2n_p(x)}{dx^2}\right) = \left(\frac{n_p(x)-n_{p0}}{\tau_n}\right)$ for electrons in the neutral p-type region and $D_p\left(\frac{d^2p_n(x)}{dx^2}\right) = \left(\frac{p_n(x)-p_{n0}}{\tau_p}\right)$ for holes in the neutral n-type region. Defining $\Delta n_p(x) = n_p(x) - n_{p0}$ and $\Delta p_n(x) = p_n(x) - p_{n0}$, we obtain homogeneous second-order equations: $\frac{d^2\Delta n_p}{dx^2} = \left(\frac{\Delta n_p}{L_n^2}\right)$ and $\frac{d^2\Delta p_n}{dx^2} = \left(\frac{\Delta p_n}{L_p^2}\right)$, where $L_n = \sqrt{D_n\tau_n}$ and $L_p = \sqrt{D_p\tau_p}$. (2) The general solutions are $\Delta n_p(x) = Ae^{x/L_n} \Rightarrow n_p(x) = n_{p0} + Ae^{x/L_n}$, and $\Delta p_n(x) = Be^{-x/L_p} \Rightarrow p_n(x) = p_{n0} + Be^{-x/L_p}$. Applying the boundary conditions $n_p(-x_p) = n_{p0}e^{qV/k_BT}$, $n_p(-d_p) = n_{p0}$, we find $A = (n_{p0}e^{qV/k_BT} - n_{p0})e^{x_p/L_n}$, so $n_p(x) = n_{p0} + (n_{p0}e^{qV/k_BT} - n_{p0})e^{(x+x_p)/L_n}$; similarly, using $p_n(x_n) = p_{n0}e^{qV/k_BT}$, $p_n(d_n) = p_{n0}$, we find $B = (p_{n0}e^{qV/k_BT} - p_{n0})e^{x_n/L_p}$, so $p_n(x) = p_{n0} + (p_{n0}e^{qV/k_BT} - p_{n0})e^{-(x-x_n)/L_p}$. (3) The diffusion current densities are $J_n(-x_p) = qD_n\frac{dn_p}{dx}|_{-x_p} = qD_n\frac{n_{p0}}{L_n}(e^{qV/k_BT} - 1)$ and $J_p(x_n) = -qD_p\frac{dp_n}{dx}|_{x_n} = qD_p\frac{p_{n0}}{L_p}(e^{qV/k_BT} - 1)$. (4) The total current density is $J(V) = J_n + J_p = J_0(e^{qV/k_BT} - 1)$, where $J_0 = q\left(\frac{D_nn_{p0}}{L_n} + \frac{D_pp_{n0}}{L_p}\right)$. (5) Using corrected expressions for effective densities of states: $N_c = N_0(m_e^*/m_0)^{3/2} \approx 2.816 \times 10^{19}\ cm^{-3}$, $N_v = N_0(m_h^*/m_0)^{3/2} \approx 1.051 \times 10^{19}\ cm^{-3}$,we compute $n_i = \sqrt{N_cN_v}e^{-E_g/2k_BT} \approx 6.86 \times 10^9$,then $n_{p0} = \frac{n_i^2}{N_A} = 47.05$ cm^{-3}, and $p_{n0} = n_i^2/N_D = 4.705 \times 10^4\ cm^{-3}$, $D_n = \frac{\mu_nk_BT}{q} = 34.931 cm^2/s$ and $D_p = \frac{\mu_pk_BT}{q} = 11.64\ cm^2/s$, $L_n = \sqrt{D_n\tau_n} = 1.868 \times 10^{-3}$ cm, $L_p = \sqrt{D_p\tau_p} = 1.078 \times 10^{-3}$ cm, and finally $J_0 = q\left(\frac{34.931\times47.05}{1.868\times10^{-3}} + \frac{11.64\times4.705\times10^4}{1.078\times10^{-3}}\right) \cong 8.142 \times 10^{-11}\mathrm{A}/cm^2$

Exercise 3:

(1) To calculate the depletion width W, we first compute the intrinsic carrier concentration using $n_i = \sqrt{N_c N_v} \cdot e^{-E_g/(2k_BT)}$, where $N_c = N_0(\frac{m_e^*}{m_0})^{3/2} = 2.508 \times 10^{19} \cdot (1.08)^{3/2} \approx 2.816 \times 10^{19}\ \text{cm}^{-3}$, and $N_v = 2.508 \times 10^{19} \cdot (0.56)^{3/2} \approx 1.051 \times 10^{19}\ \text{cm}^{-3}$, giving $n_i = \sqrt{2.816 \times 10^{19} \cdot 1.051 \times 10^{19}} \cdot e^{-1.12/(2 \cdot 0.025875)} \approx 6.86 \times 10^9\ \text{cm}^{-3}$; then the built-in potential is $V_{bi} = \frac{k_BT}{q} \ln(\frac{N_A N_D}{n_i^2}) = 0.025875 \cdot \ln(\frac{10^{18} \cdot 10^{15}}{(6.86 \times 10^9)^2}) \approx 0.79\ \text{V}$. Now using $\varepsilon_s = 11.7 \cdot 8.854 \times 10^{-14} = 1.0359 \times 10^{-12}\ \text{F/cm}$, the depletion width is $W = \sqrt{\frac{2\varepsilon_s}{q} \cdot \frac{V_{bi}}{(1/N_A + 1/N_D)}} \approx \sqrt{\frac{2 \cdot 1.0359 \times 10^{-12}}{1.602 \times 10^{-19}} \cdot \frac{0.79}{\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}} \approx 1.011\ \mu\text{m}$. (2) The photon energy is $E_{ph} = \frac{hc}{\lambda} = \frac{1.986 \times 10^{-16}}{0.55 \times 10^{-4}} = \frac{1.2424(\mu m.eV)}{0.55(\mu m)} \approx 2.25\ \text{eV}$, so the photon flux is $\Phi = \frac{P_{op}}{E_{ph}} = \frac{P_{op}}{hc.q}\lambda = \left(\frac{0.099. \times 0.55}{1.2424 \cdot 1.602 \times 10^{-19}}\right) \approx 2.74 \times 10^{17}\ \text{photons/cm}^2 \cdot \text{s}$, and the generation rate is $G = \frac{\Phi}{W} \approx \left(2.74 \times \frac{10^{17}}{1.011} \times 10^4\right) = 2.71 \times 10^{21}\ \text{cm}^{-3} \cdot \text{s}^{-1}$; thus, the photocurrent density is $J_{ph} = qGW = 1.602 \times 10^{-19} \cdot 2.71 \times 10^{21} \cdot 1.011 \times 10^{-4} \approx 4.39 \times 10^{-2}\ \text{A/cm}^2$. (3) Before calculating the open-circuit voltage, we recall the reverse saturation current density formula: $J_0 = q(\frac{D_n n_{p0}}{L_n} + \frac{D_p p_{n0}}{L_p})$, where $D_n = 34.931\ \text{cm}^2/\text{s}$, $D_p = 11.64\ \text{cm}^2/\text{s}$, $L_n = \sqrt{D_n \tau_n} = 1.868 \times 10^{-3}\ \text{cm}$, $L_p = \sqrt{D_p \tau_p} = 1.078 \times 10^{-3}\ \text{cm}$, $n_{p0} = n_i^2/N_A = 47.05\ \text{cm}^{-3}$, and $p_{n0} = n_i^2/N_D = 4.705 \times 10^4\ \text{cm}^{-3}$, yielding $J_0 = 1.602 \times 10^{-19}(\frac{34.931 \cdot 47.05}{1.868 \times 10^{-3}} + \frac{11.64 \cdot 4.705 \times 10^4}{1.078 \times 10^{-3}}) \approx 8.142 \times 10^{-11}\ \text{A/cm}^2$; finally, the open-circuit voltage is $V_{oc} = \frac{k_BT}{q} \ln(\frac{J_{ph}}{J_0} + 1) = 0.025875 \cdot \ln(\frac{4.39 \times 10^{-2}}{8.142 \times 10^{-11}} + 1) \approx 0.52\ \text{V}$.