Module: Advanced Semiconductor Physics
Series TD3 of Chapter IV: Optical Effects and Light in semiconductors

Exercise 1: Optical Generation and Carrier Dynamics in a Uniformly Doped Semiconductor
A semiconductor sample of thickness $d = 0.1\ \mu$m and absorption coefficient $\alpha = 10^3\ \text{cm}^{-1}$is illuminated at its surface ($x = 0$) with monochromatic light of intensity $I_0 = 0.08\ \text{W/cm}^2$. The photon energy corresponds to a wavelength of $\lambda = 0.6\mu\text{m}$ with $hc = 1.2424\ eV.\mu m$. The sample is uniformly doped n-type with donor concentration $N_D = 10^{16}\ \text{cm}^{-3}$, and the intrinsic carrier concentration is $n_i = 1.5 \times 10^{10}\ \text{cm}^{-3}$. Carrier lifetimes are $\tau_n = \tau_p = 10^{-7}$ s. (1) Deduce how the illumination intensity $I(x)$ is distributed along the sample depth. (2) Calculate the photon flux $\phi(x)$ within the sample. (3) Determine the generation rate $G(x)$ of electron-hole pairs within the sample. (4) Assuming the steady-state and the electric field within the sample $E(x) \cong 0\ V/cm$, compute the excess carrier concentrations δn and δp and the photoconductivity $\delta\sigma$ using $\mu_n = 1350\ \text{cm}^2/(V.s)$ and $\mu_p = 450\ \text{cm}^2/(V.s)$. (5) Calculate the total electron and hole concentrations n and p and the total conductivity σ. (6) Finally, deduce the positions of the quasi-Fermi levels E_{Fn} and E_{Fp} relative to the intrinsic level E_{fi}...

Exercise 2: Deriving the Electric Field and Einstein Relation from Equilibrium Carrier Distribution
In a semiconductor sample at thermal equilibrium, the spatial distribution of free electrons is given by $n(x) = N_C \exp\left(\frac{E_f - E_C(x)}{k_B T}\right)$, where N_C is the effective density of states in the conduction band, $E_C(x)$is the position-dependent conduction band edge, E_f is the constant Fermi level, and $k_B T$ is the thermal energy. **(1)** Considering the thermal equilibrium, determine the expression of the electric field $E(x)$ along the sample in terms of the gradient of the electron concentration. **(2)** Given that the electrostatic potential is defined as $\psi(x) = -\frac{1}{q}\left(E_C(x)\right)$, where q is the elementary charge, demonstrate the Einstein relation $\left(\frac{D_n}{\mu_n} = \frac{k_B T}{q}\right)$.

Exercise 3: Steady-State Carrier Profile Under Uniform Generation and Recombination
In a uniformly illuminated semiconductor sample, the generation rate is constant and given by $G(x) = g_0$, and the recombination rate follows the Shockley model: $R_n(x) = \frac{n(x) - n_0}{\tau_n}$, where n_0 is the equilibrium electron concentration and τ_n is the electron lifetime. The total electron current density is composed of drift and diffusion components:$J_n(x) = qn(x)\mu_n E(x) + qD_n \frac{dn(x)}{dx}$. **(1)** Write the general continuity equation for electrons, incorporating the expressions for $J_n(x)$, $G(x)$, and $R_n(x)$. **(2)** Simplify the continuity equation by assuming steady-state conditions (i.e., $\partial n/\partial t = 0$) and negligible electric field (i.e., $E(x) = 0$). **(3)** Solve the resulting second-order differential equation for the excess electron concentration $\delta n(x) = n(x) - n_0$, using the boundary conditions: $\delta n(0) = g_0 \tau_n$ and $\delta n(+\infty) = g_0 \tau_n$

Exercise 4: Simplification of Steady-State Recombination Rate in Different Semiconductor Regimes.
Consider a semiconductor with a recombination centre at energy level E_R, where the steady-state recombination rate is given by: $R = \frac{np - n_i^2}{\tau_p(n + n_i) + \tau_n(p + n_i)}$ Simplify the expression of R under the following conditions with considering $\delta n = \delta p$: (1) For an intrinsic semiconductor under illumination, where the excess carrier concentrations are δn and δp, and the total concentrations are $n = n_i + \delta n$, $p = n_i + \delta p$; (2) For an n-type doped semiconductor, where $n = n_0 + \delta n \approx n_0$ and $p = p_0 + \delta p \approx \delta p$ and $\delta p \ll n_0$; (3) For a p-type doped semiconductor, where $n = n_0 + \delta n \approx \delta n$ and $p = p_0 + \delta p \approx p_0$ and $\delta n \ll p_0$.

Solutions

Exercise 1: (1) The illumination intensity inside the semiconductor follows Beer–Lambert's law: $I(x) = I_0 e^{-\alpha x}$, and with $\alpha = 10^3\ \text{cm}^{-1}$and $d = 0.1\ \mu\text{m} = 10^{-5}\ \text{cm}$, we have $exp^{(-\alpha.d)} = exp^{(-10^{-2})} \cong 0.99$, then $I(d) \approx 0.99I_0$, indicating nearly uniform illumination across the sample $I(x) \cong 0.99I_0 \cong I_0$. (2) The photon flux is $\phi(x) = \frac{I(x)}{h\nu} = \frac{I_0 e^{-\alpha x}}{hc/\lambda}$, and using $hc = 1.2424\ \text{eV} \cdot \mu\text{m}$ and $\lambda = 0.6\ \mu\text{m}$, we get $h\nu = 2.07\ \text{eV}$, and $\phi_0 = \frac{I_0\lambda}{hc} = 2.414 \times 10^{17} cm^{-2}s^{-1}$ so $\phi(x) \approx \frac{0.08e^{-10^3 x}}{2.07\times1.6\times10^{-19}} \approx 2.41 \times 10^{17} e^{-10^3 x}\ \text{cm}^{-2} \cdot \text{s}^{-1}$ *and* $\phi(d) = 2.41 \times 10^{17} \times 0.99 = 0.99 \times \phi_0 = 2.39 \times 10^{17} cm^{-2}s^{-1}$. (3) The generation rate is $G(x) = \alpha\phi(x) \approx 2.41 \times 10^{20} e^{-10^3 x}\ \text{cm}^{-3} \cdot \text{s}^{-1}$, and since $G(d) \approx 0.99G_0$, it is nearly uniform, then $G(d) \approx 0.99\ G_0 = 2.39 \times 10^{20} cm^{-3}s^{-1}$. (4) Under steady-state and negligible electric field, the excess carrier concentrations are $\delta n = G\tau_n = \delta p = G\tau_p \cong 2.39 \times 10^{13}\ \text{cm}^{-3}$. Photoconductivity arises from the excess carriers $\delta n = \delta p = 2.39 \times 10^{13}\ \text{cm}^{-3}$, so: $\delta\sigma = q(\delta n\mu_n + \delta p\mu_p) = (1.6 \times 10^{-19})[2.39 \times 10^{13} \cdot 1350 + 2.39 \times 10^{13} \cdot 450]=\ 6.88 \times 10^{-3}\Omega^{-1}cm^{-1}$. (5) The total electron concentration is $n = N_D + \delta n = 10^{16} + 2.39 \times 10^{13} \approx 1.00239 \times 10^{16}\ \text{cm}^{-3}$, and the total hole concentration is $p = \delta p + p_0$ where $p_0 = \frac{n_i^2}{n_0} = \frac{(1.5\times10^{10})^2}{10^{16}} \approx 2.25 \times 10^4\ \text{cm}^{-3}$ and $p = 2.39 \times 10^{13} + 2.25 \times 10^4 \cong \delta p = 2.39 \times 10^{13}\ \text{cm}^{-3}$, the total conductivity is $\sigma = q(n\mu_n + p\mu_p) = (1.6 \times 10^{-19})[1.00239 \times 10^{16} \cdot 1350 + 2.39 \times 10^{13} \cdot 450] = 2.166\ \Omega^{-1}cm^{-1}$. (6) The quasi-Fermi level positions relative to the intrinsic level are $E_{Fn} - E_{fi} = k_B T \ln(n/n_i) \approx 0.025875 \ln(1.00239 \times 10^{16}/1.5 \times 10^{10}) \approx 0.347\ \text{eV}$, and $E_{fi} - E_{Fp} = k_B T \ln\left(\frac{p}{n_i}\right) = 0.025875 \ln(2.39 \times 10^{13}/1.5 \times 10^{10}) \approx 0.1907\ \text{eV}$..

Exercise2:

At thermal equilibrium, the electron concentration is given by $n(x) = N_C \exp\left(\frac{E_f - E_C(x)}{k_B T}\right)$, where the Fermi level E_f is constant and $E_C(x)$varies spatially. **(1)** The total electron current density is $J_n = qn\mu_n E(x) + qD_n \frac{dn}{dx}$, and since $J_n = 0$ at thermal equilibrium, we set the drift and diffusion terms equal and opposite: $n\mu_n E(x) = -D_n \frac{dn}{dx}$, which gives the electric field as $E(x) = -\frac{D_n}{\mu_n}\frac{1}{n(x)}\frac{dn}{dx}$. and $\frac{dn}{dx} = -\frac{n(x)}{k_B T}\frac{dE_C}{dx}$ so $E(x) = \left(\frac{D_n}{\mu_n}\right)\left(\frac{1}{k_B T}\right)\left(\frac{dE_C}{dx}\right)$ **(2)** Using the definition of electrostatic potential $\psi(x) = -\frac{1}{q}E_C(x)$, and $\frac{d\psi(x)}{dx} = \left(-\frac{1}{q}\right)\left(\frac{dE_C(x)}{dx}\right)$, and the electric field $E(x) = \left(-\frac{d\psi}{dx}\right) = \frac{1}{q}\frac{dE_C}{dx} = \left(\frac{D_n}{\mu_n}\right)\left(\frac{1}{k_B T}\right)\left(\frac{dE_C}{dx}\right)$. This leads to$\left(\frac{D_n}{\mu_n}\right)\left(\frac{1}{k_B T}\right) = \left(\frac{1}{q}\right)$ so $\frac{D_n}{\mu_n} = \frac{k_B T}{q}$, which confirms the Einstein relation.

Exercise 3 :(1) The general continuity equation for electrons is $\frac{\partial n(x,t)}{\partial t} = \frac{1}{q}\frac{\partial J_n(x,t)}{\partial x} + G(x) - R_n(x)$, where $J_n(x) = qn(x)\mu_n E(x) + qD_n \frac{dn(x)}{dx}$, $G(x) = g_0$ is the constant generation rate, and $R_n(x) = \frac{n(x) - n_0}{\tau_n}$ is the recombination rate. **(2)** Under steady-state conditions ($\partial n/\ \partial t = 0$) and negligible electric field ($E(x) = 0$), the equation simplifies to $\frac{d}{dx}\left(D_n \frac{dn(x)}{dx}\right) + g_0 - \frac{n(x) - n_0}{\tau_n} = 0$**.** Assuming constant D_n, we get

$D_n \frac{d^2 n(x)}{dx^2} + g_0 - \frac{n(x) - n_0}{\tau_n} = 0$. Rewriting in terms of excess carrier concentration $\delta n(x) = n(x) - n_0$, we obtain $\frac{d^2 \delta n(x)}{dx^2} = \frac{\delta n(x) - g_0\tau_n}{L_n^2}$. where $L_n = \sqrt{D_n \tau_n}$is the electron diffusion length. **(3)** The general solution to this nonhomogeneous second-order differential equation is $\delta n(x) = Ae^{x/L_n} + Be^{-x/L_n} + g_0\tau_n$.

Apply the boundary conditions: At $x = 0$, $\delta n(0) = A + B + g_0\tau_n = g_0\tau_n \Rightarrow A + B = 0$; At $x \to +\infty$, $\delta n(+\infty) = Ae^{+\infty} + 0 + g_0\tau_n = g_0\tau_n \Rightarrow A = 0 \Rightarrow B = 0$. Thus, the final solution is $\delta n(x) = g_0\tau_n$ and $n(x) = n_0 + g_0\tau_n$.

Exercise 4:

To simplify the steady-state recombination rate $R = \frac{np - n_i^2}{\tau_p(n+n_i)+\tau_n(p+n_i)}$, we consider three cases: (1) For an **intrinsic semiconductor under illumination**, where $n = n_i + \delta n$ and $p = n_i + \delta p$, we expand the numerator as $np - n_i^2 = n_i(\delta n + \delta p) + \delta n \delta p$, and the denominator becomes $\tau_p(2n_i + \delta n) + \tau_n(2n_i + \delta p)$, with considering $\delta n = \delta p$, the recombination rate is: $R_1 = \frac{n_i(\delta n+\delta p)+\delta n\delta p}{\tau_p(2n_i+\delta n)+\tau_n(2n_i+\delta p)} = \frac{2n_i\delta n+\delta n^2}{(\tau_p+\tau_n)(2n_i+\delta n)} = \frac{\delta n(2n_i+\delta n)}{(\tau_p+\tau_n)(2n_i+\delta n)} = \frac{\delta n}{(\tau_p+\tau_n)}$.

(2) For an **n-type doped semiconductor**, where $n \approx n_0$ and $p \approx \delta p$, the numerator simplifies to $np - n_i^2 \approx n_0\delta p - n_i^2 \cong n_0\delta p$, because $n_0 \gg n_i$ and $\delta p \gg n_i$, and the denominator becomes $\tau_p(n_0 + n_i) + \tau_n(\delta p + n_i)$, giving: $R_2 = \frac{n_0\delta p}{\tau_p(n_0+n_i)+\tau_n(\delta p+n_i)} \cong \frac{n_0\delta p}{\tau_p(n_0)+\tau_n(\delta p)} \cong \frac{n_0\delta p}{\tau_p(n_0)}$ because under low-level injection $\delta p \ll n_0$, therefore $R_2 \cong \frac{\delta p}{\tau_p}$.

(3) For a **p-type doped semiconductor**, where $n \approx \delta n$ and $p \approx p_0$, we get $np - n_i^2 \approx p_0\delta n - n_i^2 \cong p_0\delta n$ because $p_0 \gg n_i$ and $\delta n \gg n_i$, and the denominator becomes $\tau_p(\delta n + n_i) + \tau_n(p_0 + n_i)$, leading to: $R_3 = \frac{p_0\delta n}{\tau_p(\delta n+n_i)+\tau_n(p_0+n_i)} \cong \frac{p_0\delta n}{\tau_p(\delta n)+\tau_n(p_0)} \cong \frac{p_0\delta n}{\tau_n(p_0)}$ because under low-level injection $\delta n \ll p_0$, therefore $R_3 \cong \frac{\delta n}{\tau_n}$. The recombination rate is limited by excess of minority carriers in doped semiconductors.