Mohamed Khider University of Biskra
Department of Material Sciences
Module: Applied Mathematics

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Guided Work Series Number III

Exercise 2.7

Let f be the function defined by:

$$f(x) = \frac{\sqrt{1+x} - \sqrt{1+x^2}}{x}$$

1) Find the definition set $\mathcal{D}_f$ of the function f.

2) Calculate $\lim_{x \to 0} f(x)$, is it extendable continuously over $\mathbb{R}$?

Exercise 2.8

Let the function g defined on $\mathbb{R}$ be as follows:

$$g(x) = \begin{cases} \frac{1}{\ln|x|} & \text{if } x \notin \{0, -1, 1\} \\ 0 & \text{if } x = 0, -1, 1 \end{cases}$$

At which points is the function g continuous?

Exercise 2.9

1) Let the function $f : \mathbb{R} \to \mathbb{R}$ be defined as follows

$$f(x) = \begin{cases} (ax)^2 & \text{if } x \leq 1, \\ a\sin(\frac{\pi}{2}x) & \text{if } x > 1 \end{cases}$$

where $a \in \mathbb{R}$ is a real constant. What are the values of a for the function f to be continuous?

2) Find all values of the constant $\alpha, \beta, \gamma \in \mathbb{R}$ such that the following function $g : \mathbb{R} \to \mathbb{R}$ is continuous:

$$g(x) = \begin{cases} 1 & \text{if } x \leq 0, \\ \alpha e^{-x} + \beta e^{x} + \gamma x(e^{x} - e^{-x}) & \text{if } 0 < x < 1, \\ e^{2-x} & \text{if } x \geq 1. \end{cases}$$

Exercise 2.10

Let the function f defined on $\mathbb{R}\backslash\{-1\}$ as follows:

$$f(x) = \frac{1+x}{x^3+1}.$$

1) Prove that we can extend the function f by continuing at the point -1.

2) Find the value taken at -1 for this extension.

Exercise 2.11

Are the following functions differentiable at 0?

$$f(x) = \frac{x}{1+|x|}, \quad g(x) = \begin{cases} x\sin(x)\sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if} x = 0. \end{cases}, \qquad h(x) = |x|\sin x.$$

Exercise 2.12

Find $a, b \in \mathbb{R}$ such that the function f defined on $\mathbb{R}_+$ is as follows:

$$f(x) = \begin{cases} \sqrt{x} & \text{if} \quad 0 \leq x \leq 1, \\ ax^2 + bx + 1 & \text{if} \quad x > 1, \end{cases}$$

differentiable at 1.

Exercise 2.13

Study the differentiability of the following functions on $\mathbb{R}$:

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases} \qquad g(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0. \end{cases}$$

Exercise 2.14

In each case, find the definition set of the function and then its derivative:

1) $f(x) = 4x^3 - 5x^2 + x - 1,$
2) $f(x) = 5x^3 - \dfrac{1}{x} + 3\sqrt{x},$
3) $f(x) = (x^2 + 1)(x^3 - 2x),$
4) $f(x) = \dfrac{2x^2 - 3}{x^2 + 7},$
5) $f(x) = \dfrac{2x - 1}{x + 1},$
6) $f(x) = -x + 2 + \dfrac{2}{3x},$
7) $f(x) = \dfrac{1}{x + x^2},$
8) $f(x) = (2x + 1)^2,$
9) $f(x) = \sqrt{x}(5x - 3).$

Exercise 2.15 (Derivative using Definition)

Use the definition of derivative to find $f'(x), g'(x), h'(x)$:

1. $f(x) = 3x^2 - 2x.$
2. $g(x) = \frac{1}{x}.$
3. $h(x) = \sqrt{x}.$

Exercise 2.16 (Product Rule Application)

Find the derivative of the function using the product rule:

1. $f(x) = (x^2 + 1)(2x^3 - x).$
2. $g(x) = x^3 \cdot \sin x.$
3. $h(x) = e^x \cdot \ln x.$

Exercise 2.17 (Chain Rule Application)

Find the derivative using chain rule of:

1. $f(x) = \sqrt{3x^2 + 2x}.$
2. $g(x) = \sin(2x^3 - x).$
3. $h(x) = e^{x^2+3x}.$

Exercise 2.18 (Higher Order Derivatives)

Find the first three derivatives of

1. $f(x) = \sin(2x) + e^{3x}$.
2. $g(x) = x^4 - 3x^3 + 2x^2 - x + 5$.
3. $h(x) = \ln(2x + 1)$.

Exercise 2.19 (Function Monotony)

Determine intervals of increase and decrease:

1. $f(x) = x^3 - 3x^2 - 9x + 5$.
2. $g(x) = x^4 - 8x^2 + 16$.
3. $h(x) = \frac{x}{x^2+1}$.

Exercise 2.20 (Bounded Function Analysis)

Analyze boundedness and find extrema:

1. Show that $f(x) = \frac{1}{x^2+1}$ is bounded and find its maximum and minimum values.
2. The same thing about $g(x) = \sin x + 2$.
3. The same thing about $h(x) = \frac{x^2}{x^2+4}$.

Exercise 2.23 (Product Rule Application)

Find the derivative of the function using the product rule:

1. $f(x) = (x^2 + 1)(2x^3 - x)$.
2. $g(x) = x^3 \cdot \sin x$.
3. $h(x) = e^x \cdot \ln x$.