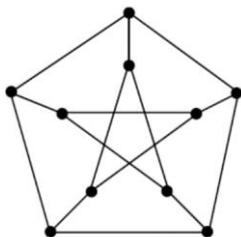


### Problems Set 3

#### Exercise 1 :

Consider the following graph G:



How many simple cycles (without edge repetition) of length 5 does this graph contain?

- Of length 6?
- Of length 8?
- Of length 9?

#### Exercise 2 :

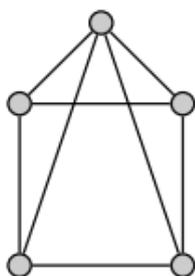
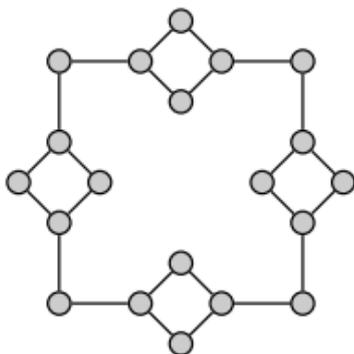
Let G be a non-Eulerian graph. Is it always possible to make G Eulerian by adding a vertex and some edges?

#### Exercise 3 :

Try to construct an undirected graph with at least two vertices such that all vertices have distinct degrees. What can you conclude?

#### Exercise 4 :

Which of the following graphs admits an Eulerian cycle? How many edges should be added to the minimum to obtain an Eulerian cycle?



#### Exercise 5:

For each of the following simple undirected graphs, give an existence example or prove non-existence.

- (a) A 3-regular bipartite graph of 8 nodes (vertices).
- (b) A 4-regular bipartite graph of 11 nodes

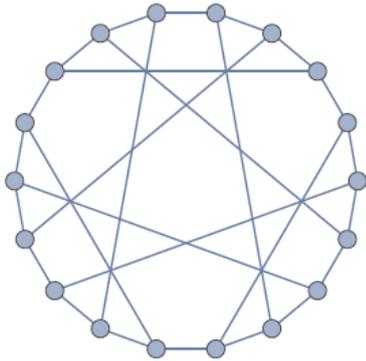
### Exercise 6:

1. For what values of  $n \in \mathbb{N} \setminus \{0\}$ , is the complete graph  $K_n$  planar?
2. Prove that there does not exist a simple planar undirected graph that has exactly 6 nodes of which at least 3 are of degree 5.

### Exercise 7 :

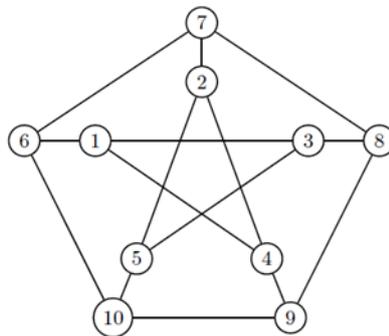
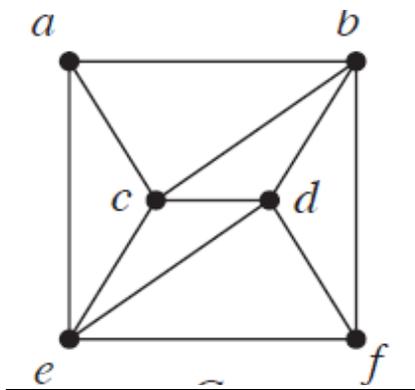
Consider the Pappus graph  $G$  shown below.

- (a) Is this graph Hamiltonian?
- (b) Is this graph Eulerian?
- (c) Show that  $G$  is a 2-colorable graph. Deduce that  $G$  is bipartite.



### Exercise 8:

Apply the graph coloring algorithm (**Welsh-Powell**) to color the vertices of these graphs. Calculate the chromatic number by varying the order of vertices (vertex list).



### Exercise 9 :

9 groups of students are learning 5 courses in a semester.

The course  $C_1$  is taken by groups 1,2,3

The course  $C_2$  is taken by groups 6,7

The course  $C_3$  is taken by groups 1,2,7,9

The course  $C_4$  is taken by groups 4,6,8

The course  $C_5$  is taken by groups 2,3,4,5

We want to schedule the exams such that no group will have more than one exam in one day, and the length of the exam period will be as short as possible,