

Worksheet N°1

Exercise 1 : Give the domains of definition of the following functions:

$$\begin{aligned} f_1(x) &= \frac{2x+3}{x^4-1}, & f_2(x) &= \frac{\sqrt{2x+3}}{x^4-1}, & f_3(x) &= \frac{2x+3}{\sqrt{x^4-1}}, & f_4(x) &= \sqrt{\frac{2x+3}{x^4-1}}. \\ f_5(x) &= \frac{x+2}{x-1}, & f_6(x) &= \sqrt{x-1}, & f_7(x) &= \frac{3}{\sqrt{x-1}}, & f_8(x) &= \sqrt{\frac{x+1}{x^2-1}} \end{aligned}$$

Exercise 02

Study the continuity of the following functions at the point $x=1$:

$$f_1(x) = \frac{x+2}{x-1}, \quad f_2(x) = \sqrt{x-1}, \quad f_3(x) = \frac{3}{\sqrt{x-1}}, \quad f_4(x) = \sqrt{\frac{x+1}{x^2-1}}.$$

Exercise 03

Give the domains of definition and calculate the limits of the following functions

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{\sqrt{x^2-1} + \sqrt{x}-1}{\sqrt{x+1}}, & \quad \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{x - \frac{\pi}{2}}, \\ \lim_{x \rightarrow 0} \frac{e^x - 1}{x}, & \quad \lim_{x \rightarrow 0} \frac{\sin(ax)}{x}. \end{aligned}$$

Exercise 04

Calculate the limits

$$\begin{aligned} \lim_{x \rightarrow \pm\infty} (\sqrt{x^2+x+1} - ax), & \quad \lim_{x \rightarrow +\infty} \frac{\ln x}{x^a}, & \quad \lim_{x \rightarrow \pm\infty} \frac{\sqrt{x^2+x-2}}{x+1+\sqrt{x^2+x-2}}, & \quad \lim_{x \rightarrow 1} \frac{e^x-1}{x}, \\ \lim_{x \rightarrow 0} \frac{1-\cos x}{x^2}, & \quad \lim_{x \rightarrow 0} \frac{\cos x - 1}{x}, & \quad \lim_{x \rightarrow +\infty} \frac{e^x}{x}. \end{aligned}$$

Exercise 05 We consider the function f defined by

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{si } x < 0, \\ 1 & \text{si } x = 0, \\ x^2 + 1 & \text{si } x > 0. \end{cases}$$

1/ Is the function f continuous on \mathbb{R} ?

2/ Determine the set of points where f is derivable ?.

3/ Calculate the derivative of f at the points where it is derivable?.