Computer Science Department **13/10/2024**

Graph Theory (L2)

Problems set 2

Exercise 1 : Let G = (V, E) be a simple, undirected, and complete graph of order n.

1- Calculate the memory size of the graph represented by the adjacency matrix.
2- Calculate the memory size of the graph represented by the incidence matrix.
3- What can be conclude?

Exercise 2 :

$$\begin{pmatrix} 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

1. Draw the graph whose adjacency matrix of the first figure.
2. Draw the graph whose incidence matrix of the second figure
3. What are the types of these graphs: (simple, multiple, directed,

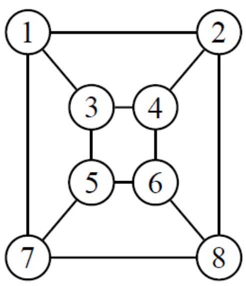
Exercise 3 :

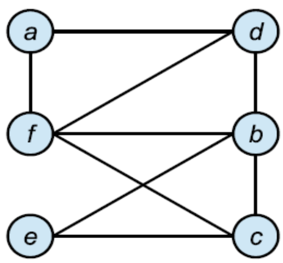
Theorem (Handshaking Lemma)

"The sum of the degrees of the vertices in a graph is equal to twice the number of edges."

1- Show that a simple graph has an even number of vertices with odd degree.
2- A graph G of order 7, with 10 edges, has six vertices with degree **a** and one vertex with degree **b**. What are the values of a and b? Furthermore, if we consider G to be simple, what is the value of b?
3- Draw a graph on six vertices with degree sequence (3, 3, 5, 5, 5, 5); does there exist a simple graph with these degrees?
4- How are your answers to question 3 changed if the degree sequence is (2, 3,3,4,5, 5)?

Exercise 4 :



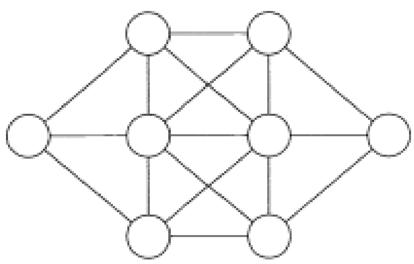


1-Show that this graph is bipartite.

2- Is this graph planar? Calculate the number of faces.

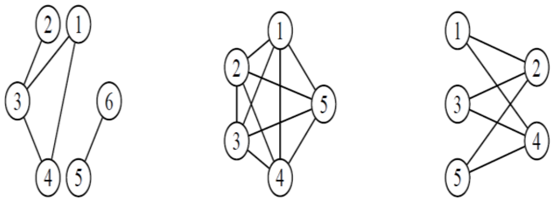
3- Let G be a simple undirected graph of order n. Show that if G is bipartite, then $m \leq n^2/4$.
G = (V, E), |V| = n, and |E| = m

Exercise 5 : (The eight circles problem)



Place the letters A, B, C, D, E, F, G, H into the eight circles in this figure, in such a way that no letter is adjacent to a letter that is next to it in the alphabet.

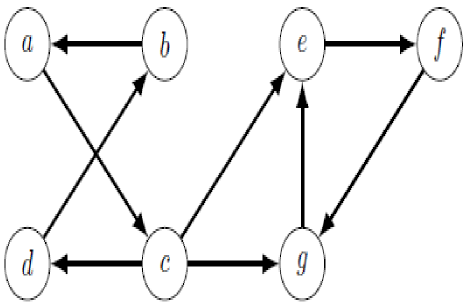
Exercise 6 :



We call an Eulerian cycle of a graph G a cycle that passes through each of the edges of G exactly once. A graph is said to be Eulerian if it has an Eulerian cycle. We call an Eulerian path of a graph G a path that passes through each of the edges of G exactly once. A graph that only has Eulerian paths is called semi-Eulerian. Put simply, a graph is Eulerian (or semi-Eulerian) if it is possible to draw the graph without lifting the pencil and without passing over the same edge twice.

Are the following graphs Eulerian (or semi-Eulerian)?

Exercise 7 :



Consider the following graph G:

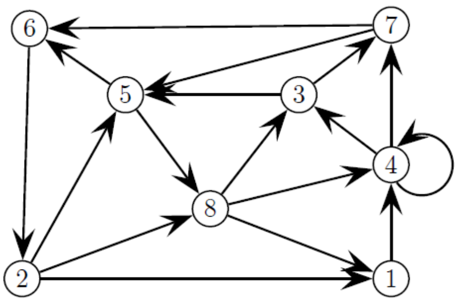
1- Is the graph G connected?

2- Find an articulation point.

3- Determine a bridge.

4- Is the graph G strongly connected? If not, determine the strongly connected components of G and the reduced graph.

Exercise 8 : Calculate the distances d(1, 8) and d(8, 1), and the diameter of the graph.



Exercise 9 :

The completion of a project involves the completion of 5 tasks, A, B, C, D, E. The precedence relationships between these tasks are represented in the following table:

Tasks	The preceding tasks or the tasks that come before
A	B,C
B	-
C	B
D	C
E	A,D

Represent the tasks of this project in the form of an

Exercise 12 :

Consider the graph G=(V,E) as follows:

V={1,2,3,4,5,6} E= { (1,6), (2,1),(2,4),(3,1),(3,5),(4,3),(5,6),(6,4) }

1- Represent the graph G in a directed form (Drawing).
2- Is G orderable? If not, determine a possible circuit.