Problem 1. The perceptron is the simplest kind of artificial neural network. It is a linear classifier used to separate data into two classes (e.g., 0 and 1, true or false). It is a supervised learning model, meaning that it learns from labeled examples (inputs and expected output). The structure of a perceptron consists of:

- Inputs $x^T = [x_1, x_2, \cdots, x_n]$
- Weights $w^T = [w_1, w_2, \cdots, w_n]$
- Biais b
- Activation function (usually a threshold or sigmoid)

The neuron calculates:

$$z = \sum_{i=1}^{n} w_1 \cdot x_i + b \tag{1}$$

Then applies an activation function:

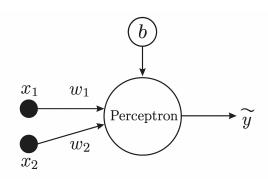
$$\sigma(z) = \begin{cases} 1 & \text{if } z > 1 \\ 0 & \text{otherwise} \end{cases}$$
 (2)

During training, weights are adjusted according to the error:

$$w_i \leftarrow w_i + \eta (y_i - \hat{y}_i) x_i$$

 $b \leftarrow b + \eta (y_i - \hat{y}_i)$

Where \hat{y}_i is the perceptron output and η is the learning rate, which is a hyperparameter taking values from 0 to 1. The structure of this perceptron can be summarized simply using the following diagram:



Solution. An example of an application is provided as a Phyton program:

```
import numpy as np
""" Dr. SAMIR KENOUCHE - simple example of supervised learning
(case of a perceptron) 25/09/2025 """
# Activation function
def activation_class(z):
   return 1 if z > 0 else 0
# Inputs and expected values
x_inputs = np.array([
   [1, 0],
   [1, 1],
   [0, 1],
   [1, 1]
])
y_target = np.array([1, 0, 0, 1]) # Targets
# Weight an biais initialisation
w = np.random.rand(2)
b = np.random.rand(1)
eta = 0.3 # Learning rate
# Learning
for epoch in range(20):
   for i in range(len(x_inputs)):
       z = np.dot(x_inputs[i], w) + b
       yhat = activation_class(z)
       error = y_target[i] - yhat
       # Update of w and b
       w = w + eta * error * x_inputs[i]
       b = b + eta * error
print("Weight :", w, "Biais :", b)
# Learning Test
X_test = np.array([
   [1, 0],
   [0, 1],
   [1, 0],
   [0, 1]
])
for i in range(len(X_test)):
   z = np.dot(X_test[i], w) + b
   yhat = activation_class(z)
   print(f"Input test : {X_test[i]}, Predicted value : {yhat}")
```