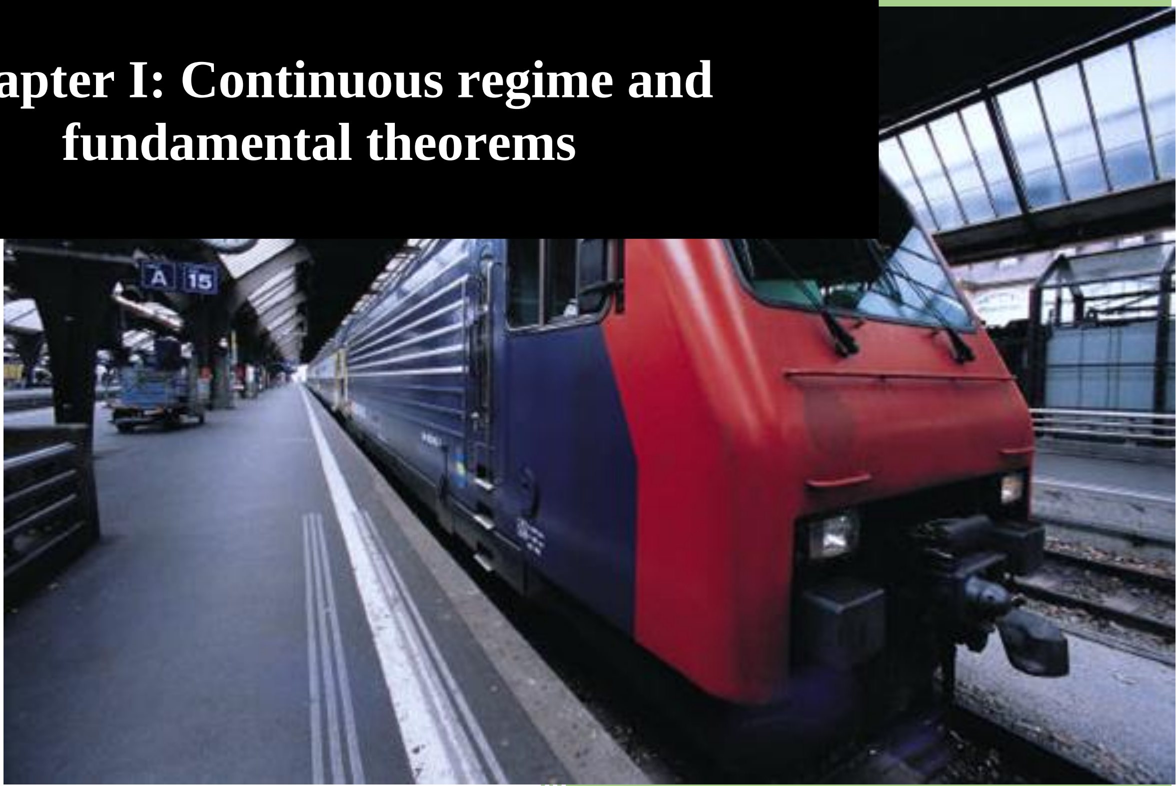
Chapter I: Continuous regime and fundamental theorems



Second year of a science and technology engineering degree

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Chapter I: Continuous regime and fundamental theorems

Chapter 1. Continuous Regime and Fundamental Theorems

K**eywords** : current voltage, passive dipoles, active dipoles, resistance coil capacitor series connection, parallel connection self-inductance capacitance receiver convention, generator convention, Kirchhoff's laws nodal law mesh, law generators direct current voltage divider bridge.

I Introduction

From the most basic assembly to the most complex system, all electrical circuits obey the same simple laws, which are ultimately few in number. To be applied effectively and easily lead to the resolution of sometimes-difficult problems, these laws must be known and used with the utmost rigour. In particular, a number of conventions must be followed, without which it would be impossible to approach this resolution. The aim of this first chapter is to familiarise the reader with the most fundamental tools, within the framework of the simplest operating regime: continuous operation.

A **continuous regime** in electricity means a state where **voltages and currents don't change over time.**

This signifies that:

- The voltage and current values are constant.
- The direction of the current is always the same (unlike alternating current, where it changes periodically).
- It is generally associated with a direct voltage source such as a cell, battery or DC power supply.

I.1 Definitions and fundamental principles

In general, any electrical circuit can be represented as an energy generator supplying a receiver responsible for converting the electrical energy received into another usable form, with the two devices connected by conductors. The operation of an electrical circuit is described by a transfer of charges between these two elements (Figure 1).

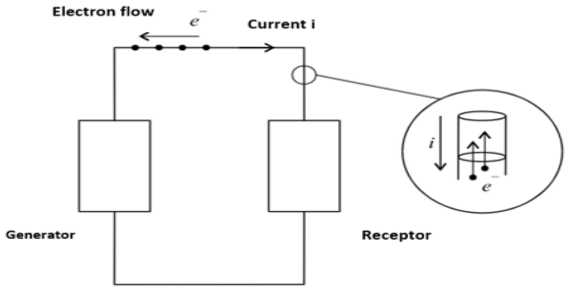


Figure I.1: Electrical circuit

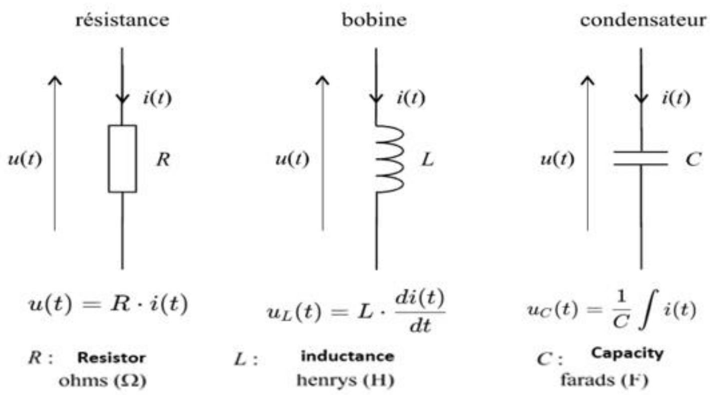
It is commonly accepted to represent this transfer as a flow of electrons, which is modelled as an electric current passing through the conductors. This electric current (expressed in amperes) represents the quantity of charges q (in coulombs) passing through a given section of the conductor per unit of time:

$$i = \frac{dq}{dt}$$

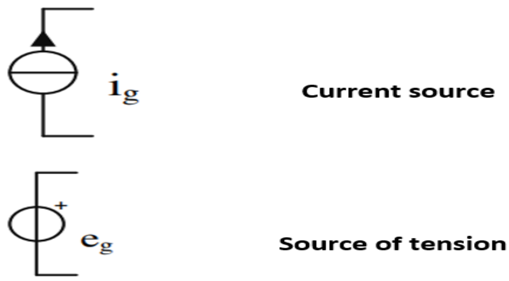
I.1.1 Electrical dipoles

Simple generators and receivers generally have two terminals. These are **electrical dipoles.**

- **Passive dipole**: This is a dipole that consumes electrical energy.



- **Active dipole**: This is a dipole that produces electric current.



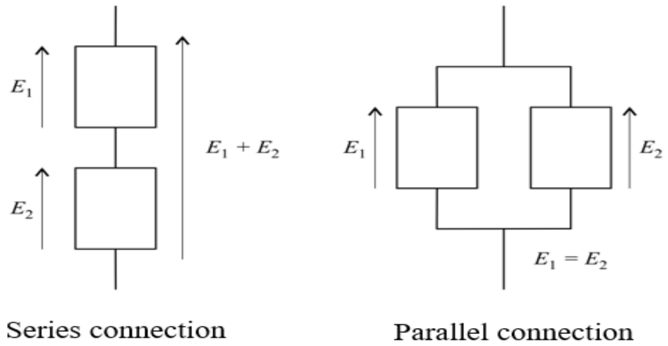
In the practical study of dipoles, two conventions are used:

- **Receiver convention:** The current and voltage arrows point in opposite directions.
- **Generator convention:** The current and voltage arrows point in the same direction.

Figure I.2 : Convention du générateur (a) et convention du récepteur (b).

- **Dipole associations**

Deux dipôles quelconques sont dits associés en série si une des bornes de l'un est reliée à une des bornes de l'autre, l'ensemble formant un nouveau dipôle. Ils sont dits associés en parallèle si les paires de bornes sont connectées deux à deux.

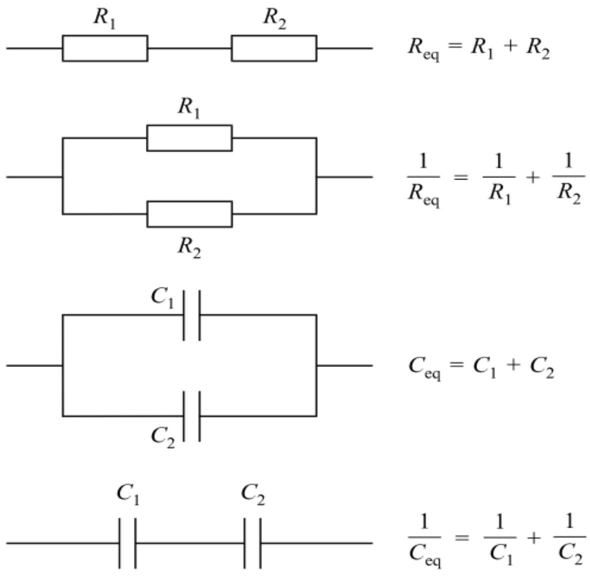


In the case of a series connection, the same current flows through both dipoles. The total voltage across the connection is equal to the sum of the two potential differences across each of the two dipoles.

In the case of a parallel connection, the same potential difference prevails across each of the two dipoles.

By connecting resistors together, we form a dipole that behaves like a resistor, whose value is called the equivalent resistance. The same applies when connecting capacitors together.

Examples :



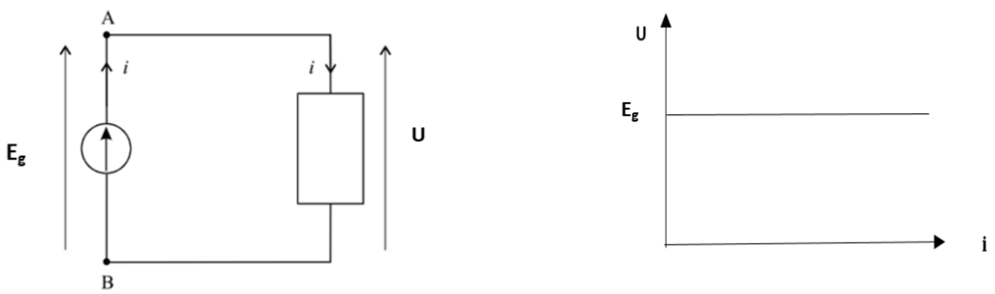
I.1.2 Sources

I.1.2.1 Ideales Sources

- **Ideal voltage source**

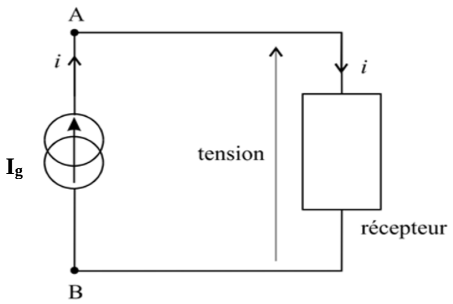
Is a dipole that delivers a voltage (or potential difference) Eg at its terminals that is independent of the current flowing through it. Eg is called the electromotive force (e.m.f.) of the voltage source.

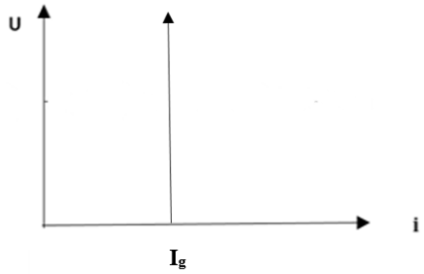
Example: a battery modelled as ideal would always supply 1.5 V, even if a device requiring a lot of current were connected to it.



- **Ideal current source**

Is a dipole that delivers a current Ig independently of the voltage across its terminals. Ig is called the electromotive force (e.m.f.) of the current source.

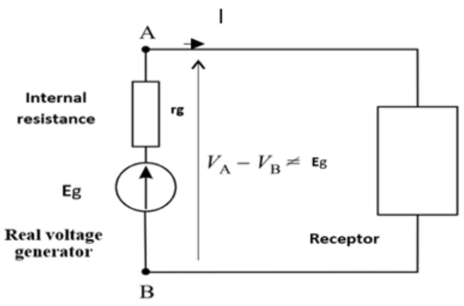


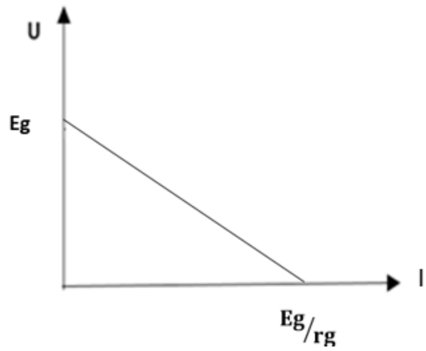


I.1.2.2 Real sources

- **Real voltage generator**

Real sources have internal resistance (or impedance).

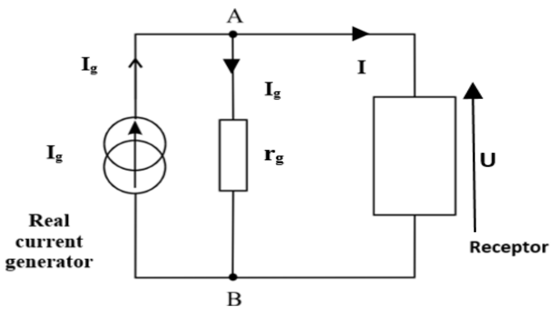


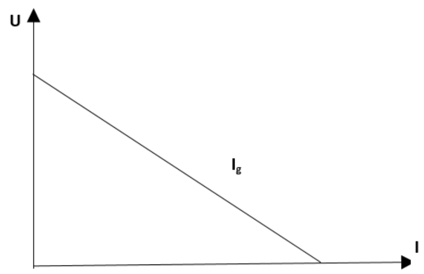


$$U = E_g - r_g\, I$$

- **Real current generator**

A real current generator has internal resistance; this resistance R_g is connected in parallel.





$$I = I_g - \frac{U}{r_g}$$

I.1.3 Kirchhoff's laws in continuous regime

I.1.3.1 Definitions Electrical network

Any simple or complex combination of interconnected dipoles, powered by a generator.

• **Branch:** Dipole part of a network through which the same current flows.

- **Node:** Any point in the network common to more than two branches**.**
- **Mesh:** Any path forming a loop and consisting of several branches**.**

Example:

In the circuit shown in the following figure, the combination of R_1, R_2, R_3, R_4 and R_5 forms the AC dipole constitutes an electrical network powered by voltage generator E.
A, B, C and D are the nodes of this network. The diagram shows three meshes. There are others; for example, starting from point A, we can define a mesh that includes R2, R3 and R5, which passes through D, then C, and rejoins A, including R1.

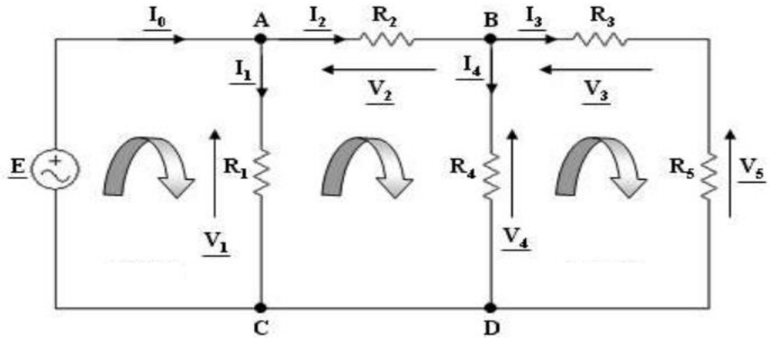


Figure I.3: Example of an electrical network.

Kirchhoff's laws express the conservation of energy and charge in an electrical circuit. They are named after the German physicist who established them in 1845: Gustav Kirchhoff.
In a circuit, Kirchhoff's laws consist of the mesh law, which deals with voltages, and the node law, which deals with currents.

I.1.3.2 Node law (Kirchhoff's first law)

The law of nodes expresses the conservation of electric charge in an electrical circuit. The law of nodes states that: 'The sum of the currents arriving at a node is equal to the sum of the currents leaving it.'

$$\sum_{i=1}^{N} I_i = \sum_{o=1}^{N} I_o$$

Note: The index 'i' is for input current and the index 'o' is for output current.

I.1.3.3 Mesh law (Kirchhoff's second law)

Mesh laws allow us to study the behaviour of voltages within an electrical circuit. Kirchhoff's second law states: 'The algebraic sum of the potential differences (or voltages) along a mesh obtained by traversing the mesh in a given direction is zero.'

$$\sum_{k=1}^{N} \Delta V_k = 0$$

ΔV_k Is an algebraic quantity.

Example: In our example, we can write :

- **Mesh (1) :** $\mathrm{E} - \mathrm{v}_1 = 0$
- **Mesh (2) :** $\mathrm{v}_1 - \mathrm{v}_2 - \mathrm{v}_4 = 0$
- **Mesh (3) :** $\mathrm{v}_4 - \mathrm{v}_3 - \mathrm{v}_5 = 0$

I.1.4 Voltage-current relationship

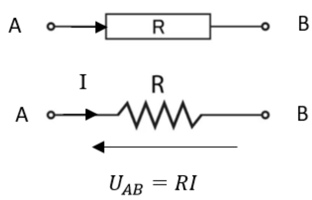
An electrical circuit often consists of passive dipoles (resistors, capacitors, diodes, etc.) and active dipoles (voltage/current generators) that power the circuit. The behaviour of a dipole in a circuit is entirely described by the voltage/current relationship.

$$V = f(I)$$

- **Case of a Resistor R**

Resistance is defined by the relationship between the voltage across its terminals and the current flowing through it, known as Ohm's law: $u(t) = R.i(t)$

The electrical energy produced by the passage of a current I through a resistor is converted into heat by the Joule effect. The instantaneous power dissipated by a resistor is expressed by the following relationship: $P(t) = u(t).i(t)$ Ou $P = RI^2$ (watt)

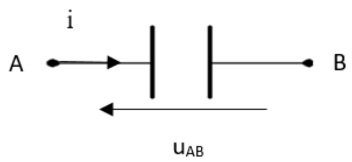


- **Case of a capacitor C**

A capacitor is an electrical component that stores energy in the form of an electrical charge in an electric field. It consists of two conductive plates separated by an insulating material called a dielectric. Its capacitance C is expressed in farads and carries a charge Q.

$$i(t) = c\frac{du(t)}{dt}$$

Shows that if $u(t) = cte$ we have therefore $i(t) = 0$



- **Case of a coil**

An inductor (L) is an electrical component made of a coiled conductive wire, which stores energy in the form of a magnetic field when an electric current flows through it

In continuous mode, a coil will always have a zero potential difference across its terminals, and the coil behaves as a short circuit.

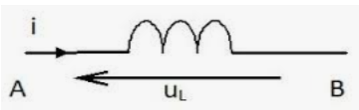
$$\phi = L.i$$

Φ = magnetic flux (in Weber, Wb)

L = coil inductance (in Henry, H)

i = current flowing through the coil (in Ampere, A)

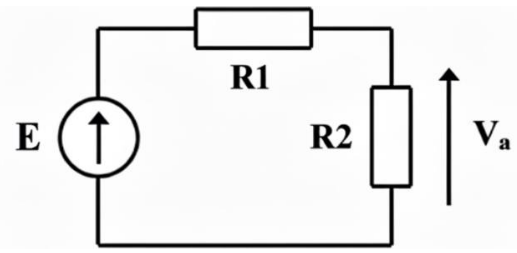
$$\boldsymbol{u}(t) = L\frac{di(t)}{dt}$$



I.1.5 Voltage and current divider

I.1.5.1 Voltage divider :

A voltage divider is a circuit made of resistors in series, which splits an input voltage across its elements.



The expression for voltage E as a function of current flowing through the resistors is

$$E = R_1.I + R_2.I$$

Where: $I = \frac{E}{R_1+R_2}$

The tension across the resistor R_2 is equal: $u_2 = R_2.I$

$$u_2 = \frac{R_2}{R_1 + R_2}.E$$

So, in a series resistance circuit connected to a voltage source E, the voltage across a resistor is calculated as follows:

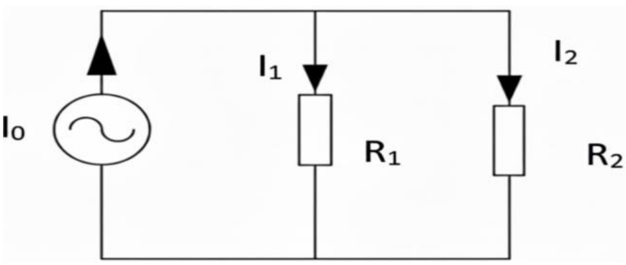
$$u_k = \frac{R_k}{R_1 + \cdots + R_m}.E$$

Where

$$E = \sum R_i.I$$

I.1.5.2 Current divider :

When we have a parallel association of resistors, we can express the current in one of them, knowing the overall current. $G_i=1/R_i$: is the conductance.



The expression of the voltage across the resistor R_1 is: $U = R_1.I_1$

$$R_{eq}.I = \frac{R_1.R_2}{R_1 + R_2}I$$

Hence the equation:

$$I_2 = \frac{R_1}{R_1 + R_2}I$$

Using conductances

$$G_1 = \frac{1}{R_1}\ et\ G_2 = \frac{1}{R_2}$$

We can rewrite the relationship as follows:

$$I_2 = \frac{G_2}{G_1 + G_2} I$$

The intensity obtained is a fraction of the total intensity I, hence the name given to this setup. The current flowing through a resistor R_k placed in a parallel circuit of m resistors, powered by a current source I, is :

$$I_k = \frac{G_k}{G_1 + \cdots + G_m} I$$

Where :

$$G = \frac{I}{u}$$

I.2 General theorems of electricity in continuous regime

In this part, we will use some tools that are more powerful than Kirchhoff's laws. Kirchhoff's laws often give many equations and unknowns, which makes calculations long and difficult. The theorems we will study help to make calculations easier and reduce mistakes. For now, we will use them with direct current, but later we will see that they can also be used with sinusoidal current.

I.2.1 Millman's Theorem

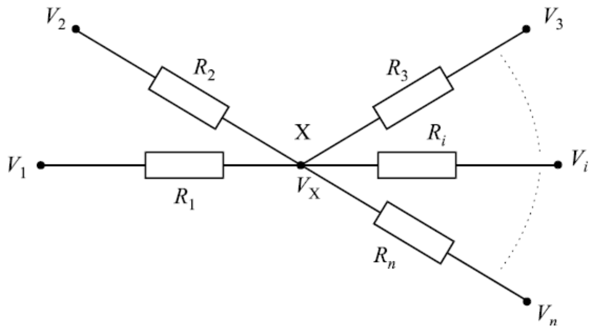
- It makes it possible to calculate the potential of a node as a function of the potentials of the neighboring nodes.
- It is a consequence of Kirchhoff's current law, but expressed only with voltages.

General formula **:**

$$V_x = \frac{\sum_{i=0}^{N} \frac{V_i}{R_i}}{\sum_{i=1}^{N} \frac{1}{R_i}}$$

Example

Let us consider an arbitrary node of a circuit (see figure). This node is connected to *n* points of the circuit through *n* branches, each having a resistance R_i. Let V_i be the voltages at the *n* neighboring points of node X.



The V_X potential is expressed as a function of the potentials at adjacent nodes as follows:

$$V_X = \frac{\dfrac{V_1}{R_1} + \dfrac{V_2}{R_2} + \cdots + \dfrac{V_n}{R_n}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots + \dfrac{1}{R_n}} = \frac{\sum_{i=1}^{n} \dfrac{V_i}{R_i}}{\sum_{i=1}^{n} \dfrac{1}{R_i}}$$

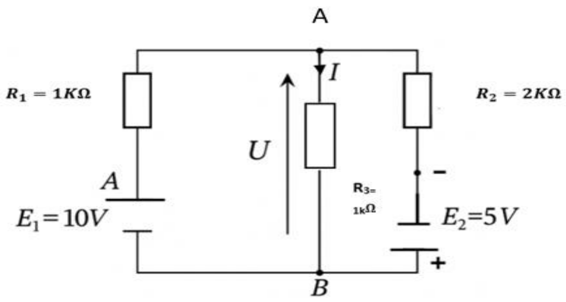
We can also define the conductance of a resistive dipole as the inverse of its resistance. **That is:** $G_1 = \frac{1}{R_1}$

Unit : siemens (S)

Thus, Millman's theorem can also be written as:

$$V_X = \frac{\sum_{i=1}^{n} G_i V_i}{\sum_{i=1}^{n} G_i}$$

Example :



Either :

$$U_{AB} = \frac{\dfrac{E_1}{R_1} - \dfrac{E_2}{R_2}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}} = 2V$$

$And\ ;\quad I = \frac{U_{AB}}{R_3} = 2A$

I.2.2 Superposition Theorem

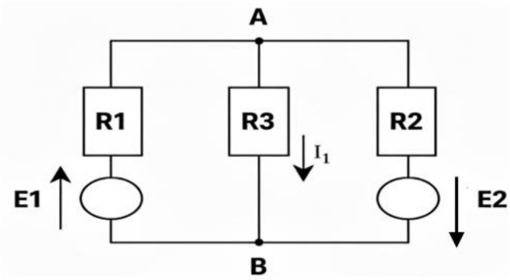
In a linear circuit containing several independent sources (voltage or current), the voltage or current in a circuit element is equal to the algebraic sum of the voltages or currents produced by each source acting alone, the others being replaced by their internal equivalents.

❖ **How to apply the theorem?**

1. Select an active source (example: a battery, a generator).
2. Cancel out the other sources:
   - A voltage source is replaced by a short circuit (U=0).
   - A current source is replaced by an open circuit (I=0).
3. Calculate the voltages/currents due to this single source.
4. Repeat the operation for each source.
5. Add up all the contributions algebraically.

Example:

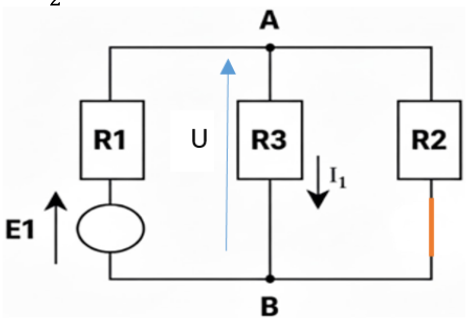
Either a circuit composed of three resistors$R_1 = R_2 = 1\text{K}\Omega$, $R_3 = 2\text{K}\Omega$ in parallel and two voltage generators $E_1 = 10V$ and $E_2 = 5V$



Calculate the current I_1

Solution:

We leave the source E_1 activated and deactivate the source E_2.



$\boldsymbol{R_{eq} = (R_3 // R_2)}$, $\boldsymbol{R_{eq} = \frac{2}{3} K\Omega}$

Using the tension divider

$\boldsymbol{I_1 = \frac{U}{R_3} = \frac{R_{eq}E_1}{R_3 + (R_{eq} + R_1)} = 2mA}$, In the second case, E_1 is deactivated and then E_2 reactivated to calculateI_2, and then the sum of the currents $I_1 + I_2$ obtained is calculated.

I.2.3 Thevenin's and Norton's theorems

I.2.3.1 Thevenin's theorem

Any linear circuit (with resistors and voltage/current sources) viewed from two terminals A and B can be replaced by:

- An equivalent voltage generator E_{th}
- In series with an equivalent resistor R_{th}

In other terms: regardless of the complexity of the circuit, in terms of what is connected between A and B, it simply 'sees' a voltage source E_{th} and a resistor .

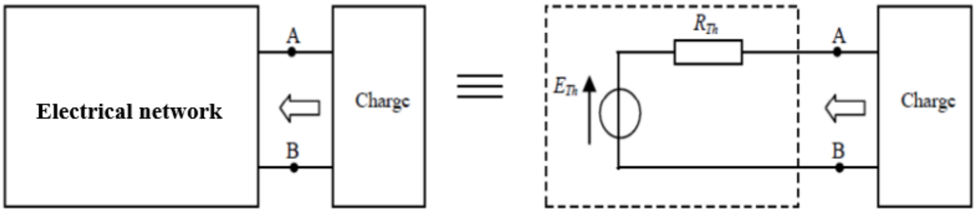
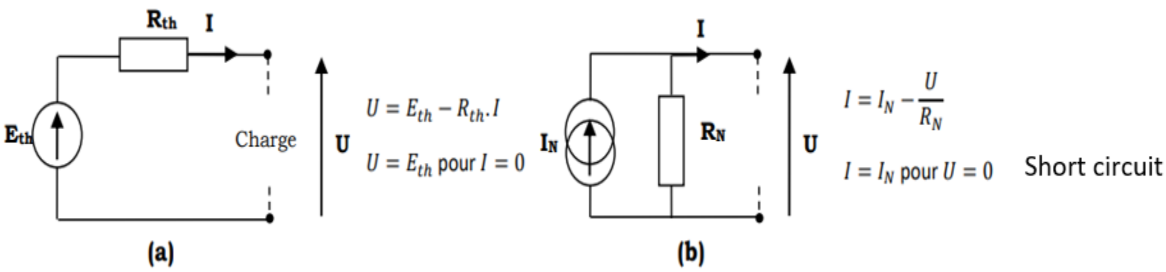


Figure I.4: Thévenin circuit equivalent to an electrical circuit

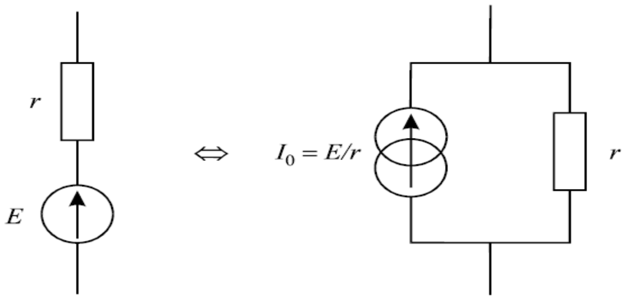
I.2.3.2 Norton's theorem

In continuous regime, any linear dipole network is equivalent to a Norton current generator, with current IN and internal resistance RN equal to the Thévenin resistance. The current I is equal to the source current IN when the latter is short-circuited..



I.2.4 Equivalence between Thevenin and Norton representations

A Thévenin voltage generator with electromotive force E and internal resistance r is equivalent to a Norton generator with current I_0 = E/R and the same internal resistance r.



I.2.5 Kennelly's Theorem

Kennelly's theorem is an electrical theorem concerning the transformation between a star (Y) and delta (Δ) resistor network, and vice versa.
It allows a complicated circuit to be replaced by a simpler equivalent to facilitate calculations.

• **Triangle transformation → star (Δ → Y)**

We do the opposite, switching from a delta configuration to a star configuration.
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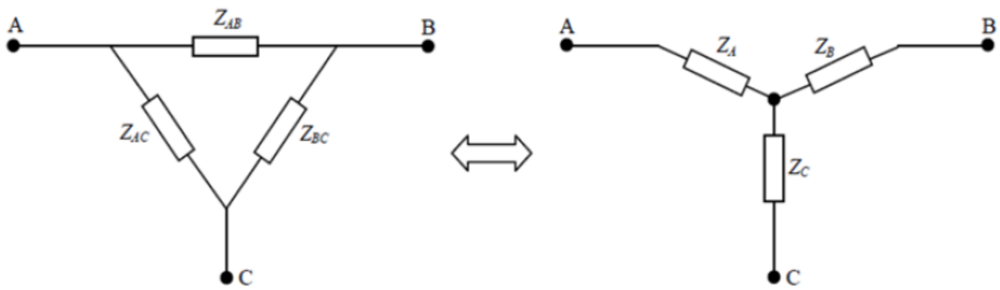


$$Z_a = \frac{Z_{ab}Z_{ac}}{Z_{ab} + Z_{ac} + Z_{bc}}$$

$$Z_b = \frac{Z_{ab}Z_{bc}}{Z_{ab} + Z_{ac} + Z_{bc}}$$

$$Z_c = \frac{Z_{bc}Z_{ac}}{Z_{ab} + Z_{ac} + Z_{bc}}$$

• **Star transformation → triangle (Y → Δ)**

Three resistors arranged in a star configuration are replaced by three equivalent resistors arranged in a triangle.

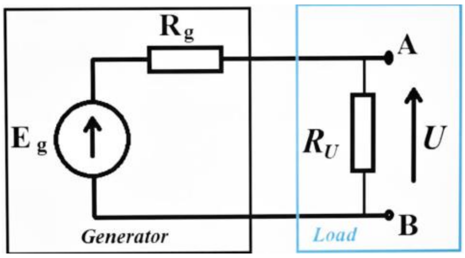
$$Z_{ac} = \frac{Z_aZ_b + Z_aZ_c + Z_bZ_c}{Z_b}$$

$$Z_{ab} = \frac{Z_a Z_b + Z_a Z_c + Z_b Z_c}{Z_c}$$

$$Z_{bc} = \frac{Z_a Z_b + Z_a Z_c + Z_b Z_c}{Z_a}$$

I.3 Maximum Power Transfer Theorem:

In an electrical network, the generator is supposed to provide the necessary energy to a receiver that accepts it. Consider the elementary network consisting of a real voltage generator and a load resistance R_U.



The power supplied by the generator is equal to:

$$P_g = E.I = R_g.I^2 + R_U.I^2 = (R_g + R_U).I^2$$

The power absorbed by the load:

$$P_U = R_U\,.I^2$$

How should R_U be chosen with respect to R_g so that the transmitted power is maximum?

We are looking for the optimal value of the utilization resistance $R_{U(opt)}$. To do this, let's calculate the P_U power as a function of R_g:

$$P_U = R_U.I^2 = R_U\left(\frac{E}{R_g + R_U}\right)^2$$

Let us study the law of variation of power by calculating its derivative:

$$\frac{dP_U}{dR_U} = \frac{E^2\left(R_g + R_U\right)^2 - E^2.R_U\left(2R_g + 2R_U\right)}{\left(R_g + R_U\right)^4} = E^2\frac{\left(R_g + R_U\right)\left(R_g - R_U\right)}{\left(R_g + R_U\right)^4} =$$
$$= E^2\frac{\left(R_g - R_U\right)}{\left(R_g + R_U\right)^3}$$

The transmitted power is maximum when this derivative is zero, that $R_g - R_U = 0$, $R_g = R_U$

$$P_{U(max)} = \frac{E^2 . R_U}{\left(R_g + R_U\right)^2} = \frac{E^2}{4R_g}$$

The transmitted power is maximum (in mathematics, we say that the curve passes through an ***Extremum***) when this derivative is zero, that is to say for $R_U = Rg$.