Module: Advanced Semiconductor Physics

Series TD2 of Chapter II: Statistics and Electronic Properties and Chapter III: Electrical Transport in Semiconductors

Exercise 1: Energy Band and Carrier Distribution

A semiconductor has a conduction band edge at $E_c = 0$ eV, and the Fermi level is located at $E_F = -0.2$ eV. Assume non-degenerate conditions and Maxwell-Boltzmann statistics. 1-Calculate the probability $f(E_c)$ that a state at the conduction band edge is occupied at T=300°K, with $k_B T = 0.025875$ eV. 2- Determine the electron concentration n with $N_c = 2.8 \times 10^{19}$ cm^{-3}. 3-Estimate the average thermal velocity of electrons $v_{\text{th}} = \sqrt{\frac{3k_B T}{m^*}}$, with $m^* = 0.26\, m_0$. 4- Compute the drift current density J assuming drift velocity $v_d = 10^7$ cm/s.

Exercise 2: Doping Effects and Carrier Concentration

1- Calculate the electron concentration in an n-type silicon sample doped with $N_D = 10^{17}$ cm^{-3}, assuming full ionization. 2-Determine the hole concentration using the mass action law at $T = 300$ K, given $n_i = 1.5 \times 10^{10}$ cm^{-3}. 3- Compute the position of the Fermi level (E_F) relative to the intrinsic level (E_{Fi}), assuming non-degenerate conditions. 4-Estimate the conductivity of the sample using $\mu_n = 1350$ cm$^2/(V.s)$

Exercise 3: Compensated n-Type Semiconductor

A silicon sample is doped with donor atoms at a concentration of $N_D = 5 \times 10^{16}$ cm^{-3}and acceptor atoms at $N_A = 1 \times 10^{16}$ cm^{-3}. Assume intrinsic carrier concentration: $n_i = 1.5 \times 10^{10}$ cm^{-3}. Effective density of states in conduction band: $N_C = 2.8 \times 10^{19}$ cm^{-3}. Thermal voltage: $k_B T = 0.025875$ eV. Temperature: $T = 300$ K.

1- Calculate the free electron concentration n .2-Determine the minority carrier (holes) concentration p. 3 –Compute the position of the Fermi level relative to the conduction band edge $(E_C - E_F)$. 4- Calculate the n_i at temperature T=500°K with assuming $N_C = N_0 \left(\frac{m_e^*}{m_0}\right)^{\frac{3}{2}} \left(\frac{T}{T_0}\right)^{\frac{3}{2}}$ and $N_V = N_0 \left(\frac{m_h^*}{m_0}\right)^{\frac{3}{2}} \left(\frac{T}{T_0}\right)^{\frac{3}{2}}$where : $N_0 = 2.508 \times 10^{19}$ cm^{-3}, $T_0 = 300$ K, $m_e^* = 1.08\, m_0$ and $m_h^* = 0.56\, m_0$, $E_g = 1.12\, eV$. $k_B = 8.617 \times 10^{-5}$ eV/K. Calculate the free electron concentration n and the free hole concentration p at this temperature.

Exercise 4: Carrier Mobility and Temperature Dependence

1- Derive the expression for mobility $\mu(T)$ assuming phonon scattering dominates, and calculate μ at $T = 300$ K given $\mu_0 = 1500$ cm$^2/(V.s)$ at $T_0 = 100$ K. 2-For a doped silicon sample, calculate the total mobility using Matthiessen's rule given $\mu_{\text{imp}} = 1200$ cm$^2/(V.s)$ and $\mu_{\text{phonon}} = 800$ cm$^2/(V.s)$. 3-Estimate the drift velocity v_d of electrons under an electric field of 10 V/cm using the mobility from part 2. 4-Compute the conductivity σ for a carrier concentration of 10^{16} cm^{-3}using the total mobility.

Exercise 5: Semi-Classical Transport and Group Velocity

1- Given $E(k) = \frac{\hbar^2 k^2}{2m^*}$, derive the group velocity $v_g = \frac{1}{\hbar} \nabla_k E(k)$.2-Calculate v_g in (cm/s) for an electron with $k = 10^8$ m^{-1}and $m^* = 0.26\, m_0$. 3-Determine the acceleration $a = dv_g/dt$ under an electric field $E = 100$ V/cm.4- Estimate the time required for the electron to reach a velocity of 10^7 cm/s.

Exercise 6: Carrier Concentration and Fermi Level in Doped Semiconductors

Consider a p-type silicon semiconductor at $T = 300$ K, doped with acceptor concentration $N_A = 5 \times 10^{16}$ cm^{-3}. Assume full ionization and non-degenerate conditions.1-Calculate the free hole concentration p. 2-Estimate the electron concentration n given $n_i = 1.5 \times 10^{10}$ cm^{-3}. 3- Determine the position of the Fermi level E_F relative to the valence band edge E_V. with assuming $N_C = N_0 \left(\frac{m_e^*}{m_0}\right)^{\frac{3}{2}} \left(\frac{T}{T_0}\right)^{\frac{3}{2}}$ and $N_V = N_0 \left(\frac{m_h^*}{m_0}\right)^{\frac{3}{2}} \left(\frac{T}{T_0}\right)^{\frac{3}{2}}$ where : $N_0 = 2.508 \times 10^{19}$ cm^{-3}, $T_0 = 300$ K, $m_e^* = 1.08\, m_0$ and $m_h^* = 0.56\, m_0$, $E_g = 1.12\, eV$. $k_B = 8.617 \times 10^{-5}$ eV/K. 4-Calculate n_i at 600°K . 5-Calculate the free hole and electron concentrations n and p at this temperature, and the resulting conductivity σ using $\mu_n = 1350$ cm$^2/(V.s)$ and $\mu_p = 450$ cm$^2/(V.s)$.

Solutions

Exercise 1: To solve this exercise, we first calculate the occupation probability at the conduction band edge using Maxwell-Boltzmann statistics: $f(E_C) = \exp\left(-\frac{E_C - E_F}{k_B T}\right) = \exp\left(-\frac{0-(-0.2)}{0.025875}\right) \approx \exp(-7.73) \approx 4.4 \times 10^{-4}$. The electron concentration is then $n = N_C \cdot f(E_C) = 2.8 \times 10^{19} \cdot 4.4 \times 10^{-4} \approx 1.23 \times 10^{16}\ \text{cm}^{-3}$. For the average thermal velocity, we use $v_{\text{th}} = \sqrt{\frac{3k_B T}{m^*}} = \sqrt{\frac{3 \cdot 1.38 \times 10^{-23} \cdot 300}{0.26 \cdot 9.11 \times 10^{-31}}} \approx 2.289 \times 10^5$ m/s. Finally, the drift current density is given by $J = q \cdot n \cdot v_d = 1.6 \times 10^{-19} \cdot 1.23 \times 10^{16} \times 10^7 \approx 1.97 \times 10^4 \text{A/cm}^2$.

Exercise2:

In an n-type silicon sample doped with $N_D = 10^{17}\ \text{cm}^{-3}$and assuming full ionization, the electron concentration is approximately $n \approx N_D = 10^{17}\ \text{cm}^{-3}$. Using the mass action law at $T = 300\,\text{K}$, the hole concentration is $p = \frac{n_i^2}{n} = \frac{(1.5 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^3\ \text{cm}^{-3}$. The position of the Fermi level relative to the intrinsic level is given by $E_F - E_{Fi} = k_B T \cdot \ln\left(\frac{n}{n_i}\right) = 0.025875 \cdot \ln\left(\frac{10^{17}}{1.5 \times 10^{10}}\right) \approx 0.025875 \cdot 15.72 \approx 0.407\,\text{eV}$, indicating that the Fermi level lies about 0.41 eV above the intrinsic level. Finally, the conductivity is $\sigma = q \cdot n \cdot \mu_n = 1.6 \times 10^{-19} \times 10^{17} \cdot 1350 \approx 21.6\ (\Omega.cm)^{-1}$. Remark: To demonstrate that $E_F - E_{Fi} = k_B T \cdot \ln\left(\frac{n}{n_i}\right)$, we start from the expressions for electron concentration in a non-degenerate semiconductor: $n = N_C \cdot \exp\left(\frac{E_F - E_C}{k_B T}\right)$ and intrinsic concentration $n_i = N_C \cdot \exp\left(\frac{E_{Fi} - E_C}{k_B T}\right)$. Dividing the two gives $\frac{n}{n_i} = \exp\left(\frac{E_F - E_{Fi}}{k_B T}\right)$, and taking the natural logarithm of both sides yields $\ln\left(\frac{n}{n_i}\right) = \frac{E_F - E_{Fi}}{k_B T}$, which rearranges directly to the desired result: $E_F - E_{Fi} = k_B T \cdot \ln\left(\frac{n}{n_i}\right)$. This equation quantifies how doping shifts the Fermi level relative to the intrinsic level.

Exercise 3

For the compensated n-type silicon sample with donor and acceptor concentrations $N_D = 5 \times 10^{16}\ \text{cm}^{-3}$and $N_A = 1 \times 10^{16}\ \text{cm}^{-3}$, the exact free electron concentration at $T = 300$ K is calculated using the quadratic formula: $n = \left(\frac{N_D - N_A}{2}\right) + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2} = 2 \times 10^{16} + \sqrt{4 \times 10^{32} + 2.25 \times 10^{20}} \approx 4 \times 10^{16}\ \text{cm}^{-3} \cong N_D - N_A$; because $n_i = 1.5 \times 10^{10}\ \text{cm}^{-3} \ll (N_D - N_A)$ at $T = 300\,\text{K}$, the minority carrier concentration is $p = \frac{n_i^2}{n} = \frac{2.25 \times 10^{20}}{4 \times 10^{16}} = 5.625 \times 10^3\ \text{cm}^{-3}$, and the Fermi level relative to the conduction band edge is $E_C - E_F = k_B T \cdot \ln\left(\frac{N_C}{n}\right) = 0.025875 \cdot \ln\left(\frac{2.8 \times 10^{19}}{4 \times 10^{16}}\right) \approx 0.169\ \text{eV}$. At $T = 500$ K, the temperature-scaled effective densities of states are $N_C = 2.508 \times 10^{19} \cdot (1.08)^{\frac{3}{2}} \cdot (500/300)^{\frac{3}{2}} \approx 6.056 \times 10^{19}\ \text{cm}^{-3}$ and $N_V = 2.508 \times 10^{19} \cdot (0.56)^{3/2} \cdot (500/300)^{3/2} \approx 2.261 \times 10^{19}\ \text{cm}^{-3}$, giving $n_i = \sqrt{N_C N_V} \cdot \exp\left(-\frac{E_g}{2k_B T}\right) \approx \sqrt{1.369 \times 10^{39}} \cdot \exp(-1.12/(2 \times 8617 \times 10^{-8} \times 500) \approx 8.38 \times 10^{13}\ \text{cm}^{-3}$; which means $n_i(500°K) \cong 8.38 \times 10^{13}\ \text{cm}^{-3}$, using this updated n_i, the exact electron concentration becomes $n = 2 \times 10^{16} + \sqrt{4 \times 10^{32} + (8.38 \times 10^{13})^2} \approx 4 \times 10^{16}\ \text{cm}^{-3}$, and the hole concentration is $p = \frac{n_i^2}{n} \approx \frac{(8.38 \times 10^{13})^2}{4.83 \times 10^{16}} \approx 1.758 \times 10^{11}\ \text{cm}^{-3}$.

Exercise 4:

Understand the dependence of mobility on temperature with phonon scattering. **Phonon scattering** causes mobility to decrease with increasing temperature, typically following a power law: $\mu_{\text{phonon}} \propto T^{-\eta}$,
where η is an exponent that depends on the scattering mechanism. For phonon scattering, η is usually around $(^3/_2)$. Assuming phonon scattering dominates, mobility varies as $\mu(T) = \mu_0 \left(\frac{T_0}{T}\right)^{3/2}$, so at $T = 300\,\text{K}$, we get $\mu(300°K) = 1500 \cdot \left(\frac{100}{300}\right)^{3/2} = 1500 \cdot (1/3)^{1.5} \approx 1500 \cdot 0.192 \approx 288\,\text{cm}^2/(V.s)$. using Matthiessen's rule, the

total mobility is $\mu_{\text{total}} = (\frac{1}{\mu_{\text{imp}}} + \frac{1}{\mu_{\text{phonon}}})^{-1} = (\frac{1}{1200} + \frac{1}{800})^{-1} = (\frac{5}{2400})^{-1} = 480\,\text{cm}^2/(V.s)$; the drift velocity is $v_d = \mu E = 480 \cdot 10 = 4800\,\text{cm/s}$; and the conductivity is $\sigma = qn\mu = (1.6 \times 10^{-19}) \cdot (10^{16}) \cdot 480 = 7.68 \times 10^{-1}\,(\Omega.cm)^{-1}$. $\sigma = 0.768\,(\Omega.cm)^{-1}$.

Exercise 5:

Starting with the energy dispersion relation $E(k) = \frac{\hbar^2 k^2}{2m^*}$, the group velocity is derived as $v_g = \frac{1}{\hbar}\nabla_k E(k) = \frac{\hbar k}{m^*}$; using $k = 10^8\,\text{m}^{-1}$, $\hbar = 1.055 \times 10^{-34}\,(J.s)$, and $m^* = 0.26\,m_0 = 0.26 \cdot 9.11 \times 10^{-31}\,\text{kg}$, we find $v_g = \frac{1.055\times10^{-34}\cdot10^8}{2.3686\times10^{-31}} \approx 4.45 \times 10^4\,\text{m/s} = 4.45 \times 10^6\,\text{cm/s}$; under an electric field $E = 100\,\text{V/cm} = 10^4\,\text{V/m}$, the acceleration is $a = \frac{qE}{m^*} = \frac{1.6\times10^{-19}\cdot10^4}{2.3686\times10^{-31}} \approx 6.75 \times 10^{15}\,\text{m/s}^2 = 6.75 \times 10^{17}\,\text{cm/s}^2$; to reach a velocity of 10^7 cm/s, the time required is $t = \frac{v}{a} = \frac{10^7}{6.75\times10^{17}} \approx 1.48 \times 10^{-11}\,\text{s}$.

Exercise 6:

For a p-type silicon semiconductor at 300 K with acceptor concentration $N_A = 5 \times 10^{16}\,\text{cm}^{-3}$, assuming full ionization and non-degenerate conditions, the hole concentration is $p = \left(\frac{N_A}{2}\right) + \sqrt{(\frac{N_A}{2})^2 + n_i^2} \approx 5.00 \times 10^{16}\,\text{cm}^{-3}$since $n_i = 1.5 \times 10^{10}\,\text{cm}^{-3}$is negligible $\ll N_A$; the electron concentration is $n = \frac{n_i^2}{p} = 4.5 \times 10^3\,\text{cm}^{-3}$; using $N_C = 2.508 \times 10^{19} \cdot (1.08)^{3/2} = 2.814 \times 10^{19}\,\text{cm}^{-3}$and $N_V = 2.508 \times 10^{19} \cdot (0.56)^{3/2} = 1.051 \times 10^{19}\,\text{cm}^{-3}$, the Fermi level relative to the valence band is given:

$E_F - E_V = k_B T \ln\left(\frac{N_V}{p}\right) = 0.025875 \cdot \ln\left(\frac{1.051\times10^{19}}{5.00\times10^{16}}\right) = 0.025875 \cdot \ln(210.2) \approx 0.138\,\text{eV}$; at 600 K, the intrinsic concentration is $n_i = 2.508 \times 10^{19} \cdot \left(\frac{600}{300}\right)^{3/2} \cdot (1.08 \cdot 0.56)^{3/4} \cdot \exp(-\frac{1.12}{2\cdot8.617\times10^{-5}\cdot600}) \approx 9.618 \times 10^{14}\,\text{cm}^{-3}$; so $n_i \cong 9.618 \times 10^{14}\,\text{cm}^{-3}$ then the hole concentration:

$$p = \frac{N_A}{2} + \sqrt{(\frac{N_A}{2})^2 + n_i^2} = 2.5 \times 10^{16} + \sqrt{6.25 \times 10^{32} + 9.25 \times 10^{29}} \approx 5.00 \times 10^{16}\,\text{cm}^{-3}$$

$p \approx 5.00 \times 10^{16}\,\text{cm}^{-3}$, $n = \frac{n_i^2}{p} \approx 1.85 \times 10^{13}\,\text{cm}^{-3}$, and the conductivity is $\sigma = q\left(n\mu_n + p\mu_p\right) = 1.6 \times 10^{-19} \cdot [(1.85 \times 10^{13} \cdot 1350) + (5.00 \times 10^{16} \cdot 450)] \approx \sigma_p = 3.60\,(\Omega.cm)^{-1}$