Module: Advanced Semiconductor Physics
Series TD1 of Chapter I: Fundamentals of Semiconductors

Exercise1: Bloch's Theorem and Crystal Periodicity

1-Given a periodic potential $V(x+a) = V(x)$, show that the electron wavefunction satisfies $\psi_k(x+a) = e^{ika}\psi_k(x)$. 2- For a 1D crystal with lattice constant $a = 0.5$ nm, calculate in (m^{-1}) the wave vector k_{max} at the first Brillouin zone boundary. given by $k_{max} = \frac{\pi}{a}$. 3- If an electron has energy $E = \frac{\hbar^2 k^2}{2m}$, compute its energy in (eV) at $k = k_{max}$, using $m = 9.11 \times 10^{-31}$ kg and $\hbar = \left(\frac{h}{2\pi}\right)$.

Exercise 2 : Band Gap and Optical Transitions

1- Calculate the energy (E_{ph}) (in joules) required to excite an electron across the band gap of GaAs $(E_g = 1.43\ eV)$. 2- Determine the corresponding photon wavelength (λ) in nm. 3- For GaP $(E_g = 2.25\ eV)$, calculate the minimum photon frequency $\big(\nu(Hz)\big)$ required for excitation. We give $h = 6.626 \times 10^{-34}$ $(J.s)$.

Exercise 3: Intrinsic Semiconductor Properties at Room Temperature

Consider intrinsic silicon at 300 K, with the following parameters: Band gap energy: $E_g = 1.11$ eV, Intrinsic carrier concentration: $n_i = 1.5 \times 10^{10}$ cm^{-3}, Electron mobility: $\mu_n = 1350$ cm$^2/(V.s)$, Hole mobility: $\mu_p = 450$ cm$^2/(V.s)$ and Elementary charge: $q = 1.602 \times 10^{-19}$ C. 1- Calculate the electrical conductivity σ_i of intrinsic silicon at 300 K knowing that the general conductivity is given by $\sigma = q(n\mu_n + p\mu_p)$. 2- Estimate the resistivity ρ_i of intrinsic silicon at 300 K. 3- calculate the number of thermally generated electron-hole pairs per (cm^{-3}).

Exercise 4: Carrier Concentrations in Doped Silicon

A silicon sample is doped with $N_D = 10^{17}$ cm^{-3}. Assuming full ionization, 1- calculate the electron concentration n. 2- Using $n_i = 1.5 \times 10^{10}$ cm^{-3}, compute the hole concentration p. 3- Calculate the conductivity $\sigma = q(n\mu_n + p\mu_p)$, with $\mu_n = 1350$, $\mu_p = 450$ cm$^2/(V.s)$. 4- Estimate the resistivity ρ.

Exercise 5: Fermi Level and Impurity States

In n-type silicon $\big(E_g = 1.11$ eV$\big)$, donor level (E_D) is 0.045 eV below E_C. 1-Estimate the Fermi level $\big(E_f\big)$ position assuming it lies halfway between E_C and E_D. 2- Using the Fermi-Dirac distribution at 300 K, $f(E) = \left(\frac{1}{1+exp\left(\frac{E-E_f}{k_B T}\right)}\right)$, Calculate the probability of electron occupancy at donor level (E_D). 3- For $N_D = 10^{16}$ cm^{-3}, estimate the number of ionized donors N_D^+ and calculate the free electron concentration n from ionized donors.

Exercise 6: Hole Dynamics in p-Type Silicon

A p-type sample has $p = 5 \times 10^{17}$ cm^{-3}, $\mu_p = 450$ cm$^2/(V.s)$, $E = 100$ V/cm. 1- Calculate the drift current density $J_{pdrift} = qp\mu_p\, E$. 2- Estimate the hole drift velocity $v_d = \mu_p\, E$.

Solution TD1 of Chapter I: Fundamentals of Semiconductors

Exercise1: Bloch's Theorem and Crystal Periodicity

1-Bloch's theorem states that in a periodic potential $V(x+a) = V(x)$, the electron wavefunction takes the form: $\psi_k(x) = u_k(x)e^{ikx}$. where $u_k(x)$ is a function with the same periodicity as the potential: $u_k(x+a) = u_k(x)$,
Then: $\psi_k(x+a) = u_k(x+a)e^{ik(x+a)} = u_k(x)e^{ika}e^{ikx} = e^{ika}\psi_k(x)$

thus, the wavefunction satisfies: $\psi_k(x+a) = e^{ika}\psi_k(x)$.

2-Calculate $k_{\text{max}} = \frac{\pi}{a}$ for $a = 0.5$ nm , Convert lattice constant to meters: $a = 0.5\text{ nm} = 0.5 \times 10^{-9}$ m
Then: $k_{\text{max}} = \frac{\pi}{a} = \frac{3.1416}{0.5\times10^{-9}} = 6.2832 \times 10^9\ \text{m}^{-1}$. So, the wave vector at the Brillouin zone boundary is:

$$k_{\text{max}} \approx 6.28 \times 10^9\ \text{m}^{-1}$$

3-Compute energy $E = \frac{\hbar^2k^2}{2m}$at $k = k_{\text{max}}$, Given: $k = k_{\text{max}} = 6.2832 \times 10^9\ \text{m}^{-1}$, $m = 9.11 \times 10^{-31}$ kg

$$\hbar = \frac{h}{2\pi} = \frac{6.626\times10^{-34}}{2\pi} = 1.0546 \times 10^{-34}\ (J.s)$$, replace in the energy formula:

$$E = \frac{(1.0546 \times 10^{-34})^2 \cdot (6.2832 \times 10^9)^2}{2 \cdot 9.11 \times 10^{-31}}$$

Compute: $\hbar^2 = 1.112 \times 10^{-68}\ \text{J}^2 \cdot \text{s}^2$and $k^2 = 3.947 \times 10^{19}$ and $\hbar^2k^2 = 4.39 \times 10^{-49}$

Divide by $2m$: $E = \frac{4.39\times10^{-49}}{2\cdot9.11\times10^{-31}} = \frac{4.39\times10^{-49}}{1.822\times10^{-30}} \approx 2.41 \times 10^{-19}$ J

Convert to eV: $E = \frac{2.41\times10^{-19}}{1.602\times10^{-19}} \approx 1.50$ eV. So, the electron energy at $k_{\text{max}}$is: $E \approx 1.50$ eV

Exercise 2: Band Gap and Optical Transitions
1- Calculate the photon energy E_{ph} in joules for GaAs, Given: Band gap of GaAs: $E_g = 1.43$ eV
And Conversion factor: 1 eV $= 1.602 \times 10^{-19}$ J
Calculation:

$$E_{\text{ph}} = E_g \times (1.602 \times 10^{-19}) = 1.43 \times 1.602 \times 10^{-19}$$

$$E_{\text{ph}} = 2.29 \times 10^{-19}\ \text{J}$$

2-: Determine the corresponding photon wavelength λ in nm
Formula: $E_{\text{ph}} = h\nu = \frac{hc}{\lambda}$

$$\lambda = \frac{hc}{E_{\text{ph}}}$$

Given: $h = 6.626 \times 10^{-34}\ (J.s)$, $c = 3.00 \times 10^8$ m/s, $E_{\text{ph}} = 2.29 \times 10^{-19}$ J
Calculation:

$$\lambda = \frac{6.626 \times 10^{-34} \cdot 3.00 \times 10^8}{2.29 \times 10^{-19}} = \frac{1.9878 \times 10^{-25}}{2.29 \times 10^{-19}} \approx 8.68 \times 10^{-7}\ \text{m}$$

Convert to nanometers: $\lambda = 868$ nm.
We have also $hc \cong 1.2424\ \mu m.eV$:so $\lambda \approx \frac{hc(\mu m.eV)}{E_{ph}(eV)} = \frac{1.2424}{1.43} = 0.868\ \mu m = 868$ nm
3- Calculate the minimum photon frequency ν for GaP. Given: Band gap of GaP: $E_g = 2.25$ eV

- $h = 6.626 \times 10^{-34}$ J.s, Convert E_g to joules:

$$E = 2.25 \times 1.602 \times 10^{-19} = 3.6045 \times 10^{-19}\ \text{J}$$

Use $\nu = \frac{E}{h}$ then $\nu = \frac{3.6045\times10^{-19}}{6.626\times10^{-34}} \approx 5.44 \times 10^{14}$ Hz .

Exercise 3: Intrinsic Silicon at 300 K

Given Parameters: Band gap energy: $E_g = 1.11$ eV, Intrinsic carrier concentration: $n_i = 1.5 \times 10^{10}$ cm^{-3}

Electron mobility: $\mu_n = 1350$ cm$^2/(V.s)$, Hole mobility: $\mu_p = 450$ cm$^2/(V.s)$

- Elementary charge: $q = 1.602 \times 10^{-19}$ C

1-Calculate the electrical conductivity σ_i

Formula: $\sigma_i = q(n_i\mu_n + n_i\mu_p) = qn_i(\mu_n + \mu_p)$

Calculation: $\sigma_i = (1.602 \times 10^{-19}) \cdot (1.5 \times 10^{10}) \cdot (1350 + 450)$

$$\sigma_i = 1.602 \times 10^{-19} \cdot 1.5 \times 10^{10} \cdot 1800$$

$$\sigma_i = 1.602 \cdot 1.5 \cdot 1800 \times 10^{-9} = 4324.86 \times 10^{-9} = 4.32 \times 10^{-6}\ (\Omega \cdot \text{cm})^{-1}$$

Answer: $\sigma_i \approx 4.32 \times 10^{-6}\ (\Omega \cdot \text{cm})^{-1}$

2- Estimate the resistivity ρ_i : $\rho_i = \frac{1}{\sigma_i}$, Calculation:$\rho_i = \frac{1}{4.32\times10^{-6}} \approx 2.31 \times 10^5\ \Omega \cdot \text{cm}$

Answer:$\rho_i \approx 2.31 \times 10^5\ \Omega \cdot \text{cm}$

3- Calculate the number of thermally generated electron-hole pairs per cm³

In intrinsic silicon, each thermally excited electron leaves behind a hole, so the number of electron-hole pairs is simply: $n = p = n_i = 1.5 \times 10^{10}$ cm^{-3}

Answer: There are 1.5×10^{10}thermally generated electron-hole pairs per cm³ at 300 K.

Exercise 4: Carrier Concentrations in Doped Silicon

Given Data:

- Donor concentration: $N_D = 10^{17}$ cm^{-3}
- Intrinsic carrier concentration: $n_i = 1.5 \times 10^{10}$ cm^{-3}
- Electron mobility: $\mu_n = 1350$ cm$^2/(V.s)$
- Hole mobility: $\mu_p = 450$ cm$^2/(V.s)$
- Elementary charge: $q = 1.602 \times 10^{-19}$ C

1-: Calculate the electron concentration n

Assumption: Full ionization of donors at room temperature.

$$n \approx N_D = 10^{17}\ \text{cm}^{-3}$$

2- Compute the hole concentration p

Use the mass action law: $np = n_i^2 \Rightarrow p = \frac{n_i^2}{n}$

Calculation:

$$p = \frac{(1.5 \times 10^{10})^2}{1.0 \times 10^{17}} = \frac{2.25 \times 10^{20}}{1.0 \times 10^{17}} = 2.25 \times 10^3\ \text{cm}^{-3}$$

$$p = 2.25 \times 10^3\ \text{cm}^{-3}$$

3-Calculate the conductivity σ

Use: $\sigma = q(n\mu_n + p\mu_p)$

Calculation:

$$\sigma = 1.602 \times 10^{-19} \cdot (10^{17} \cdot 1350 + 2.25 \times 10^3 \cdot 450)$$

We have

- $n\mu_n = 1.35 \times 10^{20}$
- $p\mu_p = 1.0125 \times 10^6 \rightarrow$ negligible compared to $n\mu_n$

So:

$$\sigma \approx \sigma_n = 1.602 \times 10^{-19} \cdot 1.35 \times 10^{20} = 21.63\ (\Omega \cdot \text{cm})^{-1}$$

$$\sigma \approx 21.63\ (\Omega \cdot \text{cm})^{-1}$$

4-Estimate the resistivity ρ

Use: $\rho = \frac{1}{\sigma} = \frac{1}{21.63} \approx 0.0462\ \Omega \cdot \text{cm}$

Then $\rho \approx 0.046\ \Omega \cdot$ cm.

Exercise 5: Fermi Level and Impurity States

Given:

- Band gap energy: $E_g = 1.11$ eV
- Donor level: $E_D = E_C - 0.045$ eV
- Temperature: $T = 300$ K

- Boltzmann constant: $k_B = 8.617 \times 10^{-5}$ eV/K
- Donor concentration: $N_D = 10^{16}$ cm^{-3}

1-: Estimate the Fermi level E_F assuming it lies halfway between E_C and E_D

$$E_D = E_C - 0.045 \text{ eV}$$

$$E_F = \frac{E_C + E_D}{2} = \frac{E_C + (E_C - 0.045)}{2} = E_C - \frac{0.045}{2} = E_C - 0.0225 \text{ eV}$$

Answer:

$$E_F = E_C - 0.0225 \text{ eV=1.11 eV−0.0225 eV=1.0875 eV}$$

2-: Calculate the probability of electron occupancy at donor level E_D

Use the Fermi-Dirac distribution: at E_D

$$f(E_D) = \frac{1}{1 + \exp\left(\frac{E_D - E_F}{k_B T}\right)}$$

Compute $E_D - E_F$:

$$E_D - E_F = (E_C - 0.045) - (E_C - 0.0225) = -0.0225 \text{ eV}$$

Compute the exponent:

$$\frac{-0.0225}{8.617 \times 10^{-5} \cdot 300} = \frac{-0.0225}{0.02585} \approx -0.87$$

replace into the formula:

$$f(E_D) = \frac{1}{1 + e^{-0.87}} \approx \frac{1}{1 + 0.42} \approx \frac{1}{1.42} \approx 0.704$$

Answer:

$$f(E_D) \approx 0.704$$

So, the probability that the donor level is occupied by an electron is approximately 70.4%.

3-: Estimate the number of ionized donors N_D^+ and the free electron concentration n

Ionized donors are those not occupied by electrons:

$$N_D^+ = N_D \cdot (1 - f(E_D)) = 10^{16} \cdot (1 - 0.704) = 10^{16} \cdot 0.296 = 2.96 \times 10^{15} \text{ cm}^{-3}$$

Assuming each ionized donor contributes one free electron:

$$n \approx N_D^+ = 2.96 \times 10^{15} \text{ cm}^{-3}$$

Answer:

- $N_D^+ \approx 2.96 \times 10^{15}$ cm^{-3}
- $n \approx 2.96 \times 10^{15}$ cm^{-3}

Exercise 6: Hole Dynamics in p-Type Silicon

Given Parameters:

- Hole concentration: $p = 5 \times 10^{17}$ cm^{-3}
- Hole mobility: $\mu_p = 450$ cm$^2/(V.s)$, Electric field: $E = 100$ V/cm
- Elementary charge: $q = 1.602 \times 10^{-19}$ C

1- Calculate the drift current density $J_{p,\text{drift}} = qp\mu_p E$

$$J_{p,\text{drift}} = (1.602 \times 10^{-19}) \cdot (5 \times 10^{17}) \cdot 450 \cdot 100$$

We have

- $5 \times 10^{17} \cdot 450 \cdot 100 = 2.25 \times 10^{22}$
- Multiply by q:

$$J_{p,\text{drift}} = 1.602 \times 10^{-19} \cdot 2.25 \times 10^{22} = 3.6045 \times 10^3 \text{ A/cm}^2$$

Answer:

$$J_{p,\text{drift}} \approx 3604.5 \text{ A/cm}^2$$

2- Estimate the hole drift velocity $v_d = \mu_p E$

Calculation:

$$v_d = 450 \cdot 100 = 4.5 \times 10^4 \text{ cm/s}$$

Answer:

$$v_d = 45\,000 \text{ cm/s}$$