

Travaux Dirigés (Série N° 1)

Exercise N° 1 :

For $n=2$ write the expression explicitly $S_1 = x_i y_i$.

For $n=3$ write the expression explicitly $S_2 = A_{ijk} B_{ij}$.

For $n=2$ write the expression explicitly $S_3 = a_{ijk} b_{ij} c_k$.

Exercise N° 2 :

Use the summation convention to write the following expressions, specifying the value of n in each case :

$$A = a_{j1} x_1 + a_{j2} x_2 + a_{j3} x_3$$

$$B = b_{11} d_{11} + b_{12} d_{12} + b_{13} d_{13} + b_{14} d_{14}$$

$$C = C^1_{11} + C^2_{12} + C^3_{13} + C^4_{14} + C^5_{15}$$

Exercise N° 3 :

Let $e_1(1,0,0)$, $e_2(0,1,0)$, $e_3(0,0,1)$ be a basis of R^3 ; we define the vectors A and B by:

$$A = 4 e_1 + 2 e_2 + e_3$$

$$B = 3 e_1 - 5 e_2 + 2 e_3$$

1°) determine the scalar product $(A \cdot B)$ of the vectors A and B .

2°) determine the vector product $(A \wedge B)$ of the vectors A and B .

3°) determine the tensor product $(A \otimes B)$ of the vectors A and B .

4°) determine the angle between the vectors A and B .

Same thing for vectors

$$A = (2, 1, 3)$$

$$B = (1, 2, 4)$$

Exercise N° 4 :

Show that : $I + (*n)^2 = {}^t(n) (n)$ with ${}^t n = (n_1, n_2, n_3)$ a unit vector

Exercise N° 5 :

We consider the basis $\{f_j\}$ of R^3 defined by the following vectors:

$$f_1 = (1, 0, 0) \quad f_2 = \left(\frac{1}{2}, \frac{3}{2}, 0\right) \quad f_3 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

and the vector : $A = 8 e_1 + 6 e_2 + 3 e_3$

1°) is the basis $\{f_j\}$ orthonormal ?

2°) determine the projections of vector A on the vector axes f_1 , f_2 et f_3 .

3°) determine the components of vector A in the basis (f_1, f_2, f_3) .

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Exercise N° 6 :

We consider the vector function defined by $F=(u_1, u_2, u_3)$ such that :

$$u_1 = 8 x_1 + 6 x_2$$

$$u_2 = 10 x_1 - 8 x_2$$

$$u_3 = 12 x_3$$

1°) Determine the tensor $\text{Grad} F$.

2°) Determine the transpose of the tensor $\text{Grad} F$ denoted $\text{Grad}^t F$.

3°) Determine the tensor A such that $A = (\text{Grad} F + \text{Grad}^t F)/2$.

4°) Determine the eigenvalues and eigendirections of the tensor A

Exercise 07

Determine the eigenvalues and eigenvectors of the following matrices :

$$M_1 = \begin{bmatrix} 4 & 3 & 0 \\ 3 & -8 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 13 & 0 & 0 \\ 0 & 12 & -2 \\ 0 & -2 & 6 \end{bmatrix} \quad \text{et} \quad M_3 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -6 & 0 \\ 1 & 0 & 4 \end{bmatrix}$$

Exercise N° 8 :

a) Demonstrate the following properties of the gradient, for scalar fields $f(x_1, x_2, x_3)$ and $g(x, y, z)$:

$$\text{Grad}(f+g) = \text{Grad}(f) + \text{Grad}(g)$$

$$\text{Grad}(af) = \alpha \text{Grad}(f)$$

$$\text{Grad}(f.g) = g \text{Grad}(f) + f \text{Grad}(g)$$

b) Let the scalar function be : $f(x_1, x_2, x_3) = x_1^2 x_2 (2 + x_2 x_3^3)$

Determine the Gradient of f .

Determine the Laplacian of f .