Tutorial exercises on Complex Numbers

Exercice 1

Let's consider z and z' as complex numbers where: $z = 3 + \sqrt{3}\, i$ and $z' = -1 + 2\, i$

Write the following complex numbers in algebric form:

$\boldsymbol{z_1 = z - \bar{z}'}$; $\boldsymbol{z_2 = z \,.\, \bar{z}}$; $\boldsymbol{z_3 = z^2}$

$\boldsymbol{z_4 = z'^3}$; $\boldsymbol{Z_5 = \frac{z}{z'}}$

Exercice 2

Write the following complex numbers in algebric form (a + ib):

1. Modulus of a complex number is 2 and the argument π/3.
2. Modulus of a complex number is 3 and the argument −π/6.

Exercice 3

Evaluate the following operations:

1. (3 + 2i) (1 − 3i).
2. Product of the complex number (modulus 2 and argument π/3) by the complex number of (modulus 3 and argument −5π/6.
3. $\frac{3+2\,i}{1-3\,i}$

Exercice 4

For a complex number z : z=x+i y, x and y are real numbers, $z \neq -1$, let consider the complex z′ defined by:

$$z' = \frac{Z - i}{Z + 1}$$

1. We note $Z' = x' + i\, y'$, where x' et y' are real numbers. Express x′ et y′ as a function of x and y ?
2. Find the set of points M with affix z such z′ is real?

Exercice 5

Find the modulus, argument and write the exponential form of each of the following complex numbers:

$z_1 = \sqrt{6} - i\sqrt{2}$, $z_2 = -\frac{1}{2} - i\frac{1}{2}$ and $z_3 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

Deduce the modulus and argument of $z_1 . z_2$, $z_1 . z_3$ et $(z_2)^2$