

Guided Work Series Number 1 (Simple and Multiple Integrals)

Exercise 1 Compute the following integrals:

$$\begin{array}{ll} 1) \int \sqrt{3x} \log x dx & 3) \int x^2 e^x dx \\ 2) \int_0^1 (7x^2 - e^x) dx & 4) \int x \sqrt{x^2 + 1} dx \end{array}$$

Exercise 2 Compute the following limits:

$$\begin{array}{ll} 1) \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{\sqrt{x^2 y^2 + 1} - 1}{x^2 + y^2} & 3) \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1} \\ 2) \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 2}} \frac{\sin xy}{x^2 + y^2} & 4) \lim_{\substack{x \rightarrow \infty \\ y \rightarrow k}} \left(1 + \frac{y}{x}\right)^x \quad k \in \mathbb{R} \end{array}$$

Exercise 3 Study the continuity of the function f at the point $(x_0, y_0) = (0, 0)$:

$$f(x, y) = \begin{cases} \frac{x^3 + y^3}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Then the function g at the point $(x_0, y_0) = (0, 1)$:

$$g(x, y) = \begin{cases} x + y, & (x, y) \neq (0, 1) \\ 0, & (x, y) = (0, 1) \end{cases}$$

Exercise 4 Compute the following multiple integrals:

$$\begin{array}{ll} 1) \iint_D (xy + y^2 + 1) dx dy & D = \{(x, y) \in \mathbb{R}^2 : 1 \leq x \leq 2, 0 \leq y \leq 3\} \\ 2) \iint_D (xy e^{x+y}) dx dy & D = \{(x, y) \in \mathbb{R}^2 : 1 \leq x \leq a, 1 \leq y \leq b\} \\ 3) \iint_D (x e^{xy}) dx dy & D = \{(x, y) \in \mathbb{R}^2 : 1 \leq x \leq 2, 1 \leq y \leq 2\} \\ 4) \iint_{[0,1] \times [0,1]} \frac{1}{x + y + 1} dx dy \end{array}$$

Exercise 5 Compute the following double integral:

$$\iint_D (xy e^{x+y}) dx dy, \quad D = \{(x, y) \in \mathbb{R}^2 : x \geq 0, y \geq 0, x^2 + y^2 \leq 1\}$$

Exercise 6 Compute the area of Δ defined as follows:

$$\Delta = \left\{ (x, y) \in \mathbb{R}^2 : \frac{x}{a^2} + \frac{y}{b^2} \leq 1 \right\}, \quad (a, b) \neq (0, 0)$$

Exercise 7 Compute the following triple integral:

$$\iiint_D x^a y^b z^c dx dy dz, \quad (a, b, c) \in (\mathbb{R}_+^*)^3$$

$$D = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq xy\}$$

Exercise 8 Compute the following triple integral:

$$\iiint_V x y z dx dy dz,$$

$$V = \left\{ (x, y, z) \in \mathbb{R}^3 : \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1 \right\} \quad (a, b, c) \neq (0, 0, 0)$$