FOUNDATIONS AND EARTH STRUCTURES

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April 2025

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CHAPTER 1: PLASTICITY AND SHEAR STRENGTH OF SOILS

1. Specific Objectives for Chapter I

Fundamental

- Define key soil mechanics terms and sources of shear strength in soils
- Explain the Mohr-Coulomb failure criterion and its role in determining soil shear strength.
- Construct Mohr's Circle to analyze 2D stress states and identify principal stresses (σ_1, σ_3).
- Determine shear strength parameters (c and φ) from laboratory test results.
- Interpret stress-strain curves from triaxial tests to identify failure conditions.

2. Introduction

The consequences of exceeding theshear strength of soils in civil engineering are seen in foundation failures, landslides, etc.

Consequences of exceeding soil shear strength

Definition: Sources of Shear Strength

The shear strength of a soil primarily arises from three sources:

1. Resistance due to particle interlocking.
2. Frictional resistance between individual soil grains, which can include sliding friction, rolling friction, or both.
3. Adhesion between soil particles, known as "cohesion."

Granular soils (e.g., sands) derive their shear strength from the first two sources, while cohesive soils (e.g., clays) rely on the second and third sources. Highly plastic clays may derive their shear strength solely from cohesion.

3. Principal Stresses — Mohr's Circle

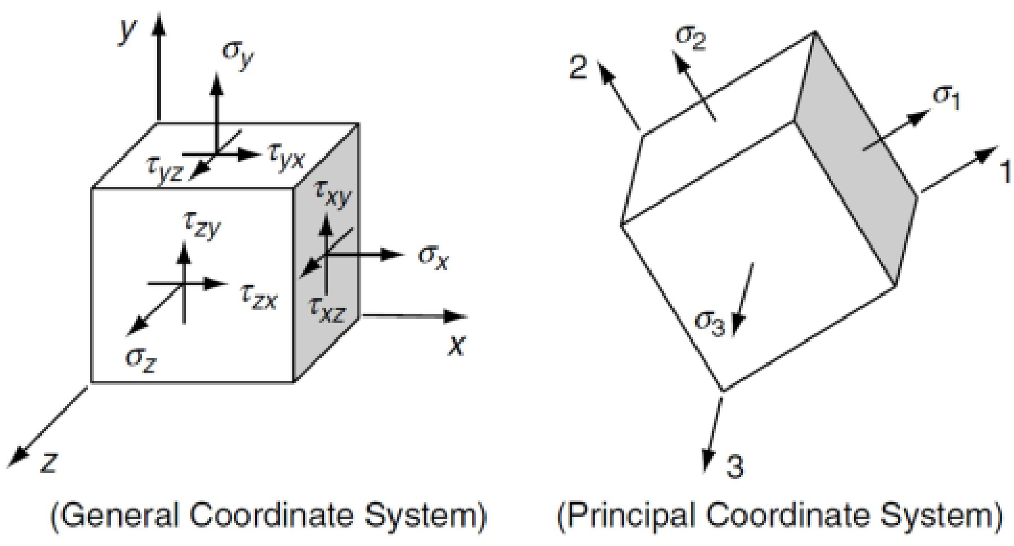
Fundamental

The state of stress at a point is typically represented by a cube with axes in the x, y, and z directions. The stress vector on each face is decomposed into a normal stress (e.g., in the x-direction) and two shear stresses (e.g., in the y and z directions).

For equilibrium, shear stresses on perpendicular planes must be equal ($\tau_{xy}=\tau_{yx}$).

Definition

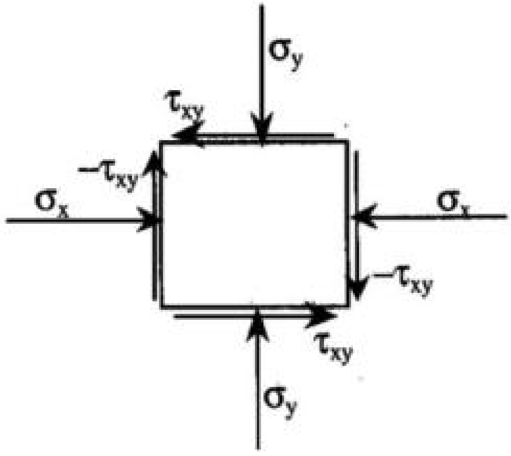
- σ_{xx} : Normal stress on the plane perpendicular to the x-direction.
- τ_{xy} : Shear stress on the plane perpendicular to the x-direction and acting in the y-direction.
- τ_{xz} : Shear stress on the plane perpendicular to the x-direction and acting in the z-direction.



Stresses on an elemental cube

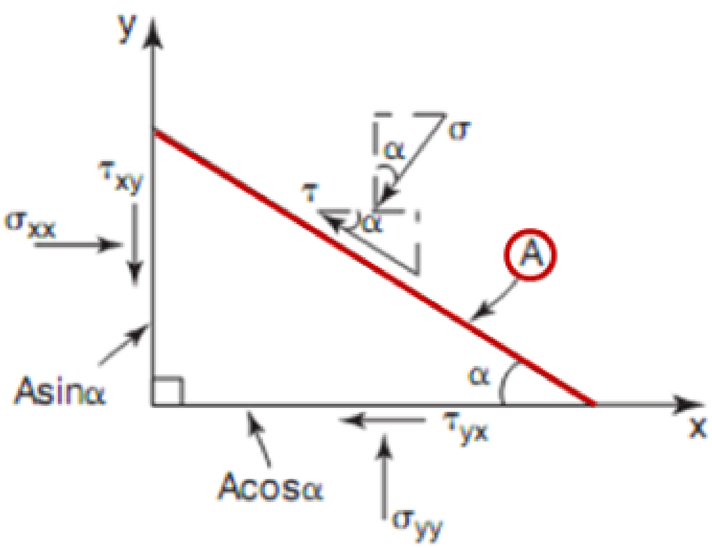
Note:Sign Convention

Compressive stresses are considered positive, as they are most common in soil mechanics. The sign convention for shear stresses is shown in the figure below .



Sign convention for positive stresses

For two perpendicular planes, the normal and shear stresses on any other plane can be related to the stresses on the perpendicular planes as follows:



Wedge in equilibrium under normal and shear stresses

From equilibrium conditions, the following equations are derived:

$$\sigma_y A\cos\alpha + \tau_{xy} A\sin\alpha + \tau A\sin\alpha - \sigma A\cos\alpha = 0$$

$$\sigma_x A\sin\alpha - \tau_{yx} A\cos\alpha - \tau A\cos\alpha - \sigma A\sin\alpha = 0$$

Where A in the surface of the inclined face.

$$\sigma = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\alpha - \tau_{xy}\sin 2\alpha$$

$$\tau = \frac{\sigma_y - \sigma_x}{2}\sin 2\alpha - \tau_{xy}\cos 2\alpha$$

If the planes perpendicular to the x and y directions are principal planes (zero shear stress), the equations simplify to:

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\alpha$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2}\sin 2\alpha$$

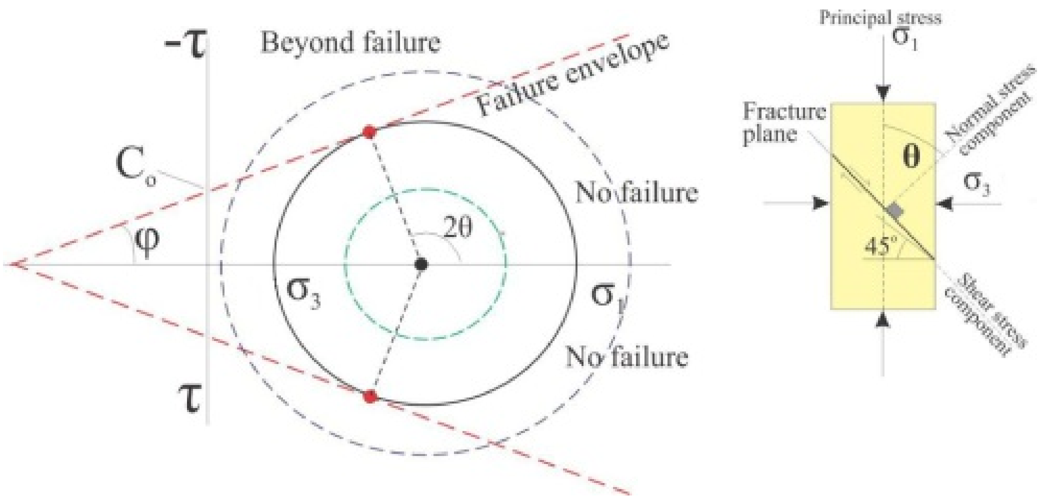
Where σ_1 and σ_3 are the major and minor principal stresses, respectively.

These equations yield the relationship:

$$[\sigma - \frac{\sigma_1 + \sigma_3}{2}]^2 + \tau^2 = (\frac{\sigma_1 - \sigma_3}{2})^2$$

This is the equation of a circle (Mohr's Circle) with center at ($\frac{\sigma_1 + \sigma_3}{2}$,0) and radius $\frac{\sigma_1 - \sigma_3}{2}$

An example of Mohr's Circle for a triaxial test at failure is shown in the figure below



Mohr's Circle for a triaxial test

4. Mohr-Coulomb Failure Criterion

Fundamental

Mohr's theory of failure postulates that rupture begins at a point in a material when the shear stress on the failure plane depends only on the normal stress, i.e., $\tau = f(\sigma)$.

The corresponding curve is called the intrinsic curve. The Mohr-Coulomb failure criterion (shear strength of a soil) is expressed as:

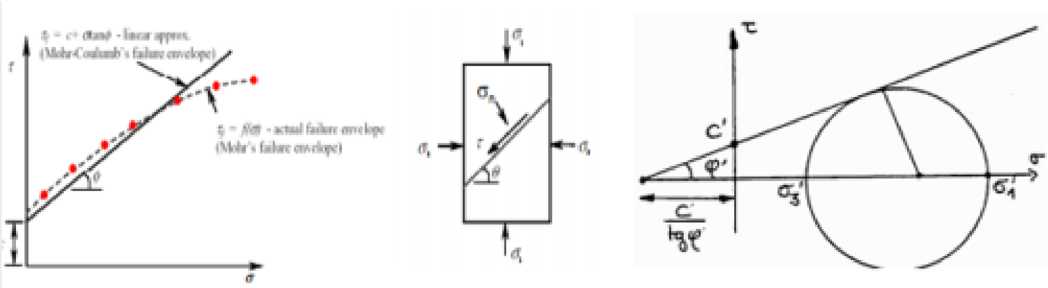
$\tau = c' + \sigma' \tan\phi'$

Where:

- c' : Cohesion (shear strength under zero normal stress).

- ϕ' : Angle of internal friction.

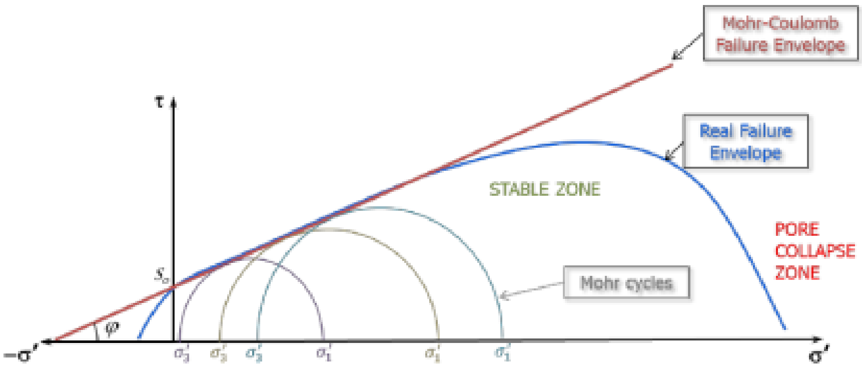
- $H = c'/\tan\phi$: Soil tensile strength.



Mohr-Coulomb failure criterion

Note

The failure envelope curve is the envelope of all Mohr circles drawn for failure under different loading conditions.



failure curve as the envelope of Mohr's Circles

5. Practical Determination of Soil Characteristics c and φ

5.1. Shear Behavior: Short-Term and Long-Term

Reminder

Soils consist of three phases: **solid, liquid, and gas**. It is essential to distinguish between **effective stresses** and **total stresses**.

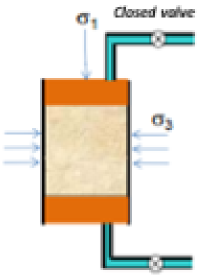
Definition

Granular Soil with high permeability allows rapid drainage under external loads. Shear behavior and strength are governed by the solid skeleton.

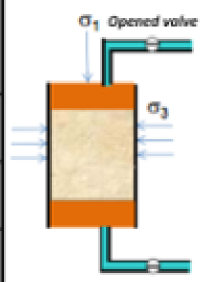
Fine Soils: with low permeability results in slow drainage. Two extreme behaviors are observed:

- **Short-Term Behavior**: Undrained conditions, where water plays a significant role in mechanical behavior.
- **Long-Term Behavior:** Drained conditions, where pore water pressures dissipate, and behavior is governed by the solid skeleton.

$$\tau_f = c + \sigma_n \tan\phi$$



Undrained (short-term behavior of saturated fine soils)	Drained (long-term behavior of saturated fine soils)
c_u	c'
ϕ_u	ϕ'
σ (Total stress)	σ' (Effective stress)



Drained and undrained conditions

Several types of tests are used to determine the plasticity characteristics; these include:

In-situ measurement tests (vane shear test, penetrometer, etc.),

Laboratory tests (unconfined compression test, direct shear test or Casagrande shear box, triaxial compression test).

5.2. Laboratory Experimental Tests

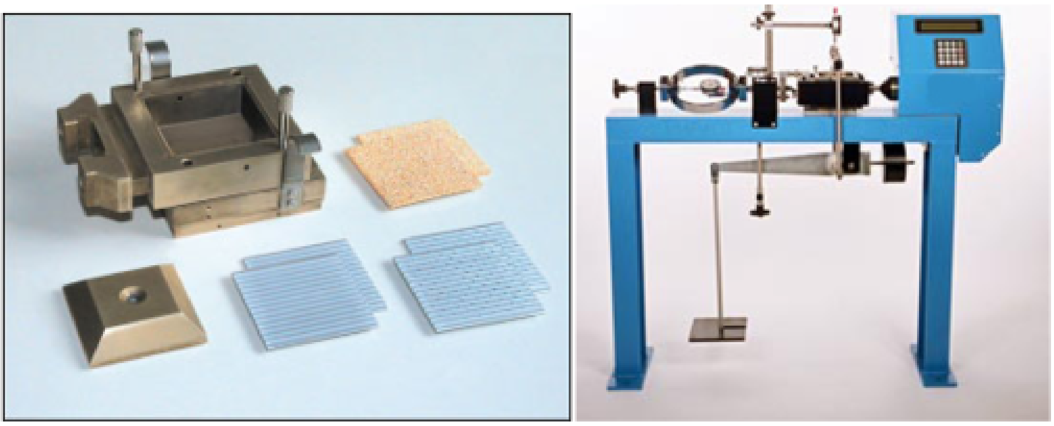
Method:Shear Box Test (Casagrande Box)

Test Procedure

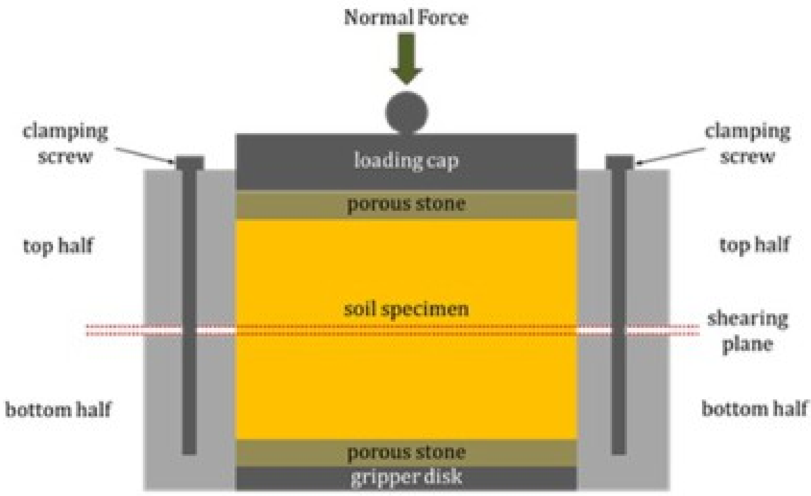
The sample is placed between two half-boxes:

- An upper half-box (C_1), which can slide horizontally,
- A lower fixed half-box (C_2).

The soil is placed between two porous stones, allowing for drainage. Alternatively, the porous stones can be replaced with solid plates, preventing drainage—at least in theory.



Shear box apparatus



Casagrande shear box diagram

The device includes a loading mechanism that applies a vertical load (P) through a piston.

The test involves pulling the upper half-box (C_1) horizontally to shear the soil along the failure plane. The horizontal force (F) is measured as a function of displacement (Δl) (see figure below). The test is performed at a controlled speed (V).

Key Parameters:

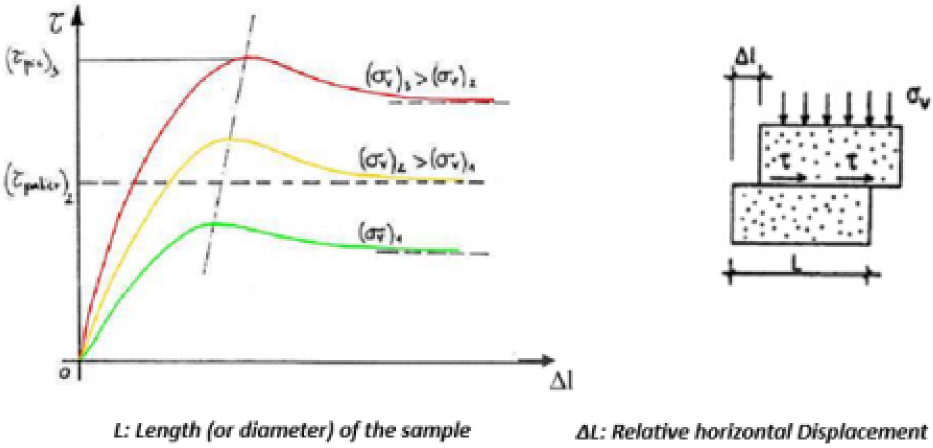
A = Cross-sectional area of the sample along plane π

$\sigma = P/A$ = Normal stress applied to the sample

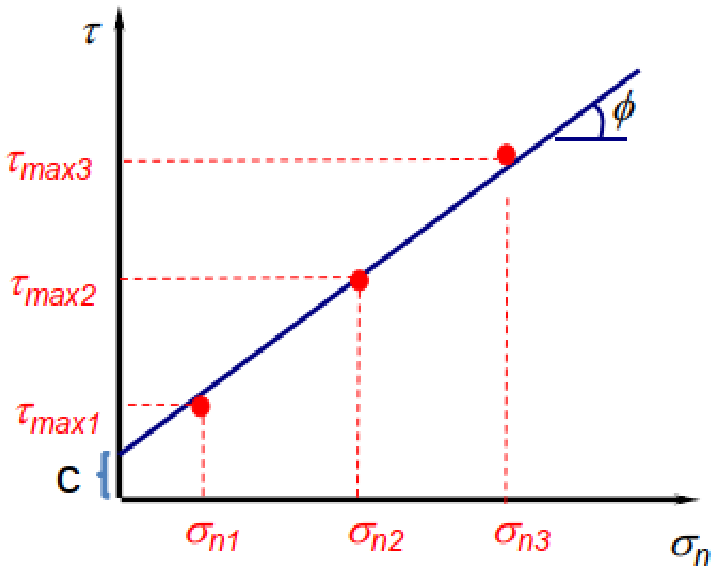
$\tau = F/A$ = Shear strength at failure

Note

If this test is repeated on multiple specimens of the same soil under different normal stresses , the intrinsic curve (failure envelope) of the soil can be determined by plotting the measured shear stresses (τ) against normal stresses(σ) on a Coulomb diagram (τ-σ plot) (as shown in the figure).



Stess-displacement curve



Determination of c and φ

Definition

The values of cohesion (c) and friction angle (φ) depend on the test conditions, including shear rate and drainage:

Shear Rate:

- Quick shear test (for undrained conditions): 0.6 mm/min (~15 minutes per test)
- Slow shear test (for drained conditions): 0.03 mm/min (~3 hours minimum per test)

Test Types:

1. **UU Test** (Unconsolidated Undrained)

  - Evaluates short-term soil behavior (immediate stability).
  - No consolidation or drainage allowed.

2. **CU Test** (Consolidated Undrained)
   - Measures the undrained cohesion (Cu) variation with preconsolidation stress.
   - Takes up to 4 days (relatively short).

3. **CD Test** (Consolidated Drained)
   - Represents long-term soil behavior (fully drained conditions).
   - Can take up to 2 weeks in some cases

Advice

Always match test conditions (drained/undrained) to field scenarios. For example, use UU tests for short-term stability in clayey slopes.

Method:Triaxial Shear Tests

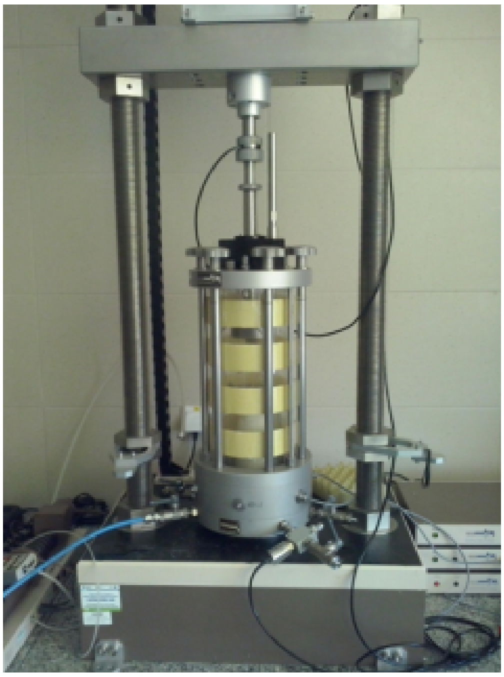
Around 1930, Casagrande developed a new compression test to overcome the limitations of the direct shear test. This led to the creation of the triaxial test, which is now widely used in geotechnical engineering.

Test Setup:

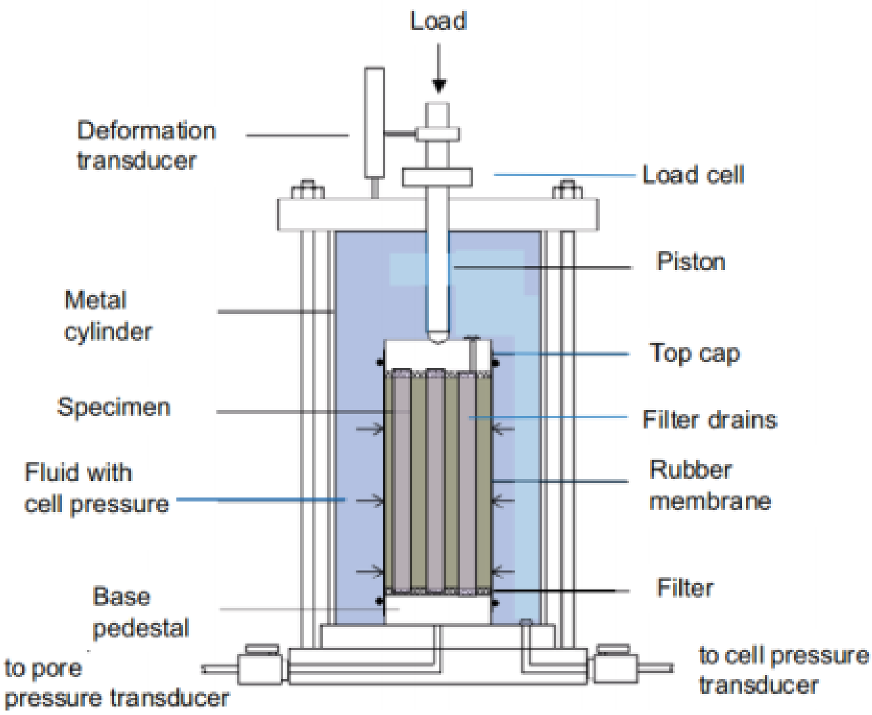
- The soil specimen is cylindrical, typically with a slenderness ratio (H/D) of 2.
- It is placed inside a pressure cell (triaxial cell) filled with a confining fluid.
- The specimen is enclosed in a flexible, impermeable membrane to prevent fluid penetration while allowing deformation.
- One or both ends (depending on the setup) are in contact with a porous stone to facilitate drainage.

Key Features:

- The test allows for controlled stress conditions (σ_1, $\sigma_2 = \sigma_3$).
- It can simulate various drainage conditions (UU, CU, CD).
- Provides more accurate and versatile results compared to the direct shear test.



Triaxial test apparatus



Triaxial test principle

There are many types of triaxial tests due to the possible combinations related to drainage and the type and sequence of stress applications. However, nearly all triaxial tests begin with a consolidation phase followed by a shearing phase.

Consolidation Phase:

The consolidation phase is designed to bring the sample to a desired stress state, often intended to match the stress conditions the sample would encounter in the field under project conditions. During the consolidation phase, the cell pressure is increased to a chosen confining pressure value. This pressure confines the sample hydrostatically and represents the minor principal stress (σ_3).

During this consolidation phase, drainage may or may not be allowed:

If drainage is not allowed, the term "unconsolidated" is used to describe the triaxial test, and the letter U is used in the acronym.

If drainage is allowed and the pore water pressure in the sample generated by the application of σ_3 can dissipate to zero, the term "consolidated" is used to describe the test, and the letter C is used in the acronym.

Shearing Phase:

During the shearing phase of the test, the vertical load (Q) on the sample is gradually increased, and the stress in the vertical direction rises. This stress is the major principal stress (σ_1):

$$\sigma_1 = \sigma_3 + F/A$$

Where:

σ_3 = Confining pressure

F = Vertical load

A = Cross-sectional area of the sample

If drainage is not permitted during the shearing phase, the term "undrained" and the letter U are used.

If drainage is allowed and the excess water (pore pressure) is maintained at zero (very slow loading), the term "drained" and the letter D are used.

Thus, the following triaxial tests are possible:

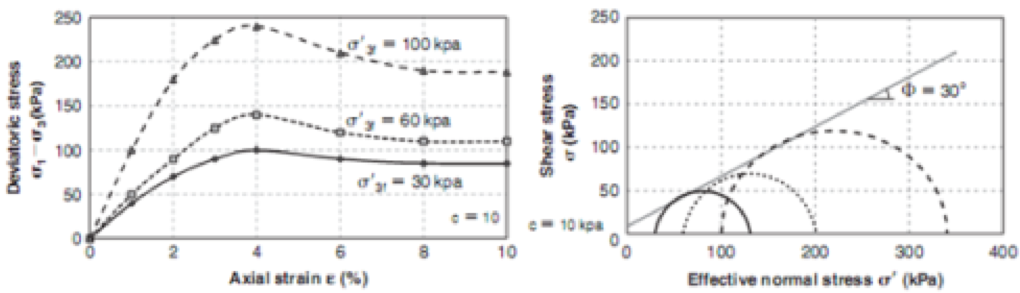
UU Test: Unconsolidated Undrained test

CU Test: Consolidated Undrained test

CD Test: Consolidated Drained test

CD and CU tests with pore pressure measurements are used to obtain the effective shear strength parameters (c' and φ').

The result of a triaxial test (as shown below) is a stress-strain curve that typically relates the deviatoric stress ($\sigma_1 - \sigma_3$) to the vertical strain ($\varepsilon = \Delta h/h$), where h is the initial height of the sample and Δh is the change in height of the sample.



Example of triaxial test results

Method:Unconfined Compression Test (UCT)

The Unconfined Compression Test (UCT) is a highly simplified form of triaxial testing in which the specimen is not subjected to lateral confining pressure ($\sigma_3 = 0$) during compression.

Test Procedure:

- A cylindrical soil specimen is placed on the platform of a compression machine.
- The specimen is compressed at a constant strain rate until failure.
- This test is only applicable to fine-grained, cohesive soils (e.g., clays).

Key Characteristics:

- The UCT is essentially a UU (Unconsolidated Undrained) test with $\sigma_3 = 0$.
- For saturated soils, the unconfined compressive strength (Rc) is determined, which is equal to the deviatoric stress at failure.
- Since $\sigma_3 = 0$, Rc = 2Cu, where Cu is the undrained shear strength.

$$R_c = 2C_u$$

II

CHAPTER 2: STABILITY OF SLOPES AND EMBANKMENTS

1. Specific Objectives for Chapter II

Fundamental

- Classify the main types of slope movements based on their mechanisms.
- Apply the Fellenius' and Bishop's methods to calculate safety factors (F) for circular failure surfaces.
- Differentiate between failure types and soil conditions.
- Compare the accuracy and limitations of the Fellenius and Bishop methods.
- Evaluate the stability of natural and artificial slopes by interpreting safety factor results.

2. Introduction

The stability of slopes is a concern for both natural slopes and artificial embankments. Failures can result in major hazards.

Definition

The failure mechanisms of natural slopes can be categorized as follows:

- **Collapse**
- **Planar, rotational, or complex sliding**
- **Mudflows or clay flows**
- **Creep**

Artificial embankments are primarily affected by sliding and sometimes by creep. They can be classified based on the type of structure:

- **Cut or fill slopes**
- **Retaining structures against deep-seated sliding**
- **Earth dams and levees**

Reminder

Any stability study must be preceded by a detailed geological and geotechnical investigation to identify factors favoring instability, such as:

- Local heterogeneities
- Favorable dip for sliding
- Cracks
- Water circulation

Since these factors are not always quantifiable, estimating the actual safety factor against the risk of failure is challenging, regardless of the approach used.

However, extensive experience has been gained in both calculation methods and stabilization techniques, allowing slope stability problems to be resolved with reasonable reliability today.

This chapter highlights the mechanisms leading to the failure of certain slopes or embankments. The most common calculation methods for assessing slope and embankment stability are described using the concept of a global safety factor.

3. Description of the Main Types of Land Movements

Land movements vary widely in nature (landslides, rockfalls, mudflows, underground collapses, subsidence, soil swelling/shrinkage, etc.) and scale (some slides can involve tens of millions of cubic meters).

Their spatial distribution is influenced by:

- Topography
- Geology (rock type, fracturing, hydrogeology)
- External loading (physical environment)

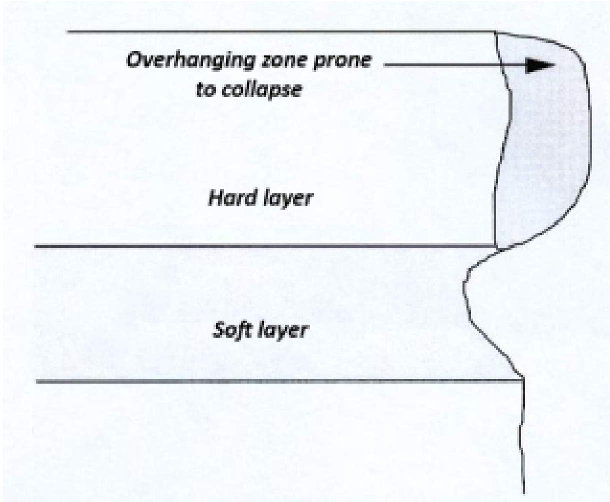
They occur not only in mountainous and coastal regions but also in areas with high densities of underground voids (natural or mined) and in clay soils sensitive to moisture changes. Their occurrence is strongly linked to climatic variations (heavy rainfall, snow melt, drought) but can also be triggered by seismic activity or human actions

3.1. Collapses (Rockfalls)

Definition

Collapses involve the sudden fall of rock masses. A typical example is the collapse of a cliff section, which can result from:

- Erosion of underlying layers
- Evolution of pre-existing discontinuities (e.g., cracks)



Collapse due to erosion of underlying layers

Example

As an example, the photo shows the rockfall at El Kantara cliffs, triggered by the widening of fissures.

Collapse of the El Kantara cliffs caused by crack evolution

3.2. Landslides

Definition

Landslides affect soils and typically involve large masses of terrain detaching and sliding down a slope or embankment. They can be triggered by:

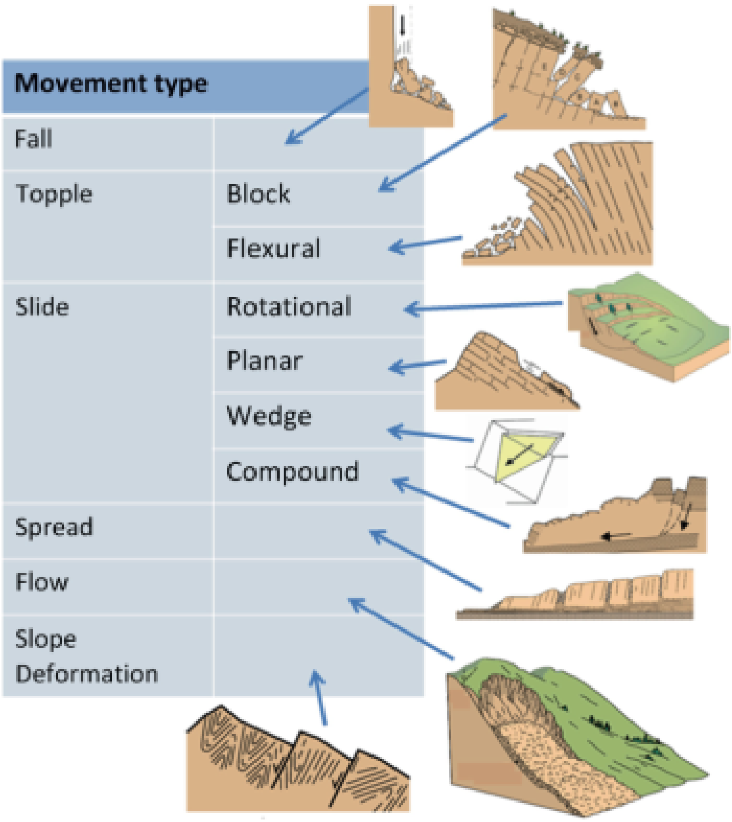
- Natural events (heavy rain, bank erosion, mechanical degradation, earthquakes)
- Human actions (earthworks, deforestation, dam/levee construction)

The speed of failure can vary greatly. Sometimes, the collapse is preceded by warning signs (e.g., cracks forming, bulging ground), but it can also occur suddenly (within seconds to minutes). The slip surface is often planar or circular.

Slide caused by earthworks (Jijel Bypass, 2008)

a. Planar Slides

The failure surface is mostly planar, following a thin layer with poor mechanical properties, often prone to water seepage (e.g., a permeable loose surface layer overlying an inclined bedrock stratum).



Schematic Representation of Different Types of Landslides

b. Rotational Slides (Single)

This is the most common type of landslide. The failure surface has a simple shape and can be approximated as a cylindrical section.

Example

The first photo shows a landslide that occurred in 2009 on the national road RN 29 in Boumerdes Province. Additionally, the second photo illustrates an embankment slope failure that occurred in 2006 during fill repair works on a section of the RN 79A roadway in Mila Province.

Rotational slide on RN 29 (Boumerdes, 2009)

Embankment slide during RN 79A repair works (Mila, 2006)

Definition: c. Complex Rotational Landslide

This type involves multiple nested rotational slide. The initial failure at the slope toe removes lateral support for upslope material, triggering a retrogressive succession of rotational failures that progress uphill through a "domino effect."

3.3. Mudflows

Definition

Mudflows behave more like fluid-transported materials than slides. They often result from slide materials mixing with large amounts of water (e.g., from rivers or heavy rain). Characteristics:

- Loose, heterogeneous clay-rich materials
- Triggered by exceeding a critical water content, creating semi-fluid behavior
- Long travel distances and potentially high velocities

Example

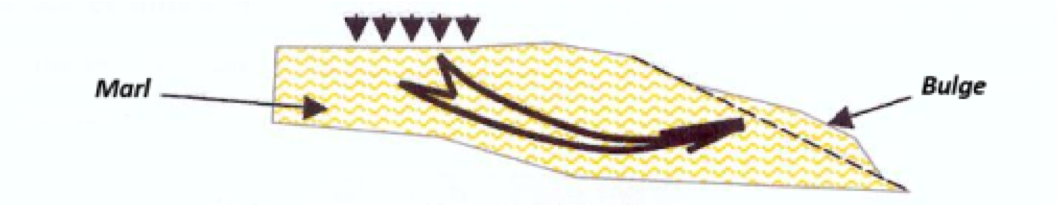
The Photo shows the 2005 clay-rich mudflow on National Road RN 52 in Mila Province. The catastrophic 2003 Beb El-Oued mudflow remains unforgettable, destroying multiple homes and claiming numerous lives.

Mudflow on RN 52 (Mila, 2005)

3.4. Creep

Definition

Creep is characterized by slow, continuous ground movement occurring at very low velocities. In cases of creep, identifying a distinct failure surface is typically difficult. Unlike landslides, this movement occurs without changes in applied stresses - the material is essentially in a near-failure state. This type of movement may either stabilize or progress to complete failure (As shown in the figure).



Example of Creep

Example

The photo illustrates the creep of a large land area toward Ben Haroune Dam, resulting in the collapse and loss of numerous structures.

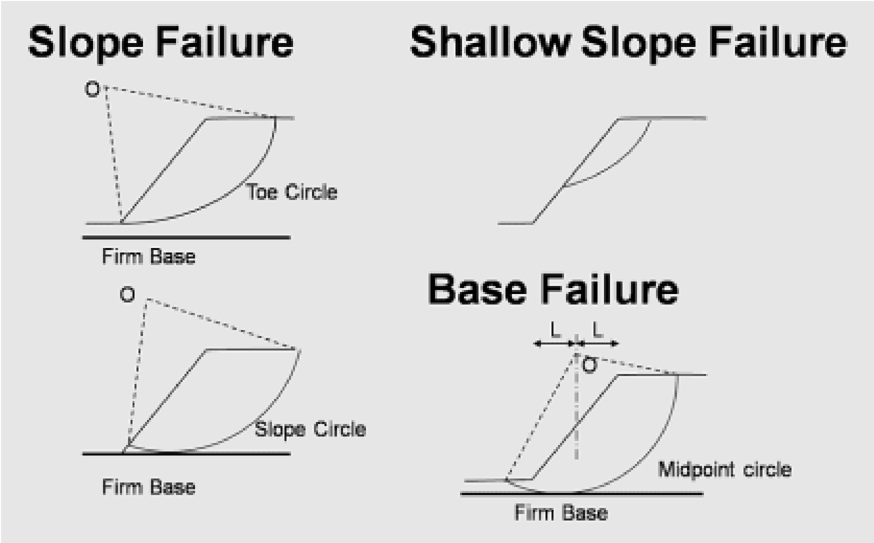
Creep of Chigara village toward Ben Haroune Dam

3.5. Cut Slopes and Fill Slopes on Non-Compressible Soils

Definition

Failure typically occurs as rotational slips with circular failure surfaces, categorized as follows:

- **Slope circles:** Occur in heterogeneous soils, with the circle base at a stronger layer.
- **Toe circles:** Most common.
- **Deep circles:** Occur when weak soil lies below the toe.



Different types of rotational slides

3.6. Embankment Slopes on Compressible Soils

Definition

Failures in compacted earth embankments (e.g., road fills) overlying soft clay, silt, or peat deposits typically exhibit the following characteristics:

Failure Mechanism:

- Rotational slip surfaces tangent to the base of the compressible layer when relatively thin
- Critical dependence on the underlying weak stratum thickness

Long-Term Stability Concerns:

Failures in compacted soil embankments (e.g., road fills) overlying soft clay, silt, or peat deposits typically exhibit the following characteristics:

Failure Mechanism:

Rotational slip surfaces tangent to the base of the soft layer when relatively thin

Potential creep-induced foundation deformation when the safety factor is marginally above 1.0, leading to:

- Excessive embankment settlement
- Lateral heaving of the soft stratum
- Strength reduction in the embankment material

Example

Case Example:

The current road embankment construction across Chott El Hodna's sebkha demonstrates these challenges. The firsrt photo shows significant sub-grade deformations during low-water periods. Despite surface water presence (second photo)), construction was enabled through:

- Filtration geotextiles
- Reinforcement geogrids (visible in photo)

Embankment on compressible soil (Chott El Hodna)

Junction of embankment sections during construction

3.7. Earth Dams and Embankments

Definition

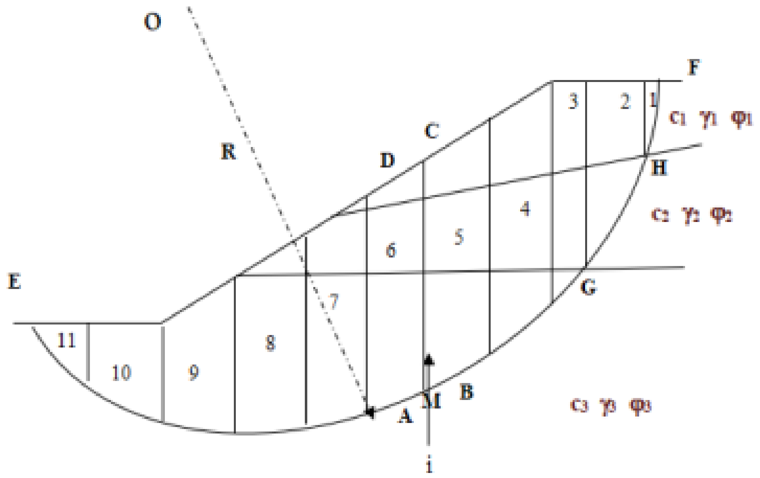
The stability analysis of upstream and downstream slopes constitutes a critical component in the design of earth dams. The structural integrity of these constructions must be verified under various loading conditions, with particular consideration given to the pore water pressure distribution within the dam body.

4. Circular Failure Stability with Global Safety Factor

4.1. Fellenius' Slice Method

Method:a. Stability for a Given Circle

A slope intersecting multiple soil layers (properties: C_i , γ_i , φ_i). The stability analysis assumes plane strain conditions (two-dimensional problem).



Division of a slope into elementary slices.

For an arbitrary slip circle with center O and radius R, we evaluate the safety factor against sliding. The method involves dividing the potentially unstable soil mass above the arc EMF into vertical slices.

Key Observations:

- The slicing should ensure that any intersection between the slip circle and layer boundaries coincides with slice boundaries
- Field experience demonstrates that satisfactory accuracy can be achieved with a limited number of slices

For any given slice ABCD with total weight W_i , the acting forces include:

Weight components:

Normal component (N_i) perpendicular to the circular arc AB

Tangential component (T_i) parallel to AB

Boundary forces:

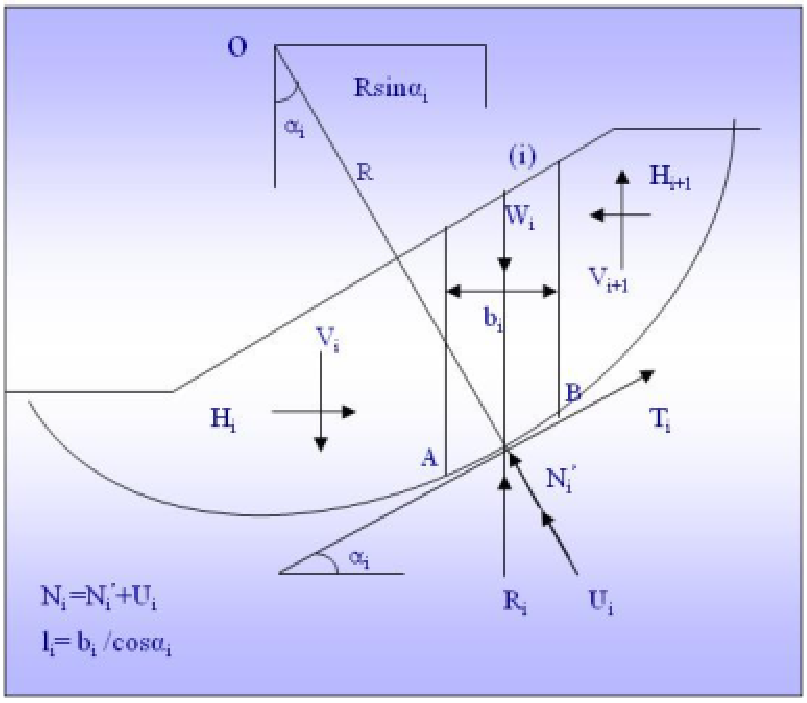
Subgrade reaction (R_i) along arc AB

Neighboring slice reactions:

Horizontal components (H_i , H_{i+1})

Vertical components (V_i , V_{i+1})

Pore water pressure force (U_i) acting along AB



Forces acting on a typical slice ABCD.

Fellenius introduced a key hypothesis that significantly simplifies calculations by considering that the internal forces H_i, H_{i+1}, V_i, and V_{i+1} become self-balancing when analyzing the complete slice system. Their moment contributions are neglected, yielding:

$$H_i - H_{i+i} = 0$$

$$V_i - V_{i+i} = 0$$

Consequently, only two forces remain active on arc AB:

The soil weight W_i

The resultant reaction R_i (where $W_i = R_i$)

leaving only $W_i = R_i$

Driving moment, Generated solely by tangential component T_i is equal to $T_i \cdot R$ (Normal component N_i passes through center O, producing zero moment).

Resisting moment: Maximum available shear resistance along AB

According to Coulomb's failure criterion:

$$(R_i)_t = C_i \cdot AB + N_i \cdot \tan\varphi_i$$

Sum of moments for all slices:

$$\sum_{i=1}^{I=m} R(C_i \cdot AB + N_i \cdot \tan\varphi_i)$$

Where:

m: Total number of slices, c_i' and φ_i': Effective shear strength parameters (cohesion and friction angle) of the layer containing arc AB.

The failure surface being bounded by points E and F, the global safety factor F is defined by the ratio:

$$F_s = \frac{\sum_{EF} Maximum\ Resisting\ Moments}{\sum_{EF} Driving\ Moments}$$

The factor of safety (F) is then defined as:

$$F_s = \frac{\sum_{i=1}^{i=m} (C_i \cdot AB + N_i \cdot \tan\varphi_i)}{\sum_{i=1}^{i=m} T_i} = \frac{\sum_{i=1}^{i=m} (C_i \cdot (\frac{b}{\cos\alpha}) + W_i \cdot \cos\alpha \tan\varphi_i)}{\sum_{i=1}^{i=m} W_i \cdot \sin\alpha}$$

Important Notes:

In the safety factor (F_s) formula, $\sum_{i=1}^{i=m} T_i$ represents an algebraic sum.

F_s can be directly applied to mechanical properties through strength reduction:

$$C_i^* = \frac{C_i}{F_{sa}}$$

$$\tan\varphi_i^* = \tan\frac{\varphi_i}{F_{sa}}$$

Where:

C_i, φ_i : Design cohesion and friction angle

F_{sa} : Required minimum safety factor

The slope stability condition then simplifies to:

$$\frac{\sum_{i=1}^{i=m} (C_i^* \cdot AB + N_i \cdot \tan\varphi_i^*)}{\sum_{i=1}^{i=m} T_i} > 1$$

Note

Suitable for rapid assessments; used to find critical slip circle with lowest safety factor (F_s).

b. Determining the Minimum Safety Factor

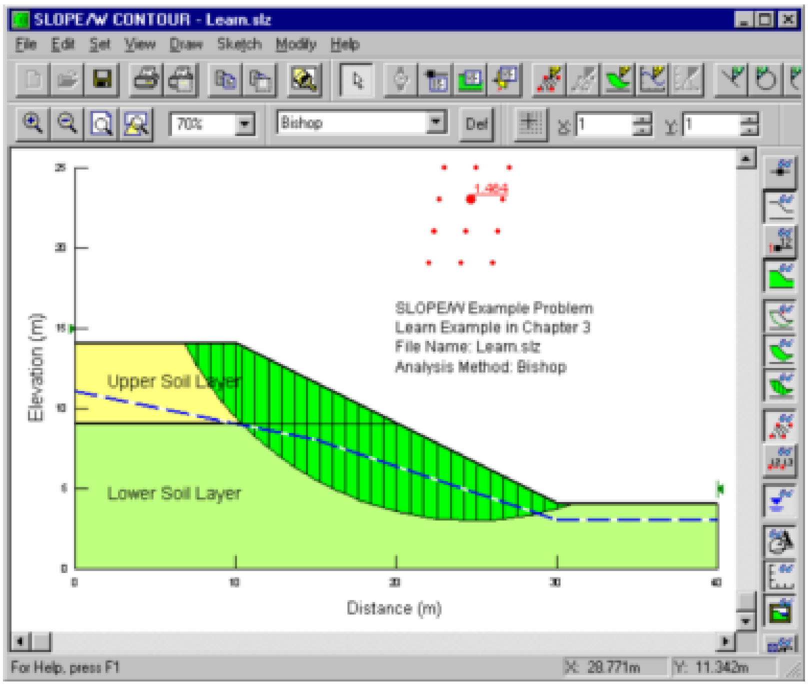
To identify the true safety factor (F_s) of a slope, the critical slip circle yielding the lowest F_s must be found, as failure is most likely to occur along this surface. There is no exact analytical method to predict the position of this critical circle a prior analysis.

Simulation

Industry-Standard Approach:

Most commercial software (e.g., Geo-Slope, shown in the figure below) uses an iterative trial-and-error process:

1. Grid Search: Computes F_s for numerous slip circles
2. Geometric Constraints: Tests only topographically plausible failure surfaces
3. Optimization: Identifies the circle with the minimal F_s as critical



Sample slope stability analysis output from Geo-Slope software.

4.2. BISHOP'S METHOD OF SLICES

Method

The components V_n , V_{n+1} , H_n , H_{n+i} contribute to the forces acting on slice (i) .

- **Vertical Force Equilibrium:**

$$W_i + \Delta V_i - (N_i' + u_i \cdot l_i)\cos\alpha_i - \left(\frac{C_i'}{F_s} l_i + N_i' \frac{\tan\varphi_i'}{F_s}\right)\sin\alpha_i = 0$$

$$N_i'\left(\cos\alpha_i + \frac{\tan\varphi_i' \sin\alpha_i}{F_s}\right) = W_i + \Delta V_i - u_i l_i \cos\alpha_i - \frac{C_i' l_i}{F_s}\sin\alpha_i$$

$$N_i' = \frac{W_i + \Delta V_i - u_i l_i \cos\alpha_i - \dfrac{C_i' l_i}{F_s}\sin\alpha_i}{\cos\alpha_i + \dfrac{\tan\varphi_i' \cdot \sin\alpha_i}{F_s}}$$

- **Moment Equilibrium:**

Driving moment:

$$\sum_1^n W_i R \sin\alpha_i$$

Resisting moment (opposing movement):

$$\sum_1^n R\left(\frac{C_i' l_i}{F_s} + N_i' \frac{\tan\varphi_i'}{F_s}\right)$$

By equating moments, substituting (N_i'), and simplifying by R:

$$F_s\left(\sum_1^n W_i \sin\alpha_i\right)=\sum_1^n C_i' l_i+\frac{W_i+\Delta V_i-u_i l_i \cos\alpha_i-\frac{C_i' l_i}{F_s}\sin\alpha_i}{\cos\alpha_i+\frac{\tan\alpha_i'}{F_s}}$$

The numerator terms can be rewritten with common denominator:

$$C_i' l_i \cos\alpha+\frac{1}{F_s}C_i' l_i \tan\varphi_i' \sin\alpha_i+(W_i+\Delta V_i-u_i l_i \cos\alpha_i-\frac{1}{F_s}C_i' l_i \sin\alpha_i)\tan\varphi_i'$$

Safety factor formula is given by:

$$F_s=\frac{1}{\sum_1^n \sin\alpha_i}\sum\frac{C_i' l_i \cos\alpha_i+(W_i+\Delta V_i-u_i l_i \cos\alpha_i)\tan\varphi_i'}{\cos\alpha_i+\frac{1}{F_s}\tan\varphi_i' \sin\alpha_i}$$

This is called the exact Bishop formula.

Solution Procedure:

- Requires iterative calculations since Fs appears on both sides
- Requires additional assumptions to define V_i

Simplified Bishop Method

Assuming $\Delta V_i=V_i-V_{i+1}=0$, the equation becomes:

$$F_s=\frac{1}{\sum_1^n \sin\alpha_i}\sum\frac{C_i' l_i \cos\alpha_i+(W_i-u_i l_i \cos\alpha_i)\tan\varphi_i'}{\cos\alpha_i+\frac{1}{F_s}\tan\varphi_i' \sin\alpha_i}$$

Or $$F_s=\frac{1}{\sum_1^n W_i \sin\alpha_i}\sum_1^n\frac{C_i' b_i+(W_i-u_i b_i)\tan\varphi_i'}{\cos\alpha_i+\frac{1}{F_s}\tan\varphi_i' \sin\alpha_i}$$

The expression for F_s is not explicit. Therefore, F_s cannot be calculated directly. An implicit method will be used in the form $F_{m+1}=f(F_m)$.

The initial value of F_s can be taken as F_{s0} , the value obtained using the Fellenius method. Convergence is generally quite fast. The process is stopped when $(F_{m+1}-F_m)$ is less than a pre-defined threshold.

Note

The Bishop method is more accurate than the Fellenius method, but it requires three to four times more computation (three iterations); the safety factors obtained are generally slightly higher.

Reminder

Most often, to avoid excessively increasing the amount of calculation, the most critical slip circle is first determined using the Fellenius method, and then it is verified that the safety factor calculated using the Bishop method is greater than that calculated using the Fellenius method.

If this is not the case, the search for the critical circle must be redone using the Bishop method. (Philipponnat G. & Hubert B, 2000)

4.3. Choosing the Safety Factor

Fundamental

A probabilistic value must be associated with the global safety factor F_s. Experience has shown that, barring major errors in the calculation assumptions:

Slopes always remain stable if F_s >1.5

Sliding is almost inevitable if F_s <1

There is an increasing risk of sliding as F_s decreases, when 1< F_s <1.51