**CHAPTER III: Calculation of isostatic beams at the service limit state**

**II.1 GENERAL**

As with any request, it is necessary to proceed, with regard to normal requests, to two categories of justification:

* At ULS to ensure the resistance of the structure,
* To the ELS to check compliance with operating and sustainability conditions.

While in the ELS, we limit ourselves to the elastic operating range of the materials, in the ELU, we admit the plasticization of the sections. This chapter concerns the study with regard to ELS, ELU will be treated in the following chapter. The principle of justification for the ELS is simple: it is enough to calculate the constraints which appear in the sections under the effect of the calculation requests and to verify that they do not exceed the regulatory limit constraints. As long as the tensile stresses in the concrete remain moderate (which we assume here), the calculations are carried out from the characteristics of the uncracked sections:

• Clear sections with regard to the stresses developed by permanent loads and by prestressing,

• Homogenized sections for stresses due to variable loads. In practice, the characteristics of the raw, net and homogenized sections are often very similar and they can be confused at the pre-sizing stage.

**III.2 DEFINITION OF CALCULATION SECTIONS**

**Raw section**

For taking into account the self-weight and rigidity calculations of the structure. Gross section= total section of the concrete without reduction of conduits and anchors

Bb =h\*b

h

 b

**Uncracked sections**

**Net sections** for calculating ELS stresses in uncracked sections. Net section = Total section of the concrete with deduction of conduits and anchors.

**Homogeneous sections**, to take into account a passive part of adherent active reinforcements.

Homogeneous section = net section + (adherent longitudinal reinforcement section) x n + passive reinforcement sections under the same conditions if they comply with articles A.6 and A.8.1 of BAEL91

n: Equivalence coefficient: take n= ni = 5 if instantaneous or n = nv=15 if long term

n=Bb-BV

**Cracked sections**

For the calculation at ELS in cracked sections: (Homogenized and reduced section) = (Section of compressed concrete alone) + (passive reinforcement section)x(nv)+ (prestressing reinforcement section)x(nv)x (ρ= 1 in case of pre-tension)

0.5 in case of post-tensioning with grout injection

0 in case of post-tensioning with greased sheathed strands.

**Coating section**

It is the surface delimited by the contour of the section and two parallel to the axis of flexion considered framing all the prestressing reinforcements, at a minimum admissible equal distance “c”. This section is used for certain checks in class II.

III.3 LOAD COMBINATION

III.3.1 Ultimate limit state (ULS)

Exceeding this state leads to the ruin of the structure. Beyond the ultimate limit state, the strength of concrete and steel materials is reached, safety is no longer guaranteed and the structure risks collapsing. We distinguish:

Limit state of resistance of one of the materials.

Limit state of static equilibrium.

Limit state of shape stability: buckling

III.3.2 Service limit state (ELS)

The service limit state reached calls into question the serviceability of the structure (cracks, leaks, various disorders). This state is defined taking into account operating and/or sustainability conditions. We distinguish:

Limit state of crack opening:

risk of crack opening.

Limit state of compression of concrete: we voluntarily limit the compressive stress to a reasonable value.

Limit state of deformation: maximum deflection.

NB: A structure must satisfy both ultimate limit state and state conditions. service limit.

* 1. III.3.3 Actions
	2. The actions are all the loads (forces, torques, etc.) applied to the structure, as well as the consequences of static or state deformations (shrinkage, settlement of supports, temperature variation, etc.) which lead to deformations of the structure.

 **III.3.3.1 Types of actions**

The three types of actions applied to the structure are as follows: Permanent actions: Permanent actions, denoted G, represent an action whose intensity is constant or varies very little over time. They understand:

The own weight of the structural elements,⎫

The weight of fixed equipment of all kinds (floor and ceiling coverings; partitions etc.),

The forces (weight, thrusts, and pressures) exerted by earth, by solids or by liquids whose levels vary little,

The differential displacements of the supports, forces due to deformations (shrinkage, creep, etc.) permanently imposed⎫ to construction, In most cases, self-weight is represented by a single nominal value, G0, calculated from project drawings and average material densities. Variable actions: variable actions, denoted Q, represent actions whose intensity varies frequently and significantly over time. They are defined by regulatory texts in force, we distinguish: operating loads (weight and related effects such as braking force, centrifugal forces, and dynamic effects),

The forces (weight, thrusts, and pressures) exerted by solids or by liquids whose level is variable, non-permanent loads applied during execution (construction site equipment, machines, material deposits, etc.),

**Climatic actions**: snow, wind, temperature, etc.

**Variable actions** are divided into two categories:

A so-called basic action noted Q1θ the other actions, called accompanying actions and denoted Qi (iθ>1)

The basic action Qi is: The unique action if this is the case

Otherwise: The most common

The highest

One or the other variable action

Accidental actions: Accidental actions, noted FA, originating from rare phenomena, and should only be considered if public documents or the market provide for it. Example: earthquakes, explosions, shocks.

**Representative values ​​of shares**

The different values ​​of the intensity of actions, called representative values, are: Qk: characteristic values ​​of the action

0i Qik: combination values ψ

1i Qik: frequent valuesψ

2i Qik: quasi-permanent values ψ

The prestress is represented by a calculation value Pd which is: the most unfavorable of two characteristic values ​​P1 and P2 for the justifications with regard to the service limit states,

P1 (x, t) = 1,02 P0 - 0,80 P (x, t)

P2 (x, t) = 0,98 P0 - 1,20 P (x, t)

* Its probable value Pm for the justifications with regard to the ultimate limit states.

Pm (x, t) = P0 - P (x, t)

P0 representing the prestress “at the origin”, corresponding to the tensionp0.

P (x, t) the loss of prestress at the point of abscissa x, at the instant t.

**III.5 JUSTIFICATION OF NORMAL CONSTRAINTS**

This verification consists of calculating the stresses in the concrete and comparing them to the authorized limit stresses. It must be established for each of the construction phases and the service phase. The stress calculation is carried out by applying the following general formula, in algebraic value: In the general case, we must have: σmin≤σ(y)≤σmax

The limiting constraints are not the same for the different load combinations, for verifications during the construction phase and for verifications during the service phases.

**III.5.1 Calculation hypotheses**

The calculations in the current section are carried out using the following two fundamental assumptions: Straight sections remain flat;

❑ The stresses of the materials are proportional to their deformations. Depending on the type of verification considered, the additional hypotheses are:

Calculation in uncracked section

❑ tensioned concrete resists traction;

❑ The materials do not suffer any relative slippage. This last hypothesis means that the normal stresses due to all actions other than permanent actions can be calculated on the entire homogeneous section. Calculation in cracked section

❑ Tensioned concrete is neglected;

❑ The materials do not undergo any relative sliding;

❑ When the deformation of the concrete is canceled at the level of reinforcement, the tension in the latter is worth:

* 0 if it is a passive reinforcement,
* σpd+niσbpd (with ni=5) if it is a prestressing reinforcement With ni=5) if it is a prestressing reinforcement

**III.5.2 Dimensioning of sections**

The objective of dimensioning the prestressing is to determine the effective force P (after subtraction of the tension losses) which must prevail in the section studied so that the limit stresses are ensured.



σs1 : upper fiber limit stress under loading 1 (P et Mm)

σs2 : upper fiber limit stress under loading 2 (P et MM)

σi1 : limit stress at lower fiber under loading 1(P et Mm)

σi2 : limit stress at lower fiber under loading 2(P et MM)

**II.5.2.1 Pressure center and line**

Under any real load case, a section is subjected to the following stresses:

•A force N = P due to the prestress,

•A bending moment M = P e0 + Mext. These reduction elements can be considered as generated by a single force P of eccentricity

*e*=*eo*+*Mf*

 P

P

Mf

e

eo

P

This eccentricity defines the ordinate of a point C called the depression center.

In a given section, when the external moment varies between Mm and MM, the center of pressure moves between the ordinate m and the ordinate M em emeeM with

**III.5.2.2 Limit core, limit spindle**

In a given section, the limit core is the interval in which the pressure center must be located to comply with the admissible stresses. When the section describes the entire length of the beam, the pressure line must therefore be located inside the limit spindle.

**III.5.4 Passage core, passage spindle**

In a given section, the passage core is the interval in which the cable must be located to comply with the limit core. When the section describes the entire length of the beam, the passage core generates the passage spindle.

**III.5.5 Concept of critical section**

**Subcritical section**: If all the passage segments are inside the zone that allows sufficient cover, the section is said to be subcritical.

**Critical section**: In the case where it is possible that the passage segment is reduced to a point, the section is critical.

**Supercritical section**: If the passage segment at one of its boundaries cuts the coating zone (open segment), the section is said to be supercritical.

 **Subcritical section Critical section Supercritical section III.6 EVALUATION OF THE PRESTRESS**

**Case of subcritical and critical section**

The passage segment is limited to a point





* + **Section case on criticism**

**Positive moment**



**Negative moment**



**Note**

If PI>PII the section is under critical

If PI< PII the section is over critical

**Special case** If we assume σs1=σi2=o, then we has:

**Subcritical section:** PI= ΔM

ρ

 L h

**Section on criticism**

**Positive moment**

*PII=Vi*+*MM*−*di*

ρ*Vs*

**Negative moment**

*PII*= −*Mm*

 Vs+ρVi-ds

By comparison, we can see the savings obtained on the prestressing force when tensile stresses are tolerated in the concrete. (σs1=σi2<o).

 **III.7 MINIMUM CONCRETE SECTION**

The minimum concrete section is obtained when the compression limit stresses are reached. In the following, it is assumed that we systematically adopt the minimum values ​​previously found for the prestressing (PI, PII).

**Cas d’une section sous critique**

*I*≥*MM* −*Mm*=Δ*M*

*Vs* σ*s2*−σ*s1*Δσ*s*

*I*≥*MM* −*Mm*=Δ*M*

*Vi* σ*i1*−σ*i2* Δσ*i*

**Case of a section on criticism**

 **Positive moment**

*I*≥ ρ*Ph*

*Vs*σ*s2*+*Vs*σ*i2*

 Vi

*I*≥ *MM* −*Mm*=Δ*M*

*Vi* σ*i1*−σ*i2* σ*fi*

Negative moment

*I*≥*MM* −*Mm*=Δ*M*

*Vs* σ*s2*−σ*s1*Δσ*s*

*I*≥ ρ*Ph*

*Vi*σ*i1*+*Vi*σ*s1*

Vs

III.8 LONGITUDINAL PASSIVE REINFORCEMENT

They result from the most severe of the following requirements:

III.8.1 Skin reinforcements µ

The purpose of these reinforcements is essentially to limit the cracking of the concrete before the application of the prestressing force under the action of phenomena such as differential shrinkage. The section of the skin reinforcements must be at least 3 cm2 per meter of length, without being able to be less than 0.10% of the concrete section.

**III.8.2 Reinforcement of tensioned areas**

In the parts of the section where the concrete is tensioned, it is necessary to have a minimum reinforcement section As

With : 

Bt: the area of ​​the part of the concrete in tension

NBt: the resultant of the corresponding tensile stresses.

σBt: the absolute value of the maximum tensile stress.

**III.9 JUSTIFICATION OF TANGENTIAL STRESSES**

A beam subjected to a shear force must be the subject of the following justifications:

In all areas of the beam with respect to:

The serviceability limit state,

The ultimate limit state

In the simple support and end areas of the beam. , additional justifications relating to the equilibrium of the shear force connecting rod and possibly the lower corner. The presence of the prestress induces a new data in the calculation of the prestressed elements. Thus, to the effects of permanent loads and operating loads is added that of prestressing: V = Vg + Vq + Vp

For the case of a prestress of force P inclined at an angle "α" relative to the average fiber, the action of the prestressing force on the section can be broken down into two forces: one "N" normal and the other "Vp" perpendicular.

**N=Pcosα N>0**

**Vp=-Psinα Vp<0**

Consequently, the value of the shear force to be considered is a reduced shear force defined by: **Vréd=(Vg+Vq)-Psinα**

**N.B**: Depending on the sign of "sinα", the shear force can be favourable or unfavourable.

**III.9.1 Justification at SLS**

The justifications are carried out for a given section of the beam from the stresses σx, σt and τt, calculated for the element considered at the verification level, assuming elastic and linear deformations of the materials and assuming the concrete is not cracked. In the general case of a beam element comprising transverse prestressing reinforcements of unit tensile force Ft inclined at α' on the mean fibre and spaced at st' (Figure III.1), we have:

Ft

st

'

α’

Figure III.1: Beam with transverse reinforcement [5]



σx: normal stress at the section;

σt: normal stress at the cross-section;

τ: shear stress of the element;

τred: shear stress due to the shear force reduces the element which can be calculated by the formula:

**τ*réd*= *VrédS***

 ***bnIn***

Vréd: reduced shear force

S: static moment

bn: net width of the section

In: net moment of inertia of the section under the effect of service stresses in the case of the most unfavourable loads; and whatever the sections considered, the following conditions are verified:



**III.10 EXERCISES**

**EXERCISE 1:**

Let a beam of rectangular section (60x120) cm subjected to the moments Mmin = 1.3MNm and Mmax = 3.3MNm with a value of the cover such that di = 0.15m.

* Determine the value of the prestress (P1 and P2).
* Give an observation on the nature of the section.
* Determine the value of the eccentricity eO.

**Solution**:

In subcritical section, the value of the prestress is determined by the equation:

PI=ΔM

 ρh

With:

ΔM=2 MNm

ρ=0.33 ,h=1.20m

From where: P1=5.05MN

In supercritical section (positive moment), the value of the prestress is determined by the equation:

PII = MM / Vi + ρ.Vs- di

With:

Mmax=3.2MNm

ρ=1/3

Vs=Vi=0.60m

di =0.15m

From where: P2=5.09MN

We note that P1>P2 hence the section is subcritical The value of the eccentricity eO is given by:

−Ci−Mm=eo=Cs−MM

PI PI

With:

Cs= ρVs= 0.2m P1=5MN

From where: e0=-0.44m

**Exercise 2**

Let us consider a beam with a rectangular section (60x130) cm subjected to the moments Mmin=1.8 MNm and Mmax= 2.5 M N m with a value of the cover such that di = 0.16m.

Determine the value of the prestress (P1 and P2).

Sketch the stress diagram

**Exercice 3**

Consider a slab (1 m, h) with a span of 16 m, subject to an operational load q=0.06 MN/m2 with no value of the coating as i = 0.108 m. The concrete used has strength of 25 MPa. Tensile limit stress σ t = 0, Compressive limit stress σb= 15 MPa

Determine the height h

Determine the value of the pre-stress P and the value of the centrifugal stress eO.

Give an observation on the nature of the section.