Series 4

Exercise 1:

A proton that has a velocity $\vec{V} = 3 \times 10^6 \vec{\imath} + 4 \times 10^6 \vec{\jmath})$ (m/s) moves through a magnetic field $\vec{B} = (0.02\vec{\imath} + 3\vec{\jmath})$) (T).

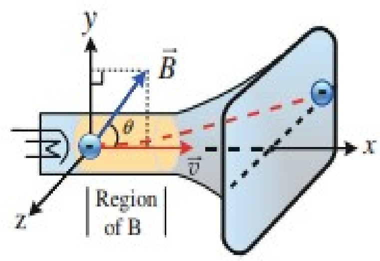
Find the vector of magnetic force exerted by the field on the proton, and then find the magnitude and direction of this force.

Exercise 2:

An electron in a television tube moves along the x-axis with a speed v of 10^7 m/s, see the sketch in Fig. A uniform magnetic field in the xy plane has a magnitude 0.02 T and is directed at an angle of 30° from the x-axis.

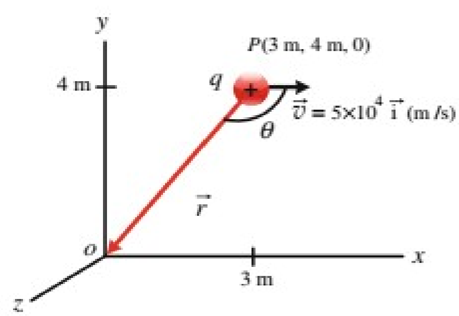
a- Calculate the magnitude of the magnetic force on the electron.

b- Find the vector expression of the magnetic force on the electron in terms of the unit vectors $\vec{\imath}$, $\vec{\jmath}$, and $\vec{k}$ along x, y, and z axes.



Exercise 3:

A point charge q = 6 μC is moving in a straight line with a velocity $\vec{V} = 5 \times 10^4 \vec{\imath}$ (m/s). When the charge is at the location P(3 m, 4 m, 0), find the magnetic field produced by this point charge at the origin o, see Fig.



Exercise 3:

Two very long parallel straight wires carry currents that are perpendicular to the page. Wire ©1 carries a current I_1 = 3 A out of the page and passes through the origin o of the x-axis, while wire ©2 carries a current I_2 = 2 A into the page and passes through the x-axis at a distance d = 0.6 m from the origin. (a) On the x-axis, show the directions of the magnetic fields, to right of wire ©2 , between the two wires, and to the left of wire ©1 . (b) To the right of wire ©2 , find a distance a at which the resultant magnetic field is zero.

Solution of Series 4

Exercise 1:

1- **Finding the vector magnetic force exerted by the field on the proton:**

Using the relation $\vec{F} = q(\vec{V} \times \vec{B})$, we have:

$\vec{F} = q(\vec{V} \times \vec{B}) = +e \begin{vmatrix} \vec{\imath} & -\vec{\jmath} & \vec{K} \\ 3 \times 10^6 & 4 \times 10^6 & 0 \\ 0.02 & 3 & 0 \end{vmatrix} = 1.6 \times 10^{-19} [(3 \times 10^6 \times 3) - (4 \times 10^6 \times 0.02)]\vec{K}$

$\vec{F} = 14.272 \times 10^{-1} \vec{K}$ (N)

2- **Finding the magnitude of this force:**

$\|\vec{F}\| = |F_z| = |14.272 \times 10^{-19}| = 14.272 \times 10^{-14}$ (N)

3- **Finding the direction of this force:**

The direction of this force is directed along the positive z-axis

Exercise 2:

a- **Calculating the magnitude of the magnetic force on the electron:**

using equation of the magnitude of the magnetic force we find that:

$F_B = |q|\, vB \sin\theta = |-e|\, vB \sin\theta = e\, vB \sin\theta = (1.6\times10^{-19}\ C)(10^7\ m/s)(0.02\ T)(\sin 30°) = 1.6\times10^{-14}$ N

b- **Find the vector expression of the magnetic force on the electron in terms of the unit vectors $\vec{\imath}$, $\vec{\jmath}$, and $\vec{k}$ along x, y, and z axes:**

We first express the velocity and the magnetic field in terms of the unit vectors as follows:

$\vec{V} = V_x\vec{\imath} = (10^7\vec{\imath})$ (m/s)

$\vec{B} = (Bco\quad \vec{\imath} + Bsin\theta^{\rightarrow})$ (T)= $(0.02cos30^{\circ}\vec{\imath} + 0.02sin30^{\circ}\vec{\jmath})$ (T)=. $(0.017\vec{\imath} + 0.01\vec{\jmath})$ (T)

$$\vec{F} = q(\vec{V} \times \vec{B}) = -e \begin{vmatrix} \vec{\imath} & -\vec{\jmath} & \vec{K} \\ 10^7 & 0 & 0 \\ 0.017 & 0.01 & 0 \end{vmatrix} = -1.6 \times 10^{-19} \, [(10^7 \times 0.01) - (0 \times 0.017)]\vec{K}$$

$$\vec{F} = -1.6 \times 10^{-14} \vec{K} \text{ (N)}$$

$$\|\vec{F}\| = |F_z| = |-1.6 \times 10^{-14}| = 1.6 \times 10^{-1} \text{ (N)}$$

So: The magnetic force on the electron $\vec{F}$ has a magnitude that agrees with the result of part (a) and is directed along the negative z-axis.

Exercise 3:

From the Biot-Savart law, we have:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \overrightarrow{u_r}}{r^2}$$

From the figure, we can find $\vec{r}$and $\overrightarrow{u_r}$ (from the point charge) as follows:

$$\vec{r} = (-3\,\vec{\imath} - 4\vec{\jmath})$$

$$r = \sqrt{(-3)^2 + (-4)^2} = 5m$$

Thus :

$$\overrightarrow{u_r} = \frac{\vec{r}}{r} = \frac{(-3\,\vec{\imath} - 4\vec{\jmath})}{5} = -0.6\,\vec{\imath} - 0.8\vec{\jmath}$$

Substituting the above results into the equation for $\vec{B}$ we obtain:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \overrightarrow{u_r}}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{q(v\vec{\imath}) \times (-0.6\,\vec{\imath} - 0.8\vec{\jmath})}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{qv[-0.6(\vec{\imath} \times \vec{\imath}) - 0.8(\vec{\imath} \times \vec{\jmath})]}{r^2}$$

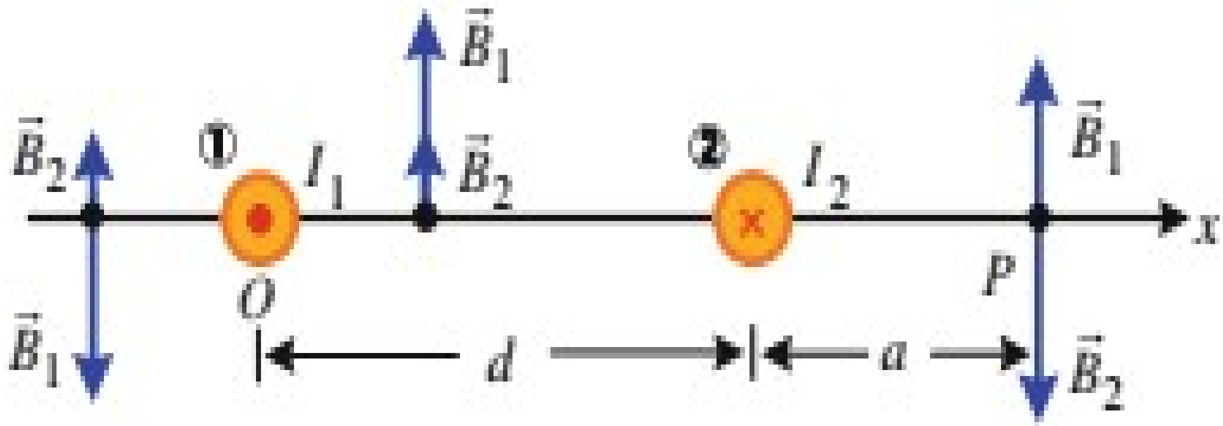
$$= -\frac{\mu_0}{4\pi} \frac{qv[0.8\vec{k}]}{r^2}$$

$$= -\frac{4\pi \times 10^{-7}}{4\pi} \frac{6 \times 10^{-6} \times 5 \times 10^4 [0.8\vec{k}]}{5^2}$$

$$= -9.6 \times 10^{-10} \vec{k} (T)$$

Exercise 4:

Using the right hand rule, we can draw the direction of $\overrightarrow{B_1}$ of wire 1 and $\overrightarrow{B_2}$ of wire 2 on the three regions of the
x-axis as shown in Fig:



When the magnitudes of the opposite two vectors $\overrightarrow{B_1}$ and $\overrightarrow{B_2}$ are equal, the resultant magnetic field becomes zero. Therefore, we have:

$$\frac{\mu_0 I_1}{2\pi(d+a)} = \frac{\mu_0 I_2}{2\pi a} \Rightarrow \frac{I_1}{(d+a)} = \frac{I_2}{a} \Rightarrow aI_1 = (a+d)I_2 \Rightarrow a\left(\frac{I_1}{I_2} - 1\right) = d$$

Thus:

$$a = \frac{d}{\left(\frac{I_1}{I_2} - 1\right)} = \frac{0.6}{\left(\frac{3}{2} - 1\right)} = 1.2m$$