Series 3

Exercise 1:

The charge that passes a cross-sectional area $A = 10^{-4}\ m^2$ varies with time according to the relation $Q = 4 + 2t + t^2$, where Q is in coulombs and t is in seconds.

a- Find the relation that gives the instantaneous current at any time, and evaluate this current at time t = 2 s.

b- Find the relation that gives the current density at any time, and evaluate this current density at time t = 2 s.

Exercise 2:

Estimate the drift speed of the conduction electrons in a copper wire that is 2 mm in diameter and carries a current of 1 A. The density of copper is 8.92×10^3 kg/m3.

[Hint: Assume that each copper atom contributes one free conduction electron to the current.

Exercise 3:

A metallic rod has a length L = 1.5 m and a diameter D = 0.2 cm. The rod carries a current of 5 A when a potential difference of 75 V is applied between its ends.

a- Find the current density in the rod.

b- Calculate the magnitude of the electric field applied to the rod.

c- Calculate the resistivity and conductivity of the material of the rod.

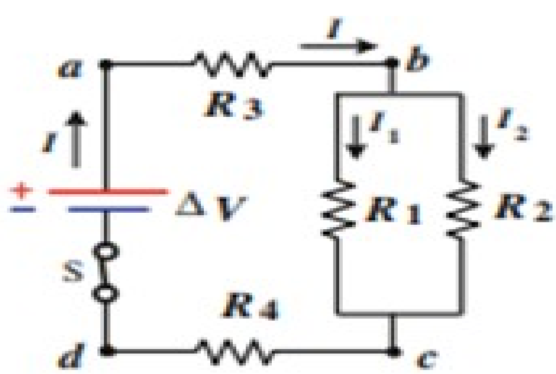
d- Find the resistance of the rod.

e- **Exercise 4:**

In Fig., let $R_1 = 3\ \Omega$, $R_2 = 6\ \Omega$, $R_3 = 1\ \Omega$, $R_4 = 7\ \Omega$, and $V_{da} = V_a - V_d = 30$ V.

a- What is the equivalent resistance between points a and d? b

b- Evaluate the current passing through each resistor.

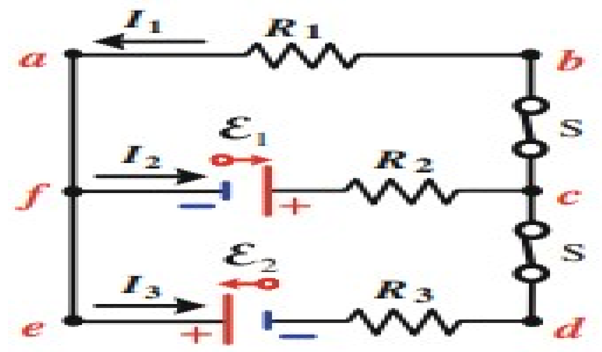


Exercise 5:

In Fig, let $R_1 = 2\Omega$, $R_2 = 6\ \Omega$, $R_3 = 4\ \Omega$,

$E_1 = 10$ V, and $E_2 = 14$ V.

Find the currents I_1, I_2, and I_3 in the circuit.



Solution 3.1

a-)

a-1- Finding the instantaneous current at any time.

$I=(dQ/dt)=[d(4 + 2t + t^{2})/ dt]$ (A)

$I= 2+ 2t$

a-2- Finding the instantaneous current at time t = 2 s.

$I(2s)= 2+ 2(2)=6(A)$

b-)

b-1- Finding the current density at any time.

$i=(I/A)=(2+2t)/ A=(2+2t)/ 10^{-4}$

$i= 10^{+4}(2+2t)$ (A/m^2)

b-2- Finding the current density at time t = 2 s.

$i(2s)= 10^{+4}(2+2(2))=6*10^{+4}$ (A/m^2)

Solution 3.2

- **Finding the drift speed:**

From the two equations of the current density , the drift speed is given by

$$\begin{matrix} \boldsymbol{i} = \boldsymbol{n}|\boldsymbol{q}|\mathrm{v} \\ \boldsymbol{i} = \frac{\boldsymbol{I}}{\boldsymbol{A}} \end{matrix} \Rightarrow \frac{\boldsymbol{I}}{\boldsymbol{A}} = \boldsymbol{n}|\boldsymbol{q}|\mathrm{v} \Leftrightarrow \mathrm{v} = \frac{\boldsymbol{I}}{\boldsymbol{An}|\boldsymbol{q}|} --------- (*)$$

To get the drift speed v, we need to find the free-electron density n and area.

The **area** is given by

$A=\pi r^2$ where r is the radius ($r=2mm/2=1mm=10^{-3}m$)

And the free-electron density n is given by

$$\boldsymbol{n} = \frac{\boldsymbol{N_A} \times \boldsymbol{d}}{\boldsymbol{M}}$$

where

$NA=6.022 \times 10^{23}$ atoms/mol

$d=8.92 \times 10^3\ kg/m^3$

$M(Cu) =63.546$ g/mol$= 63.546 \times 10^{-3}$ kg/mol

Substituting n and A into Eq. *) gives

$$\mathrm{v} = \frac{\boldsymbol{I}}{\pi r^2 \frac{\boldsymbol{N_A} \times \boldsymbol{d}}{\boldsymbol{M}} |\boldsymbol{q}|} = \frac{\boldsymbol{I}}{\pi r^2 |\boldsymbol{q}|} \times \frac{\boldsymbol{M}}{\boldsymbol{N_A} \times \boldsymbol{d}}$$

$$\mathrm{v} = \frac{\boldsymbol{MI}}{\pi r^2 |\boldsymbol{q}| \boldsymbol{N_A d}}$$

$$\mathrm{v} = \frac{63.546 \times 10^{-3} \times \mathbf{1}}{3.14 \times (10^{-3})^2 \times |\mathbf{1.6} \times 10^{-1}| \times 6.022 \times 10^{23} \times 8.92 \times 10^3} = \mathbf{2.35} \times \mathbf{10}^{-5} \left(\frac{\boldsymbol{m}}{\boldsymbol{S}}\right)$$

Solution 3.3

a- **Finding the current density in the rod:**

The current density in a rod of diameter D = 0.2 cm is

$$\boldsymbol{i = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{I}{\pi \left(\frac{D}{2}\right)^2} = \frac{4 \times I}{\pi D^2}}$$

$$\boldsymbol{i = \frac{4 \times 5}{3.14 \times (0.2 \times 10^{-2})^2} = 1.59 \times 10^6 \left(\frac{A}{m^2}\right)}$$

b- **Calculating the magnitude of the electric field applied to the rod.**

The magnitude of the electric field applied to the rod is

$$\boldsymbol{E = \frac{\Delta V}{L} = \frac{75}{1.5} = 50 \left(\frac{V}{m}\right)}$$

c- **Calculating the resistivity and conductivity of the material of the rod.**

The resistivity (ρ):

From the equation of the current density $(i = \frac{E}{\rho})$, the resistivity is given by

$$\rho = \frac{E}{i} = \frac{50}{1.59 \times 10^6} = 3.14 \times 10^{-5} (\Omega.m)$$

The conductivity (σ):

The inverse of resistivity is called the conductivity σ, thus:

$$\sigma = \frac{1}{\rho} = \frac{1}{3.14 \times 10^{-5}} = 3.18 \times 10^4 (\Omega.m)^{-1}$$

d- Finding the resistance of the rod:

There are three methods for calculating resistance

1-From the equation of the Ohm's law ($\Delta V = R \times I$) , the resistivity is given by

$$R = \frac{\Delta V}{I} = \frac{75}{5} = 15\Omega$$

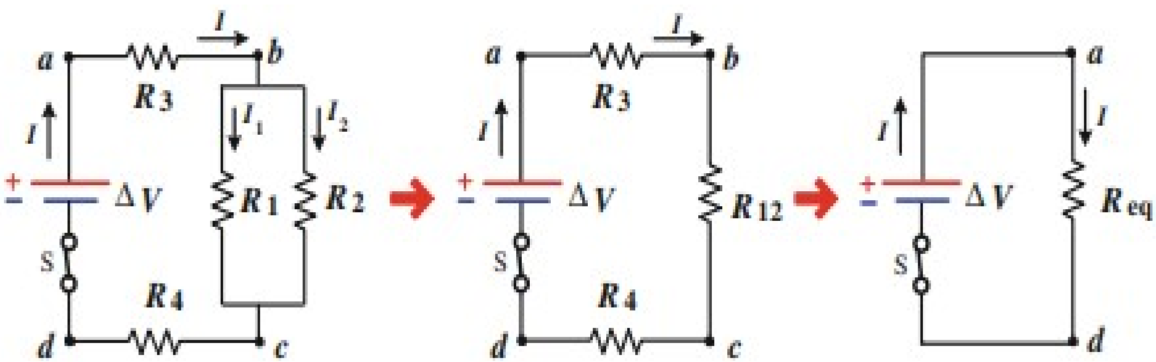
2)-Using the resistivity expression $R = \frac{\rho L}{A}$, we find that:

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = \frac{\rho L}{\pi\left(\frac{D}{2}\right)^2} = \frac{4\rho L}{\pi D^2} = \frac{4 \times 3.14 \times 10^{-5} \times 1.5}{3.14(0.2 \times 10^{-2})^2} = 15\Omega$$

3)-Using the resistivity expression $R = \frac{\rho L}{A}$, we find that:

$$R = \frac{L}{\sigma A} = \frac{L}{\sigma \pi r^2} = \frac{L}{\sigma\pi\left(\frac{D}{2}\right)^2} = \frac{4L}{\sigma \pi D^2} = \frac{4 \times 1.5}{3.18 \times 10^4 \times 3.14(0.2 \times 10^{-2})^2} = 15\Omega$$

Solution 3.4



a- We can simplify the circuit by the rule of adding resistances in series and in parallel in steps. The resistors R_1 and R_2 are in parallel and their equivalent resistance R_{12} between b and c is:

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{6\Omega} + \frac{1}{3\Omega} = \frac{1}{2\Omega}$$

Then : $R_{12} = 2\Omega$

Now R_3, R_{12}, and R_4 are in series between points a and d. Hence, their equivalent resistance Req is:

$$R_{eq} = R_3 + R_{12} + R_4 = 1\ \Omega + 2\ \Omega + 7\ \Omega = 10\ \Omega$$

b - The current I that passes through the equivalent resistor also passes through R_3 and R_4. Thus, using Ohm's law, we find that:

$I = \frac{\Delta V_{da}}{R_{eq}} = \frac{30}{10} = 3A$ (Current through the battery, R_3 and R_4)

Since $\Delta V_{cb} = IR_{12} = I_1R_1 = I_2R_2$, then we find I_1 and I_2 as follows:

$$I_1 = \frac{IR_{12}}{R_1} = \frac{3 \times 2}{3} = 2A \quad and \quad I_2 = \frac{IR_{12}}{R_2} = \frac{3 \times 2}{6} = 1A$$

Solution 3.5

Applying Kirchhoff's junction rule at point b, we get:

(1) Junction f : $I_1 - I_2 - I_3 = 0$
Junction c : $I_2 + I_3 - I_1 = 0$ ⇒ $I_1 = I_2 + I_3$

We have three loops in this circuit, but we need only two loop equations to determine the three unknown currents. Applying Kirchhoff's loop rule to the loops abcfa and fcdef and traversing these loops clockwise, we obtain the following equations

(2) Loop abcfa: $-I_1R_1 - I_2R_2 = -E_1 \Rightarrow I_1R_1 + I_2R_2 - E_1 = 0 \Rightarrow 2I_1 + 6I_2 - 10 = 0$
(3) Loop fcdef : $+I_2 R_2 - I_3 R_3 = E_2 + E_1 \Rightarrow E_1 - I_2 R_2 + I_3 R_3 + E_2 = 0 \Rightarrow 24 - 6 I_2 + 4 I_3 = 0$

Substituting Eqs. (1) $I_1 - I_2 = I_3$ into (3) gives:

$24 - 6 I_2 + 4 I_3 = 0 \Rightarrow 24 - 6 I_2 + 4 (I_1 - I_2) = \Rightarrow 24 - 6 I_2 + 4 I_1 - 4I_2 = 0$

$24 - 10 I_2 + 4 I_1 = 0$

Dividing this equation by 2 gives:

(4) $12 - 5 I_2 + 2 I_1 = 0$

Subtracting Eqs. (4) from (2) gives:

$(2 I_1 + 6 I_2 - 10) - (12 - 5 I_2 + 2 I_1) = 0 \Rightarrow 11I_2 = 22 \Rightarrow I_2 = 22/11 \Rightarrow I_2 = 2A$

Using this value of I_2 in Eq. (4) gives I_1 a value of:

$12 - 5 \times 2 + 2 I_1 = 0 \Rightarrow 2I_1 = -2 \quad 0 \Rightarrow I_1 = -2/2 = -1$ A

Finally, from Eq. (1) we have:

$I_3 = I_1 - I_2 = -1$ A $- 2$ A $= -3$ A

Thus ($I_1 = -1A, I_2 = 2A$ and $I_3 = -3A$)