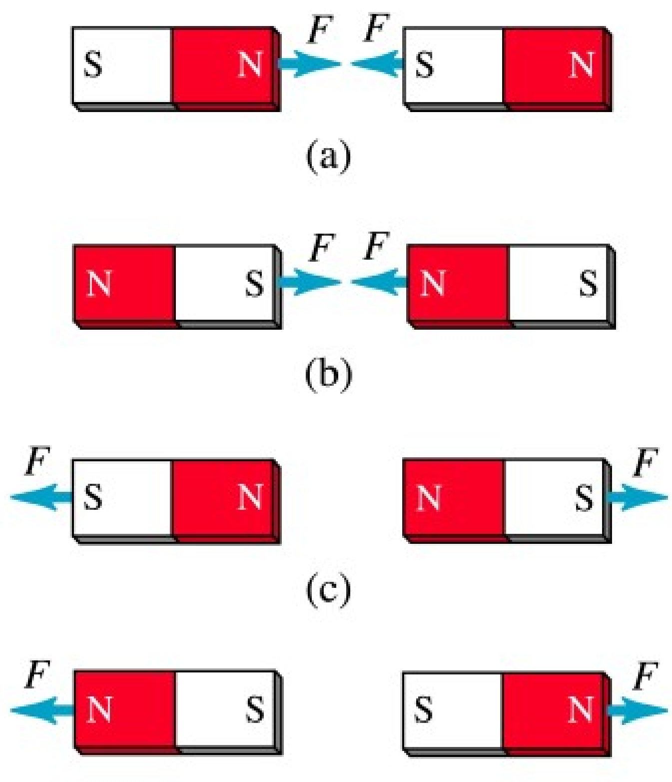
Chapter 4:

a. **Definition of magnetic field**
b. **Magnetic force**
c. **The Biot-Savart Law**

Electromagnetism

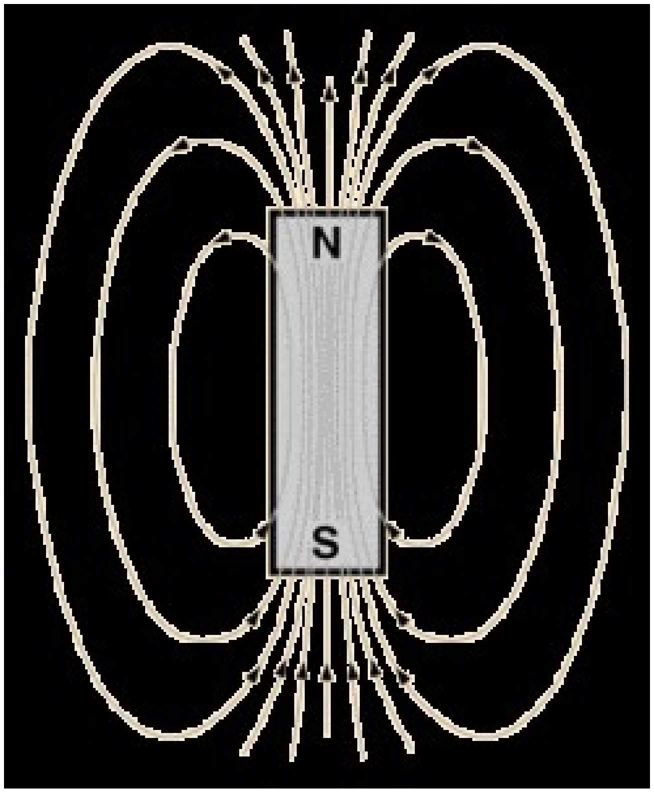
a-Definition of magnetic field

Facts about Magnetism



- Magnets have 2 poles (north and south)
- Like poles repel
- Unlike poles attract
- Magnets create a MAGNETIC FIELD around them

Magnetic Field



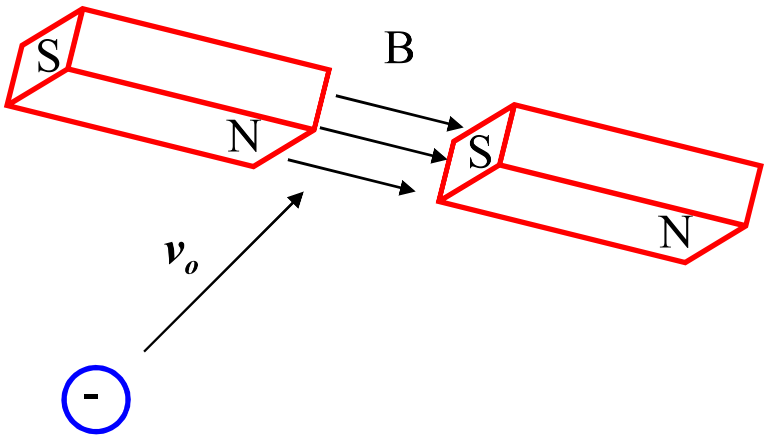
A bar magnet has a magnetic field around it. This field is 3D in nature and often represented by lines LEAVING north and ENTERING south

To define a magnetic field you need to understand the MAGNITUDE and DIRECTION

We sometimes call the magnetic field a B-Field as the letter "B" is the SYMBOL for a magnetic field with the TESLA (T) as the unit.

b- Magnetic Force
(Lorentz' Law and Laplace's law)

Magnetic Force on a moving charge



If a MOVING CHARGE moves into a magnetic field it will experience a MAGNETIC FORCE. This deflection is 3D in nature.

$$\vec{F} = q\vec{v} \wedge \vec{B}$$ **Or** $$\vec{F} = q(\vec{V} \times \vec{B})$$

The conditions for the force are:

- Must have a magnetic field present
- Charge must be moving
- Charge must be positive or negative

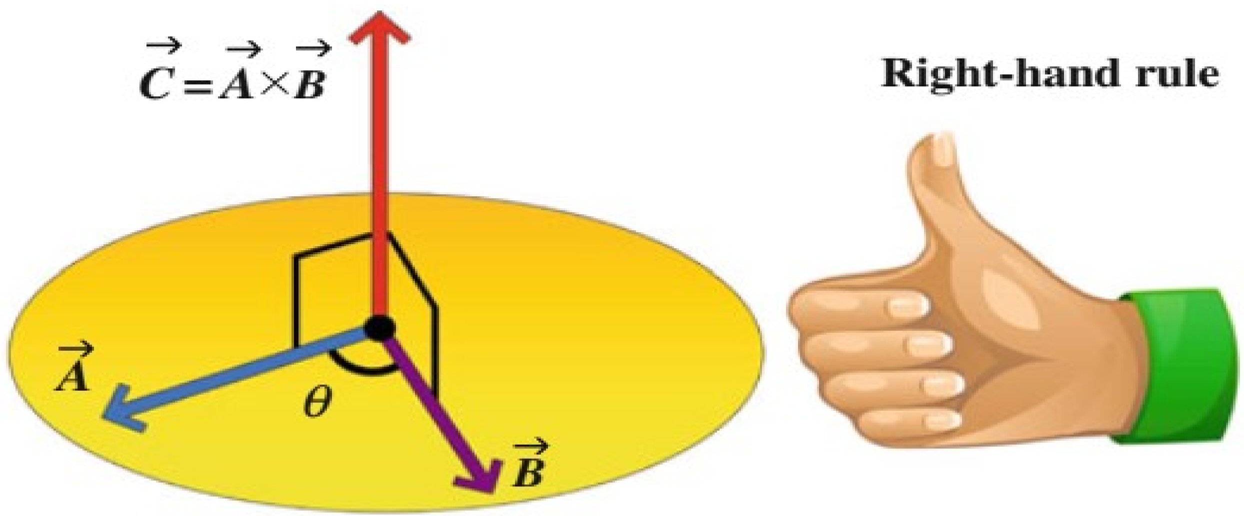
Magnitude of Magnetic Force

$$F = qvB\sin\theta$$

The Vector Product (or the Cross Product)

The **vector product** of the two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A} \times \vec{B}$ and defined as a third vector $\vec{C}$ whose magnitude is: $C = AB \sin\theta$

where θ is the smaller angle between $\vec{A}$ and $\vec{B}$ (hence, $0 \leq \sin\theta \leq 1$). The direction of $\vec{C}$ is perpendicular to the plane that contains both $\vec{A}$ and $\vec{B}$, and can be determined by using the **right-hand rule**, see Fig.



The vector product definition leads to the following properties:

1. The order of vector product multiplication is important; that is:

$$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$$

which is unlike the scalar product and can be easily verified with the right-hand rule.

2. If $\vec{A}$ is parallel to $\vec{B}$ (that is, $\theta = 0°$) or $\vec{A}$ is antiparallel to $\vec{B}$ (that is, $\theta = 180°$), then:

$$\vec{A} \times \vec{B} = 0 \quad \text{(if } \vec{A} \text{ is parallel or antiparallel to } \vec{B}\text{)}$$

3. If $\vec{A}$ is perpendicular to $\vec{B}$, then:

$$|\vec{A} \times \vec{B}| = AB \quad (\text{if } \vec{A} \perp \vec{B})$$

4. The vector product obeys the distributive law, that is:

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C} \quad \text{(Distributive law)}$$

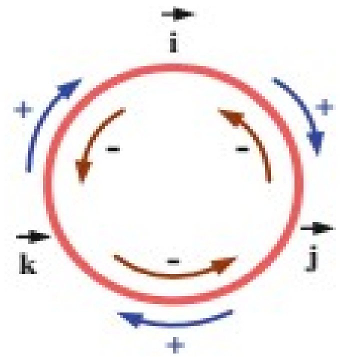
5. The derivative of $\vec{A} \times \vec{B}$ with respect to any variable such as t is:

$$\frac{d}{dt}(\vec{A} \times \vec{B}) = \vec{A} \times \frac{d\vec{B}}{dt} + \frac{d\vec{A}}{dt} \times \vec{B}$$

6. From the definition of the vector product and the unit vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$, we get the following relationships:

$$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$$

$$\vec{i} \times \vec{j} = \vec{k}, \quad \vec{j} \times \vec{k} = \vec{i}, \quad \vec{k} \times \vec{i} = \vec{j}$$



The last relations can be obtained by setting the unit vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$ on a circle, see Fig. , and rotating in a clockwise direction to find the cross product of one unit vector with another. Rotating in a counterclockwise direction will involve a negative sign of the cross product of one unit vector with another, that is:

$$\vec{i} \times \vec{k} = -\vec{j}, \quad \vec{k} \times \vec{j} = -\vec{i}, \quad \vec{j} \times \vec{i} = -\vec{k}$$

7. When two vectors $\vec{A}$ and $\vec{B}$ are written in terms of the unit vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$, then the cross product will give the result:

$$\begin{aligned} \vec{A} \times \vec{B} &= (A_x \vec{i} + A_y \vec{j} + A_z \vec{k}) \times (B_x \vec{i} + B_y \vec{j} + B_z \vec{k}) \\ &= (A_y B_z - A_z B_y)\vec{i} + (A_z B_x - A_x B_z)\vec{j} \\ &\quad + (A_x B_y - A_y B_x)\vec{k} \end{aligned}$$

This result can be expressed in determinant form as follows:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \vec{i} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \vec{j} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \vec{k} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

Example

A proton moves with a speed of $1.0x10^5$ m/s through the Earth's magnetic field, which has a value of 55μT at a particular location. When the proton moves eastward, the magnetic force is a maximum, and when it moves northward, no magnetic force acts upon it. What is the magnitude of the magnetic force acting on the proton?

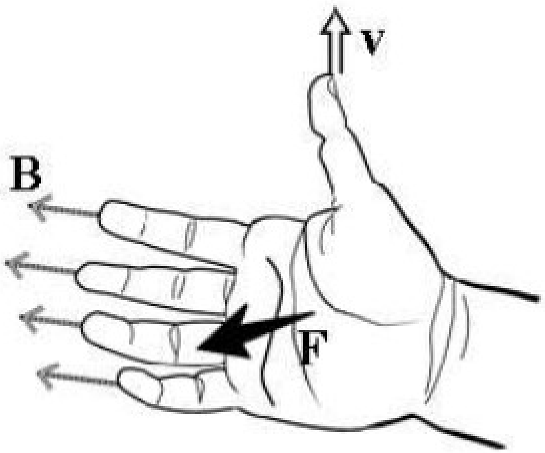
SOLUTION

$$F_B = qvB, \theta = 90, \sin 90 = 1$$

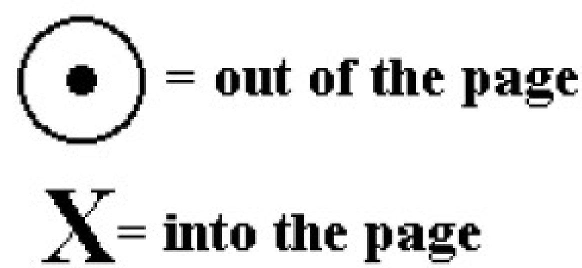
$$F_B = (1.6x10^{-19})(1.0x10^5)(55x10^{-6})$$

$$F_B = 8.8x10^{-19} \text{ N}$$

Direction of the magnetic force?
Right Hand Rule



To determine the DIRECTION of the force on a **POSITIVE** charge we use a special technique that helps us understand the 3D/perpendicular nature of magnetic fields.



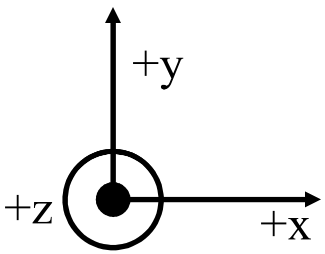
Basically you hold your right hand flat with your thumb perpendicular to the rest of your fingers

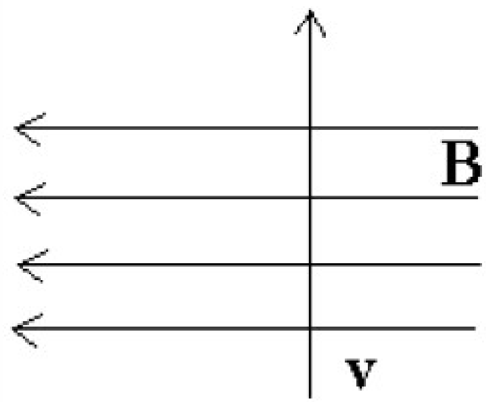
- The Fingers = Direction B-Field
- The Thumb = Direction of velocity
- The Palm = Direction of the Force

For **NEGATIVE** charges use left hand!

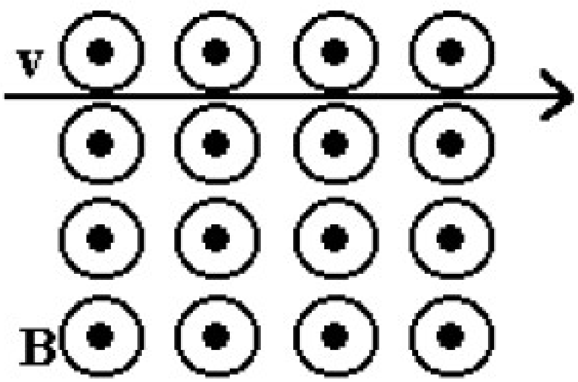
Example

Determine the direction of the unknown variable for a proton moving in the field using the coordinate axis given

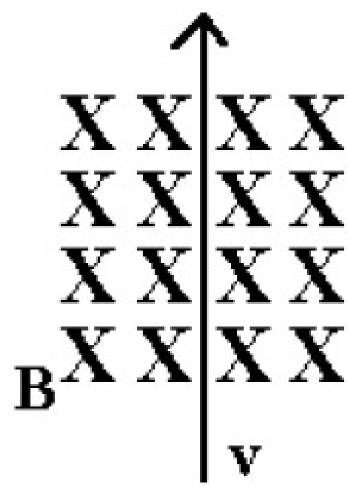




B = -x
v = +y
F = **+z**

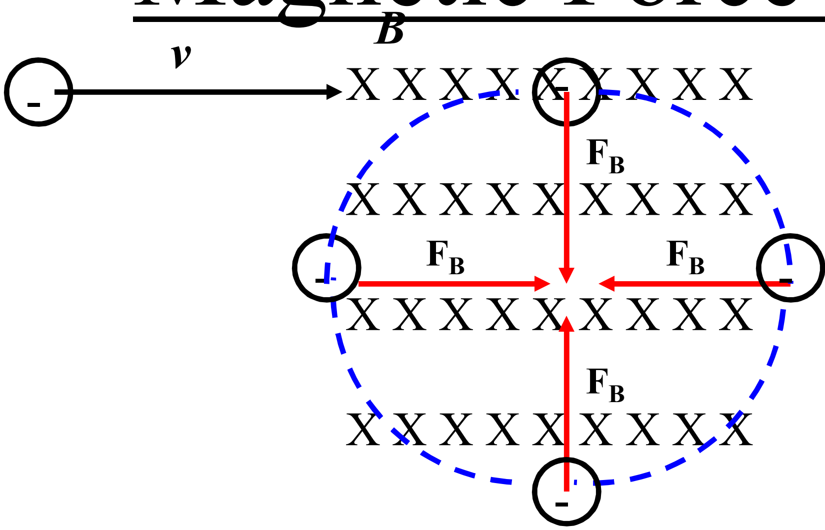


B =+Z
v = +x
F = **-y**



B = -z
v = +y
F = **-x**

Magnetic Force and Circular Motion



Suppose we have an electron traveling at a velocity , ***v***, entering a magnetic field, ***B***, directed into the page.

The magnetic force is equal to the centripetal force and thus can be used to solve for the circular path. Or, if the radius is known, could be used to solve for the MASS of the ion. This could be used to determine the material of the object.

$$F_B = qvB,\ F_c = \frac{mv^2}{r},\ F_B = F_c$$

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

There are many "other" types of forces that can be set equal to the magnetic force.

$$F_B = qvB$$

$$mg = qvB$$

$$ma = qvB$$

Example

A singly charged positive ion has a mass of 2.5 x 10^{-26} kg. After being accelerated through a potential difference of 250 V, the ion enters a magnetic field of 0.5 T, in a direction perpendicular to the field. Calculate the radius of the path of the ion in the field.

$q = 1.6x10^{-19}\,C$

$m = 2.5x10^{-26}\,kg$

$\Delta V = 250V$

$B = 0.5T$

$r = ?$

$$F_B = F_c \quad qvB = \frac{mv^2}{r} \quad r = \frac{mv}{qB}$$

We need to solve for the velocity!

$$\Delta V = \frac{W}{q} = \frac{\Delta K}{q} = \frac{\frac{1}{2}mv^2}{q}$$

$$v = \sqrt{\frac{2\Delta Vq}{m}} = \sqrt{\frac{2(250)(1.6x10^{-19})}{2.5x10^{-26}}} =$$ **56,568 m/s**

$$r = \frac{(2.5x10^{-26})(56{,}568)}{(1.6x10^{-19})(0.5)} =$$ **0.0177 m**

Charged Particles in an Electric and Magnetic Fields

In the presence of both an electric field $\vec{E}$ and a magnetic field $\vec{B}$, the total force $\vec{F}$ exerted on a charge q moving with velocity $\vec{v}$ is

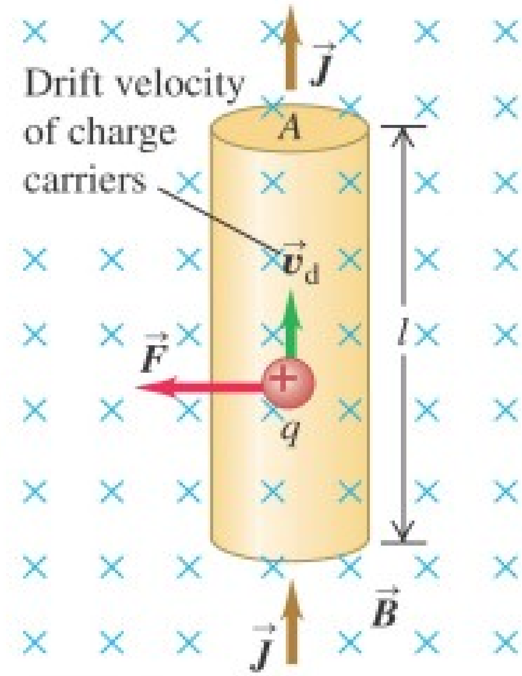
$$\vec{F} = q\vec{F} + q\left(\vec{V} \times \vec{B}\right) \quad or \quad \vec{F} = q\vec{F} + q\left(\vec{V} \wedge \vec{B}\right)$$

which is often called the Lorentz force.

Magnetic Force on a Current-Carrying Conductor

$$\vec{F}_m = q\vec{v}_d \times \vec{B}$$

$$F_m = qv_d B$$ Force on one charge

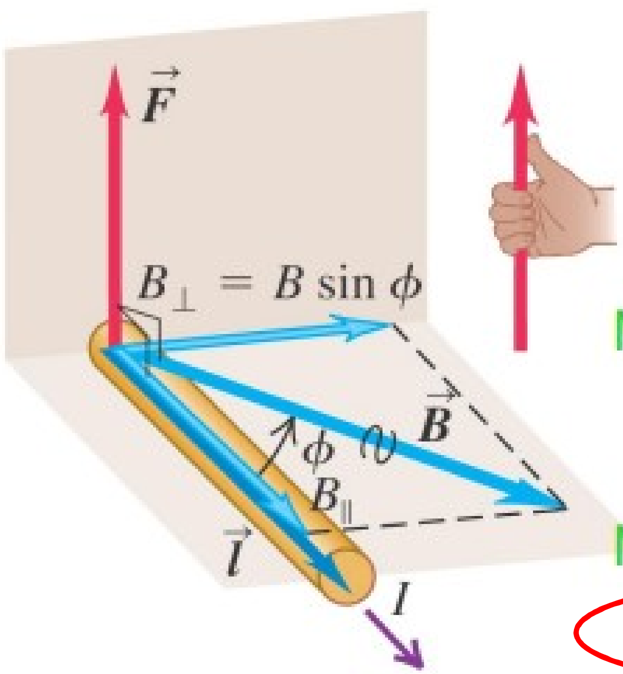


- Total force: $$F_m = (nAl)(qv_d B)$$

n = number of charges per unit volume

A l = volume

$$F_m = (nqv_d)(A)(lB) = \boxed{\text{iA}}(lB) = IlB$$ (B ⊥ wire)



In general:

$$F = IlB_\perp = IlB \sin \varphi$$

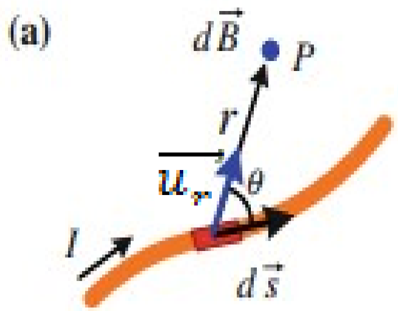
Magnetic force on a straight wire segment:

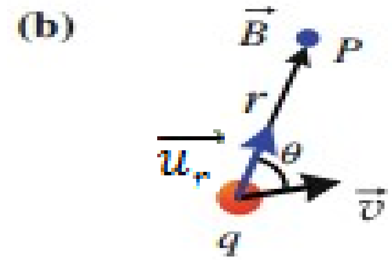
$$\boxed{\vec{F} = I\vec{l} \times \vec{B}}$$

Magnetic force on an infinitesimal wire section:

$$d\vec{F} = Id\vec{l} \times \vec{B}$$ ⇐ **Laplace's law**

The Biot-Savart Law





Based on quantitative experiments, Biot and Savart were able to arrive at a mathematical expression that describes the magnetic field at any point in terms of the current or the charge that produces the field.

Consider a point P at a distance r from: (a) an element $d\vec{s}$ chosen in the direction of a *steady* current I, (b) a point charge q moving with velocity $\vec{v}$, see Fig. Biot and Savart proposed that the magnetic field produced by the element, or by the charge, would be:

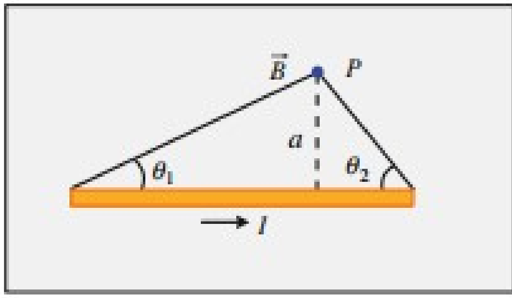
$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{I d\vec{s} \times \overrightarrow{u_r}}{r^2} \ and\ \vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \overrightarrow{u_r}}{r^2} \quad \text{(Biot-Savart law)}$$

where $\overrightarrow{u_r}$ is a unit vector directed from $d\vec{s}$ or q toward point P. The product $I\,d\vec{s}$ is called the *differential current element*, and μ_o is a constant called the **permeability of free space'** which has the exact value:

$$\mu_o = 4\pi \times 10^{-7}\ \text{T.m/A}$$

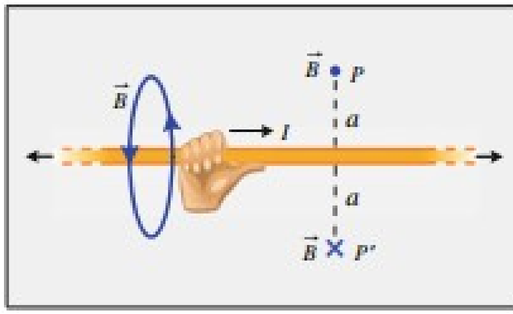
Some Applications of the Biot-Savart Law

2. Magnetic Field Surrounding a Thin Straight Wire



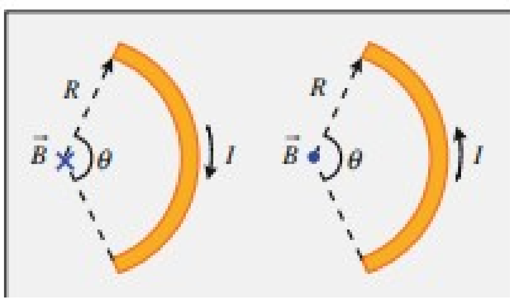
$$B = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 + \cos\theta_2)$$

3. Magnetic Field Surrounding a Very Long Straight Wire



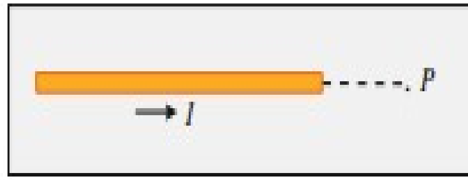
$$B = \frac{\mu_0 I}{2\pi a}$$

4. Magnetic Field Due to a Curved Wire Segment



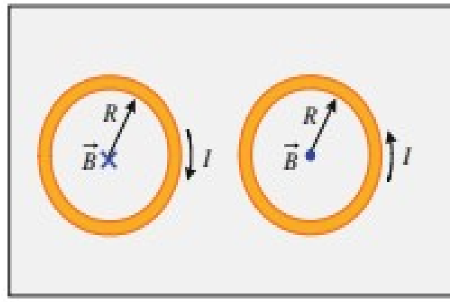
$$B = \frac{\mu_0 I}{4\pi R}\theta$$

1. Magnetic Field on the Extension of a Straight Wire



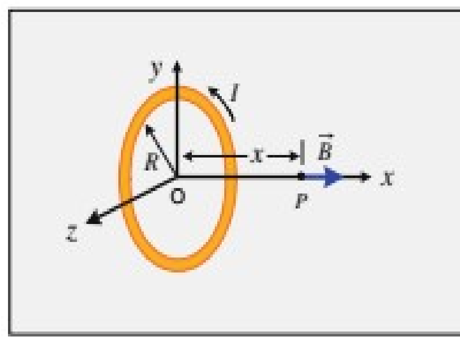
$$\vec{B} = 0$$

5. Magnetic Field at the Center of a Circular Wire Loop



$$B = \frac{\mu_0 I}{2R}$$

6. Magnetic Field on the Axis of a Circular Wire Loop



$$B = \frac{\mu_0 I}{2}\frac{R^2}{(R^2 + x^2)^{3/2}}$$

$$B = \frac{\mu_0 I R^2}{2x^3} \quad (\text{for } x \gg R)$$