Continuous regime and fundamental theorems

5. Fundamental theorems:

5.1 Dipole Associations

Dipoles are said to be in series if they are traversed by the same intensity of electric current. And they are said to be in parallel if they have the same potential difference at their terminals

5.1. 1 Series connection of resistors

Let there be n resistors connected in series and carrying the same current I (figure 7).

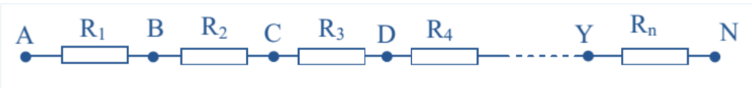


Figure 13. Series resistors

If we consider that the resistances between A and N behave as a single resistance and as the resistances are in series; then the same current I which passes from A to N therefore we can write:

$$U_{AN}=R_{eq}.\,\boldsymbol{I}$$

By applying Ohm's law to each of these resistances we can write the following relationships:

$$U_{AB}=R_1 I;\ U_{BC}=R_2 I; U_{CD}=R_3 I; U_{DE}=R_4 I;\ldots;U_{YN}=R_n I;$$

The ddp between the ends A and N i.e. of the circuit is equal to the sum of the ddp between A and B, between B and C, between C and D, …, and between Y and N. ie U_{AN} U_{AB} U_{BC} U_{CD} U_{YN}

$U_{AN}=R_1I+R_2I+R_3I+R_4I+\cdots+R_nI$

$U_{AN}=(R_1+R_2+R_3+R_4+\cdots+R_n)$

So by comparison we will have:

$R_{eq}=R_1+R_2+R_3+R_4+\cdots+Rn$

So we can conclude that the resistances of a branch (connected in series) are equivalent to a single resistance equal to the sum of these resistances of the latter.

5.1. 2 Parallel or shunt connection of resistors

Let us place several resistances (for example four resistances, figure 9) between two points N and M. The current I in the circuit creates several derived currents, the intensity of which is equal to the sum of the intensities of these derived currents.

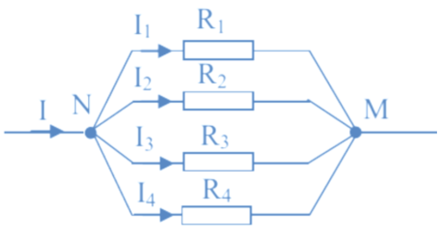


Figure 14. Resistors in parallel.

If we consider that the resistances between M and N behave like a single resistance, we can write: $\mathbf{U_{MN}=R_{eq}\,.\,I}$

So $$I = \frac{U_{MN}}{R_{eq}} = \left(\frac{1}{R_{eq}}\right) U_{MN}$$

We know that $I=I_1+I_2+I_3+I_4+.....+I_n$

Applying Ohm's law between nodes M and N to each of the resistors, knowing that the voltage between M and N is constant, we can write the following relations:R_1; R_2; R_3;R_4;….;R_n

$$U_{MN}=R_1I_1=R_2I_2=R_3I_3=R_4I_4=.......=R_nI_n$$

$$I_1 = \frac{U_{MN}}{R_1}; I_2 = \frac{U_{MN}}{R_2}; I_3 = \frac{U_{MN}}{R_3}; I_4 = \frac{U_{MN}}{R_4}; \dots\dots; I_n = \frac{U_{MN}}{R_n}$$

We replace the values of currents in the previous equation we will have ***I***

$$I = \frac{U_{MN}}{R_1} + \frac{U_{MN}}{R_2} + \frac{U_{MN}}{R_3} + \frac{U_{MN}}{R_4} + \cdots + \frac{U_{MN}}{R_n}$$

$$I = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \cdots + \frac{1}{R_n}\right) U_{MN}$$

Therefore $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \cdots + \frac{1}{R_n}$$

From the last relation, we can conclude that the inverse of the equivalent resistance is equal to the sum of the inverses of the resistances connected in parallel

The inverse of resistance is known as: conductance we can write the relationship G (G =1/R).

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \cdots + \frac{1}{R_n}$$

In the following manner using the conductance

$$G_{eq} = G_{1+}G_2 + G_3 + G_4 + \cdots + G_{eq}$$

5.2 Voltage divider:

When we have a series association of resistors, we can express the voltage across one of them, knowing the voltage across the whole.

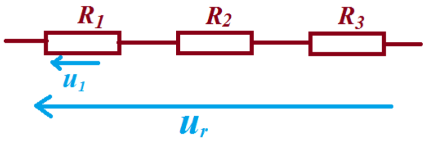


Figure 15

$$u_1 = u_t \cdot \frac{R_1}{R_1 + R_2 + R_3} = u_t \cdot \frac{R_1}{\sum R_i} \qquad u_t = \sum R_i \cdot I$$

5.3 Current divider:

When we have a parallel association of resistors, we can express the current in one of them, knowing the overall current. $G_i = 1/R_i$: is the conductance

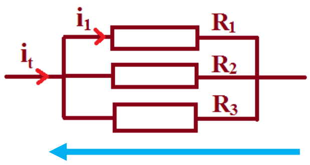


Figure 16

$$i_1 = i_t \cdot \frac{G_1}{G_1 + G_2 + G_3} = i_t \cdot \frac{G_1}{\sum G_i} = i_t \cdot \frac{1/R_1}{\sum 1/R_i}$$

we have $I_t = V(\sum^n{}_{k=1} G_k)$ and $I_j = V\,G_j$

The Curtrent in branche j is . . . $I_j = \dfrac{\frac{1}{R_j}}{\left(\sum_{k=1}^{n} \frac{1}{R_k}\right)} I$.

In terms of resistance . . . . $G_j I_j = \dfrac{G_j}{\left(\sum_{k=1}^{n} G_k\right)} I$

5.4 Superposition theorem:

Let us consider a linear network comprising n independent voltage and current sources that we can denote: S_1, S_2, . . ., S_n, and a quantity to be calculated, such as I_K the current in branch K. Let us call I_{K1}, I_{K2}, . . . , I_{Kn}, the values of this quantity created individually in this branch by each source acting alone assuming that the other sources being passivated or deactivated.

$I_K = I_{K1} + I_{K2} + \bullet \bullet \bullet + I_{Kn}$

NB: Pacify a source means replacing it with its internal resistance. In other words, this means short-circuiting the voltage sources and opening the current sources.

Example for the circuit below: U_1 and U_2 known, we wish to determine the voltage U_3

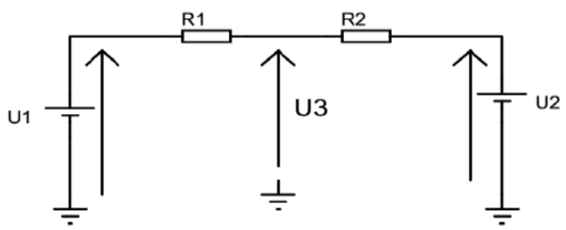


Figure 17

The value of the potential U_3 can be found in two steps:

- We turn off the source U_2 (replaced by a short circuit) and we calculate U_{31}, as a function of U_1, R_1 and R_2.
- We turn off the source U_1 (replaced by a short circuit) and we calculate U_{32}, as a function of U_2, R_1 and R_2.
- ➢ The potential difference U_3 is then $U_{31}+U_{32}$.

5.5 Thevenin's theorem

A linear network, comprising only independent sources of voltage, current and resistances, taken between two terminals behaves like a voltage generator E_0 in series with a resistance R0. The emf E_0 of the equivalent generator is equal to the voltage existing between the two terminals considered when the network is in open circuit. The resistance R_0 is that of the circuit seen from the two terminals when all the sources are off.

5.6 Norton's Theorem

Similarly, any linear network, not including controlled sources, taken between two of its terminals can be replaced by a current source I_0 in parallel with a resistance R_0. The intensity I_0 is equal to the short-circuit current, the two terminals being connected by a perfect conductor. The resistance R_0 is that of the circuit seen from the two terminals when all the sources are off.

5.7 Equivalence between Thevenin and Norton representations

The respective application of Thevenin and Norton's theorems allows us to show the equivalence of the following two circuits:

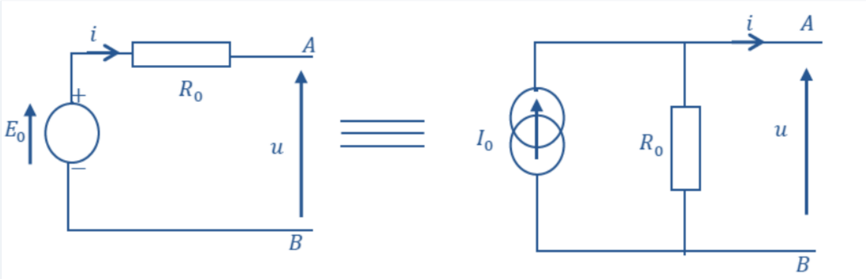


Figure 18. Thevenin equivalence Norton

With : $E_0 = R_0 I_0$

Transformations de source : Cas particulier

Il existe deux cas particuliers lorsqu'on fait des transformations de source. Pour le premier cas, on a une résistance en parallèle avec la source de tension. On peut ignorer cette résistance parallèle, comme `a la figure 1.

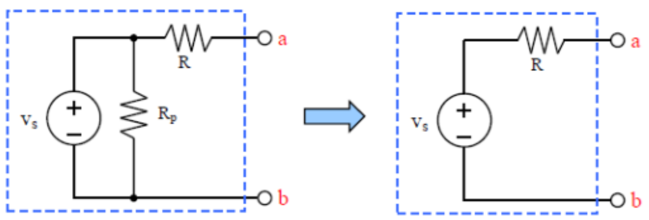


FIGURE 1 – Transformation de source : cas particulier 1

Pour le deuxième cas, il s'agit d'une résistance en série avec la source de courant. On peut ignorer la résistance série, comme `a la figure 2.

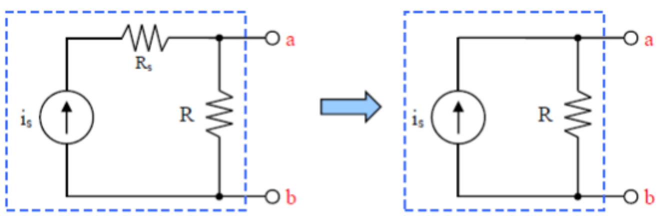


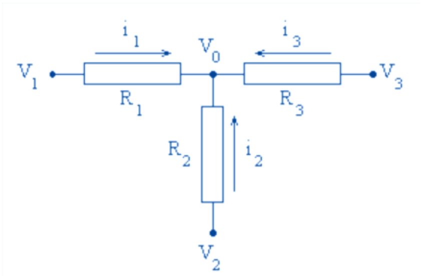
FIGURE 2 – Transformation de source : cas particulier 2

Pour les deux cas particuliers, un voltmètre placé entre a et b mesurera la même tension, et un ampèremètre placé entre a et b mesurera le même courant.

5.8 Millman's Theorem

For each of the branches we can write:

$$\begin{cases} V1-V0=R1I1 \\ V2-V0=R2I2 \\ V3-V0=R3I3 \end{cases} \qquad \text{Or again:} \qquad \begin{cases} I_1 = \frac{V_1-V_0}{R_1} \\ I_2 = \frac{V_2-V_0}{R_2} \\ I_3 = \frac{V_3-V_0}{R_3} \end{cases}$$



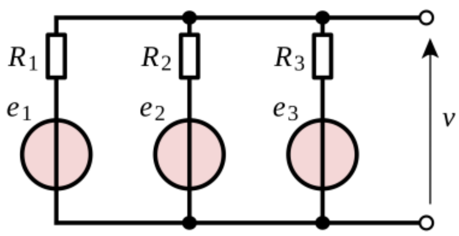
By adding up these relations we get: $I_1 + I_2 + I_3 = \frac{V_1-V_0}{R_1} + \frac{V_2-V_0}{R_2} + \frac{V_3-V_0}{R_3}$

And since $I_1+I_2+I_3=0$

we have therefore $V_0\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$ or $V_0 = \frac{\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$

This result generalizes to any number of branches: $V_0 = \frac{\sum_1^n \frac{V_k}{R_k}}{\sum_1^n \frac{1}{R_k}} = \frac{\sum_1^n G_k V_1}{\sum_1^n G_k}$

The voltage at the node is the average of the voltages across all dipoles weighted by the respective conductance.
Specifically, Millman's theorem is used to compute the voltage at the ends of a circuit made up of only branches in parallel.



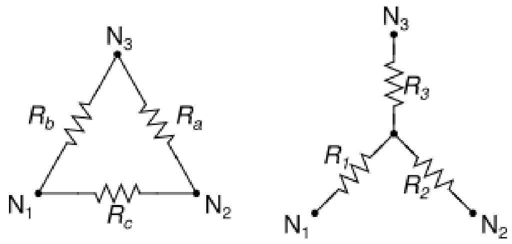
5.9 Kennelly's Theorem

This theorem, named in homage to Arthur Edwin Kennelly, allows you to move from a "triangle" configuration (or Δ, or Π, depending on how you draw the diagram) to a "star" configuration (or, similarly, Y or T). The diagram opposite is drawn in the "triangle-star" form; the diagrams below in the T-Π form.

This theorem is used in electrical engineering or in power electronics in order to simplify three-phase systems. It is also commonly used in electronics to simplify the calculation of filters or attenuators. The two circuits in Figure 17 are equivalent if the values of their resistances are related by the relationships shown below.

5.9. 1 Equations for the transformation from Δ to Y

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$
$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



5.9. 2 Equations for the transformation from Y to Δ

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$
$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

Or, if using admittance instead of resistance

$$Y_a = \frac{Y_3 Y_2}{\sum Y_Y}$$
$$Y_b = \frac{Y_3 Y_1}{\sum Y_Y}$$
$$Y_c = \frac{Y_1 Y_2}{\sum Y_Y}$$

3- <u>Maximum Power Transfer Theorem:</u>

In an electrical network, the generator is supposed to provide the necessary energy to a receiver that accepts it. Consider the elementary network consisting of a real voltage generator and a load resistance RU.

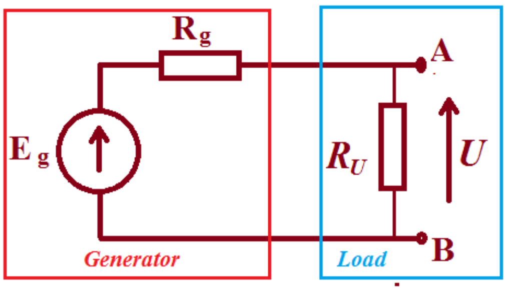


Figure 19

The power supplied by the generator is equal to:

$P_g = E.I = Rg.I_2 + R_U.I_2 = (Rg + R_U).\ I_2$

The power absorbed by the load:

$Pu = Ru\ .I_2$

How should R_U be chosen with respect to Rg so that the transmitted power is maximum?

We are looking for the optimal value of the utilization resistance $R_{U(opt)}$. To do this, let's calculate the P_U power as a function of Rg:

$$P_U = R_U.I^2 = R_U \left(\frac{E}{R_g + R_U}\right)^2$$

Let us study the law of variation of power by calculating its derivative:

$$\frac{\mathrm{d}P_U}{\mathrm{d}R_U} = \frac{E^2\left(R_g + R_U\right)^2 - E^2.R_U\left(2R_g + 2R_U\right)^2}{\left(R_g + R_U\right)^4} = \frac{E^2\left(R_g + R_U\right) \times \left(R_g + R_U - 2R_U\right)}{\left(R_g + R_U\right)^4}$$

The transmitted power is maximum (in mathematics, we say that the curve passes through an ***Extremum***) when this derivative is zero, that is to say for R_U = Rg.

It is then worth:

$$\boxed{P_{U(\max)} = \frac{E^2.R_U}{\left(R_g + R_U\right)^2} = \frac{E^2}{4R_g^2}}$$