Chapter 2:

a. Conductors in Electrostatic Equilibrium
b. **The Capacitor**

Conductors in Electrostatic Equilibrium

Conductors in Electrostatic Equilibrium

Conductors contain free electrons that can move freely. When there is no net motion of electrons within the conductor, the conductor is in electrostatic equilibrium

A conductor in electrostatic equilibrium has the following properties:

The excess charge on an isolated conductor lies on its outer surface

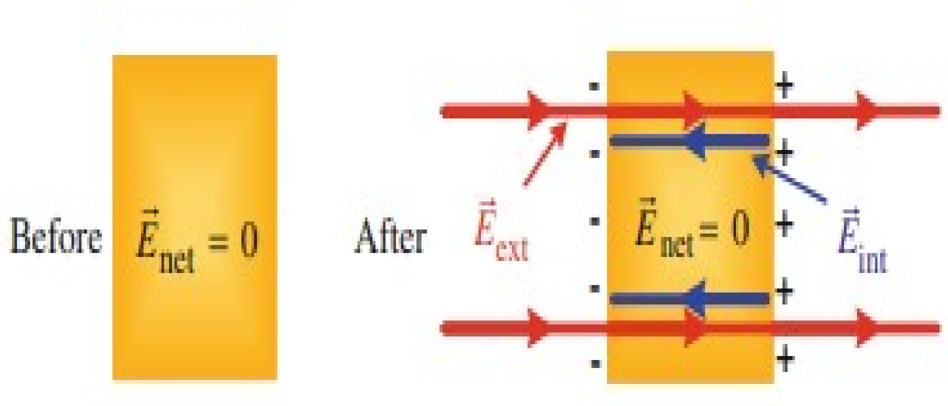
The electric field inside the conductor is zero and the electric potential inside the conductor is constant.

The electric field just outside a charged conductor at any point is perpendicular to its surface and has a magnitude $E = \sigma / \varepsilon_0$, where σ is the surface charge density at that point.

The surface charge density is greatest at locations where the radius of curvature of the surface is smallest.

The electric field is zero everywhere inside the conductor

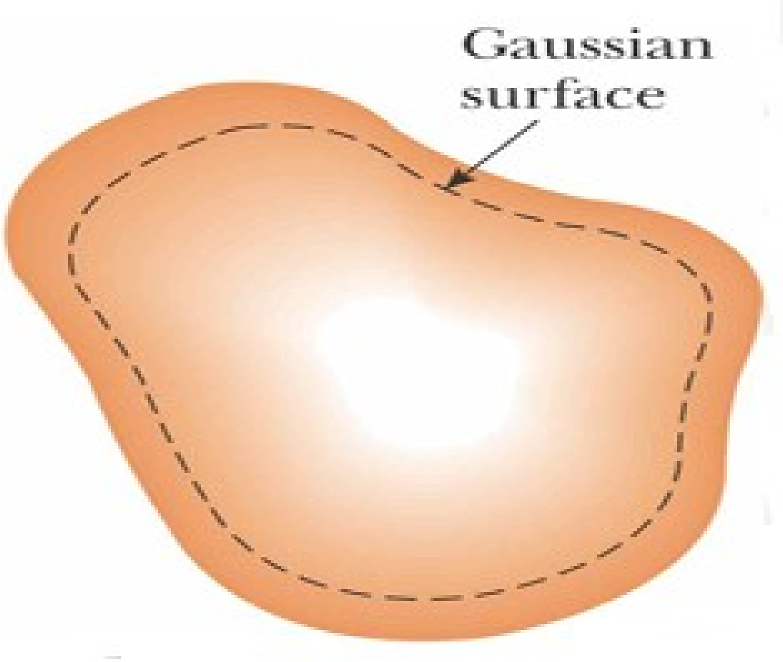
- The electric field inside the conductor must be zero under the assumption that we have electrostatic equilibrium



- If the field were not zero, free electrons in the conductor would experience an electric force(F=qE) and would accelerate due to this force (this motion means conductor is not in electrostatic equilibrium)
- When the external field is applied, the free electrons accelerate to the left, causing a plane of negative charge to be present on the left surface. The movement of electrons to the left results in a plane of positive charge on the right surface. These planes of charge create an additional electric field inside the conductor that opposes the external field. As the electrons move, the surface charge densities on the left and right surfaces increase until the magnitude of the internal field equals that of the external field, resulting in a net field of zero inside the conductor

If an isolated conductor carries a charge, the charge resides on its surface

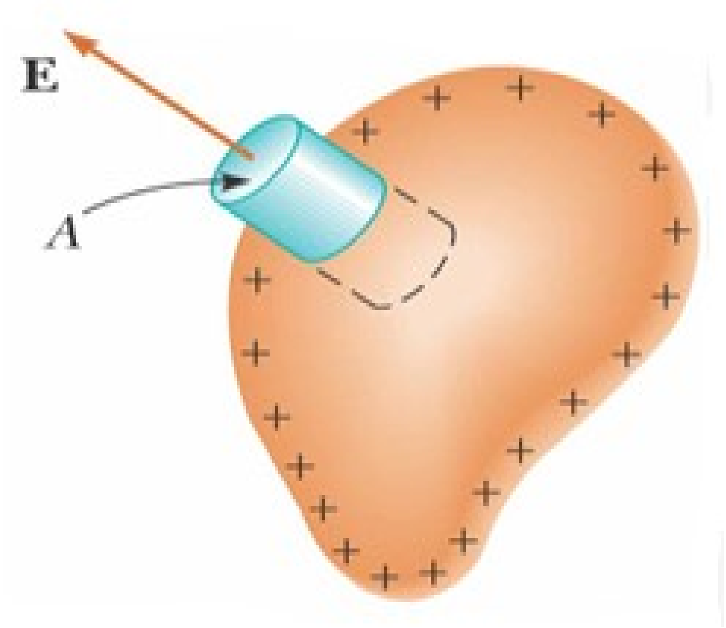
From Gauss's law, we conclude that the net charge inside the gaussian surface is zero.



Because there can be no net charge inside the gaussian surface (which is close to the conductor's surface), any net charge on the conductor must reside on its surface.

The electric field just outside a charged conductor is perpendicular to the conductor's surface

To determine the magnitude of the electric field, we draw a gaussian surface in the shape of a small cylinder whose end faces are parallel to the surface of the conductor

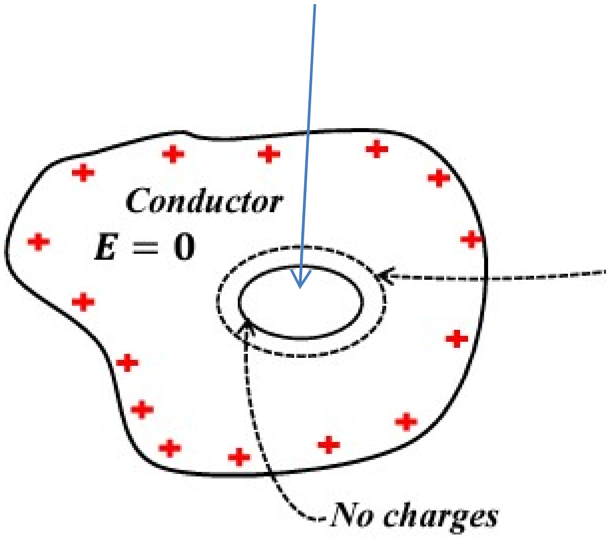


$$\Phi_E = EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

Charged conductor with a hole inside

Are there any charges on an interior surface?



Let's apply Gauss's law
Place a Gaussian surface around the hole.

$$\Phi_E = \oint \cancelto{0}{\vec{E}} \cdot d\vec{A} = \boxed{\frac{Q_{in}}{\varepsilon_0} = 0}$$

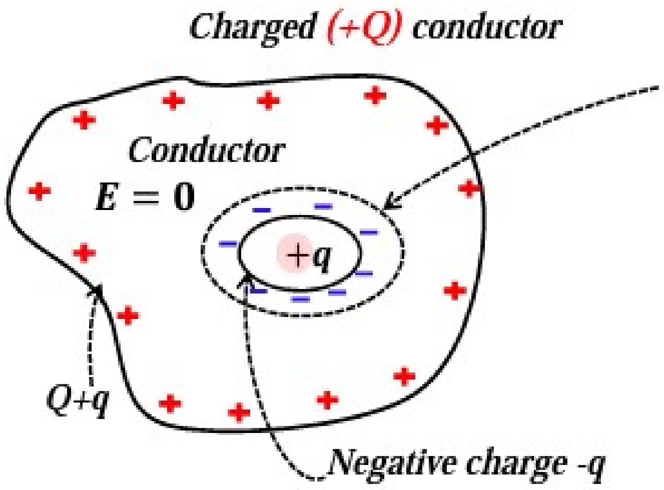
The electric flux is zero through the Gaussian surface, since E=0 inside the conductor

So Qin=0 (inside the Gaussian surface), there is no charge on the surface of the hole

Let’s put a charge inside the hole

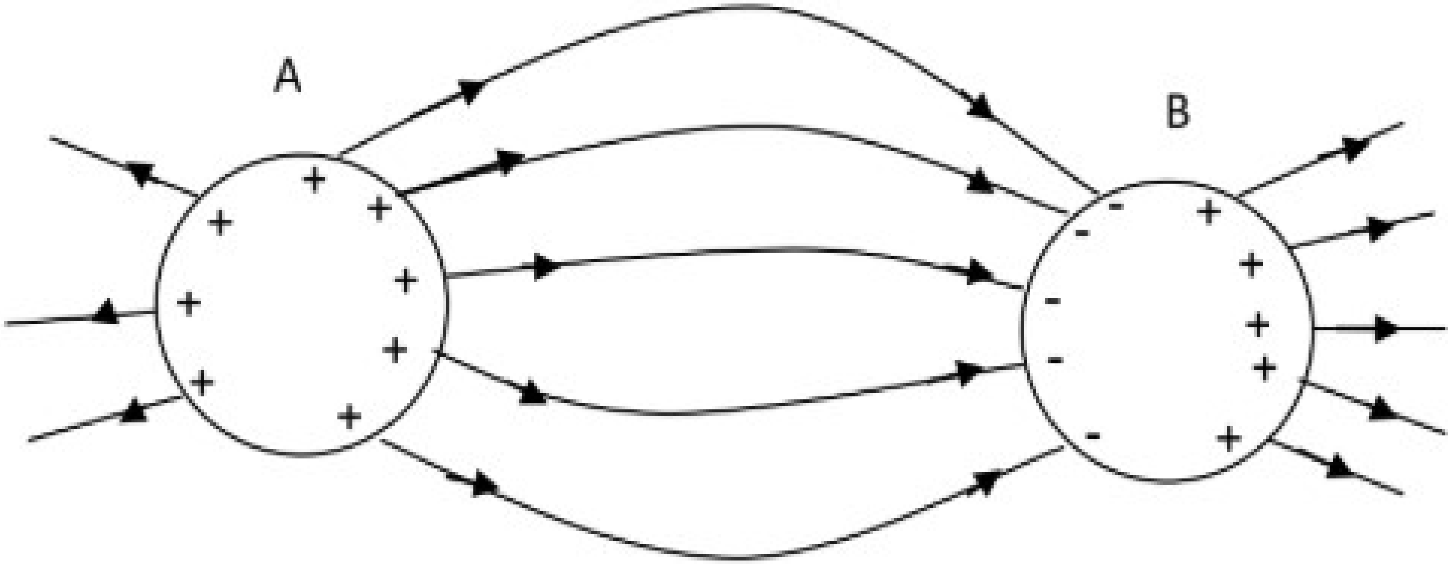
But, we know that there is +q inside, it means that there must be –q on the interior surface (+q charge induced –q on the surface)

Let’s count all charges inside the conductor to find the amount of charge on the exterior surface

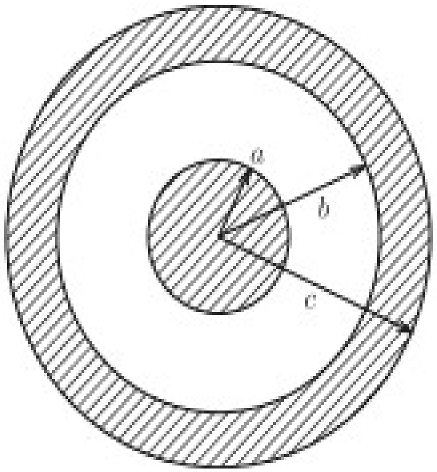


$$\underset{\text{Inner surface}}{-q} + Q_{outer\ surface} = \underset{\text{The conductor Is charged to } +Q}{+Q} \Rightarrow Q_{outer\ surface} = +Q + q$$

Partial effect

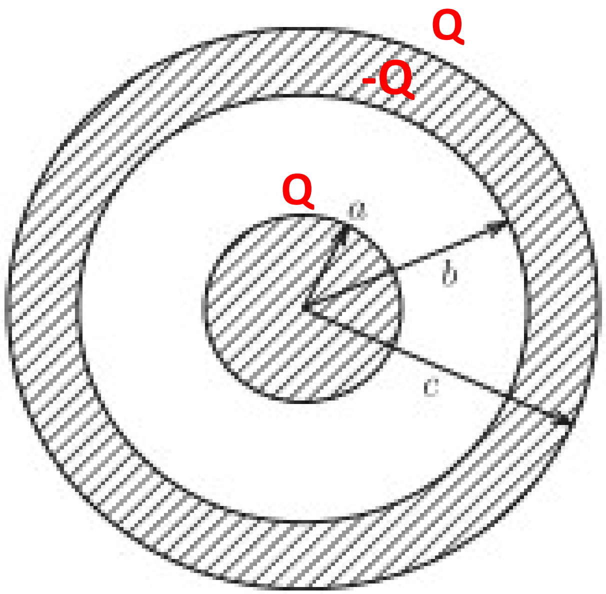


Only part of the electric field lines ends at the conductor B, and therefore this effect is called partial effect.



(Total effect): A conducting sphere of radius a carries a net positive charge . A conducting spherical shell of inner radius b(b > a) and outer radius c. This shell is concentric with the conducting sphere. Determine the distribution of the electric charges on conductors (conducting spherical and conducting spherical shell) . Determine the electric field strength and electric potential in all regions when the electric charge, Q, is given on the inner conductor.

1- Distribution of electric charges



1- A conducting sphere

We can assume that Q is uniformly distributed on the surface (r = a) of the inner conductor because of the spherical symmetry

2- A conducting spherical shell

- The electric charge appears on the inner surface (r = b), is denoted by Qb and is equal to opposite the charge of the inner conductor (Qb=-Q).
- The electric charge appears on the outer surface (r = c), is denoted by Qc and is equal The electric charge of a conducting spherical shell - The electric charge of the inner surface Qc=0-Qb=0-(-Q)=Q

2- Finding the electric field

We apply Gauss' law to a spherical surface, A, of radius r with the same center as that of the conductors

$$\Phi_E = \oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}} = \mathbf{E}\oint d\mathbf{A} = \mathbf{EA} = \mathbf{E4\pi r^2} = \frac{Q_{enclosed}}{\varepsilon_0} \Leftrightarrow \mathbf{E} = \frac{Q_{enclosed}}{4\pi r^2 \varepsilon_0} = \frac{KQ_{enclosed}}{r^2}$$

case	**radius r**	**the total electric charge inside the spherical surface A**	**The electric field**
1	$0 < r < a$	Q1=0	$\mathbf{E}_1 = 0$
2	$a < r < b$	Q2=Q	$\mathbf{E} = \frac{KQ}{r^2}$
3	$b < r < c$	Q3=Q+(-Q)=0	$\mathbf{E}_3 = 0$
4	$c < r$	Q4=Q+(-Q)+Q=Q	$\mathbf{E} = \frac{KQ}{r^2}$

3- Finding the electric potential

We determine the electric potential from $V=-\int Edr$

$4)\ c \le r$

$$V_4 = -\int E_4 dr = -\int \frac{\mathrm{KQ}}{r^2} dr = \frac{\mathrm{KQ}}{r} + C_4$$

Using the condition of electric potential is zero at infinity. This gives

$$\lim_{r \to \infty} \mathrm{V}_4 = 0 + C_4 = 0 \Leftrightarrow C_4 = 0$$

we write the electric potential as $V_4 = \frac{\mathrm{KQ}}{r}$

$3)\ b \le r \le c$

$$V_3 = -\int E_3 dr = -\int 0\, dr = C_3$$

Using the condition of continuity of potential. This gives

$$\mathrm{V}_3(\mathrm{c}) = \mathrm{V}_4(\mathrm{c}) \Leftrightarrow \mathrm{C}_3 = \frac{\mathrm{KQ}}{c}$$

we write the electric potential as $V_3 = \frac{\mathrm{KQ}}{c}$

$2)\ a \le r \le b$

$$V_2 = -\int E_2 dr = -\int \frac{\mathrm{KQ}}{r^2} dr = \frac{\mathrm{KQ}}{r} + C_2$$

Using the condition of continuity of potential. This gives

$$\mathrm{V}_3(\mathrm{b}) = \mathrm{V}_2(\mathrm{b}) \Leftrightarrow \frac{\mathrm{KQ}}{b} + C_2 = \frac{\mathrm{KQ}}{c} \Leftrightarrow C_2 = \frac{\mathrm{KQ}}{c} - \frac{\mathrm{KQ}}{b} \Leftrightarrow C_2 = \mathrm{KQ}\left(\frac{1}{c} - \frac{1}{b}\right)$$

we write the electric potential as

$$V_2 = \frac{\mathrm{KQ}}{r} + \mathrm{KQ}\left(\frac{1}{c} - \frac{1}{b}\right) \Leftrightarrow V_2 = \mathrm{KQ}\left(\frac{1}{r} + \frac{1}{c} - \frac{1}{b}\right)$$

$1)\ 0 \prec r \le A$

$$V_1 = -\int E_1 dr = -\int 0\, dr = C_1$$

Using the condition of continuity of potential. This gives

$$\mathrm{V}_1(\mathrm{a}) = \mathrm{V}_2(\mathrm{a}) \Leftrightarrow C_1 = \mathrm{KQ}\left(\frac{1}{a} + \frac{1}{c} - \frac{1}{b}\right)$$

we write the electric potential as

$$V_1 = \mathrm{KQ}\left(\frac{1}{a} + \frac{1}{c} - \frac{1}{b}\right)$$

Here, we suppose that the outer conductor in Example is grounded

Grounding is a method to make the electric potential of a conductor zero by connecting it to the ground. It sometimes accompanies transfer of electric charge. In the above case, the electric charge on the outer surface (r = c) of the outer conductor transfers to the ground through the grounding.

The electric field

1) $0 \leq r \prec a$

$E_1 = 0$

2) $a \prec r \prec b$

$$E_2 = \frac{KQ}{r^2}$$

3) $b \prec r$

$E_3 = 0$

The electric potential

1) $0 \prec r \leq a$

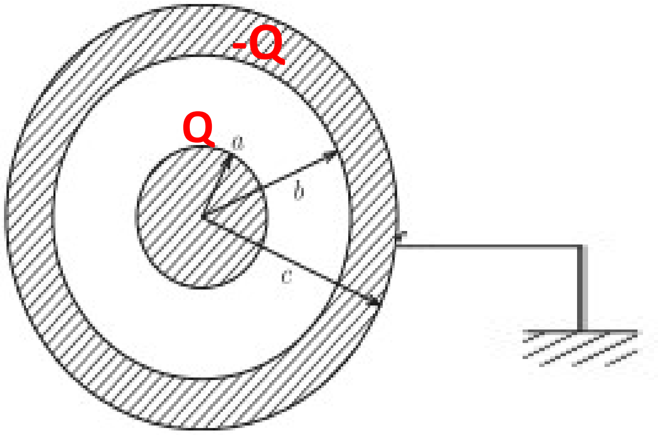
$$V_1 = KQ\left(\frac{1}{a} - \frac{1}{b}\right)$$

2) $a \leq r \leq b$

$$V_2 = KQ\left(\frac{1}{r} - \frac{1}{b}\right)$$

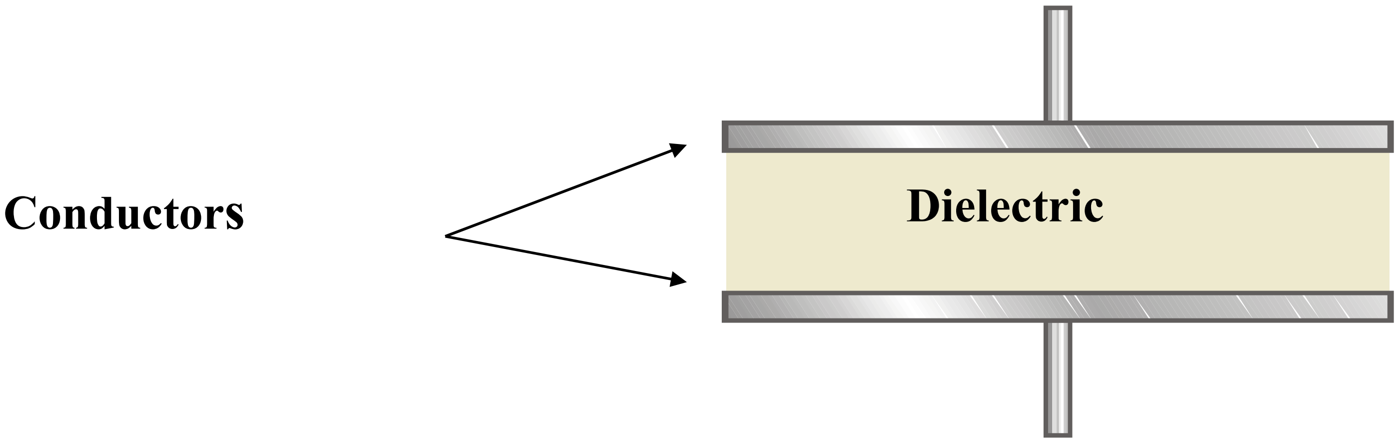
3) $b \leq r$

$V_3 = 0$



The Capacitor

Capacitors are one of the fundamental passive components. In its most basic form, it is composed of two plates separated by a dielectric. The ability to store charge is the definition of capacitance.



Capacitance

Capacitance: is the ratio of charge to voltage

$$C = \frac{Q}{V}$$

V is really |ΔV|, the potential difference across the capacitor

So the amount of charge on a capacitor can be determined using the following formula

$$Q = C \times V$$

capacitance C is a device property, it is always positive

unit of C: farad (F)

1 F is a large unit, most capacitors have values of C ranging from pico farads to microfarads (pF to μF). micro ⇒10^{-6}, nano ⇒10^{-9}, pico ⇒10^{-12}

The energy of a charged capacitor is given by the equation

$$W = \frac{1}{2}CV^2$$

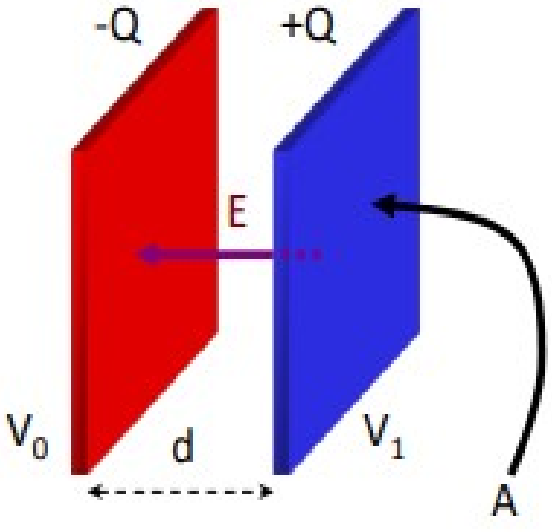
Other expressions for energy

$$W = \frac{1}{2}QV$$

$$W = \frac{1}{2}\frac{Q^2}{C}$$

Capacitance of parallel plate capacitor

electric field between two parallel charged plates:



$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \ .$$

Q is magnitude of charge on either plate.

potential difference:

$$\Delta V = V_1 - V_0 = -\int_0^d \vec{E} \cdot d\vec{\ell} = E\int_0^d dx = Ed$$

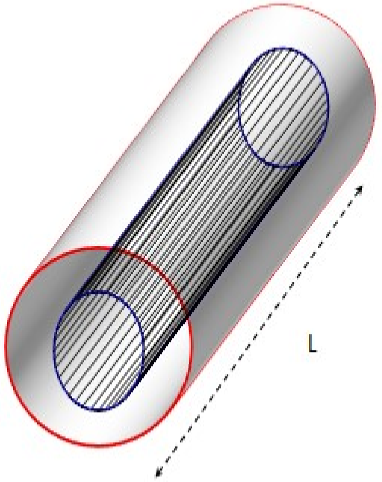
capacitance:

$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed} = \frac{Q}{\left(\dfrac{Q}{\varepsilon_0 A}\right)d} = \frac{\varepsilon_0 A}{d}$$

Capacitance of coaxial cylinder and Concentric Spheres

- **capacitors do not have to consist of parallel plates, other geometries are possible**

- **capacitor made of two coaxial cylinders:**



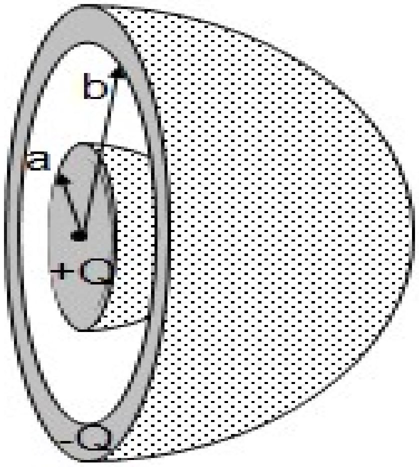
Calculating the capacitance of a concentric spherical capacitor of charge Q…

In between the spheres (Gauss' Law)

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

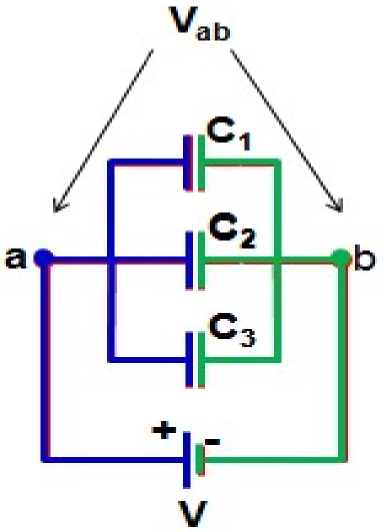
$$|\Delta V| = \frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$C = \frac{Q}{|\Delta V|} = \frac{4\pi\varepsilon_0}{\left[\frac{1}{a} - \frac{1}{b}\right]}$$



Circuits Containing Capacitors in Parallel

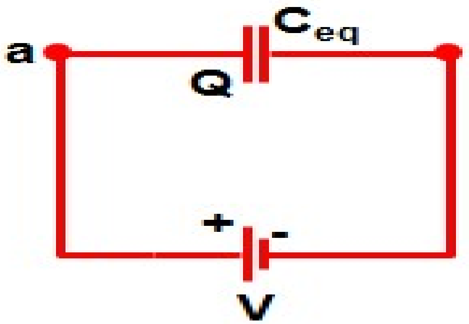
Capacitors connected in parallel:



all three capacitors must have the same potential difference (voltage drop) $V_{ab} = V$

$\Rightarrow Q_1 = C_1 V$
and $Q_2 = C_2 V$
and $Q_3 = C_3 V$

Imagine replacing the parallel combination of capacitors by a single equivalent capacitor



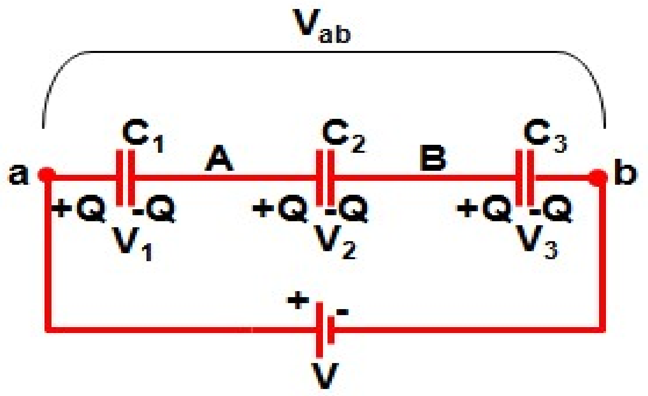
“equivalent” means “stores the same total charge if the voltage is the same.”

$Q_{total} = C_{eq} V = Q_1 + Q_2 + Q_3$ Using $Q_1 = C_1 V$, etc., gives

$C_1 V + C_2 V + C_3 V = C_{eq} V$ $C_1 + C_2 + C_3 = C_{eq}$ (after dividing both sides by V)

Generalizing: $C_{eq} = \sum_i C_i$ (capacitances in parallel add up)

Circuits Containing Capacitors in Series



The charges on C_1, C_2, and C_3 are the same, and are

$$Q = C_1 V_1 \qquad Q = C_2 V_2 \qquad Q = C_3 V_3$$

The voltage drops across C_1, C_2, and C_3 add up

$$V_{ab} = V_1 + V_2 + V_3$$

Substituting for V_1, V_2, and V_3:

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$Q = C_{eq} V$ Substituting for V:

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

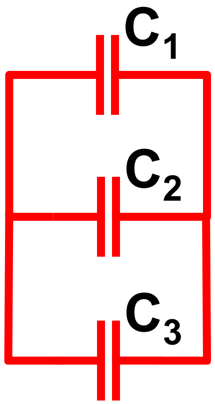
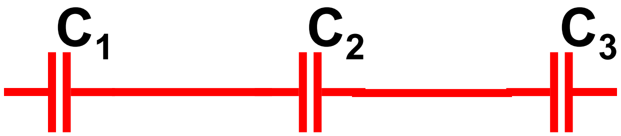
Dividing both sides by Q:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Generalizing:

$$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$$

Summary

	Parallel 	Series 
equivalent capacitance	$C_{eq} = \sum_i C_i$	$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$
charge	Q’s add	same Q
voltage	same V	V’s add