Chapter 1:

a. Electric force
b. Electric Field
c.Electric Potential
d. Electric Dipole
E. Gauss's theorem

Electrostatic

Introduction

Electrostatics is a branch of physics that deals with the phenomena and properties of stationary or slow-moving electric charges with no acceleration.

Electric Charge

- There are 2 types basically, positive (protons) and negative (electrons)
- LIKE charges REPEL and OPPOSITE charges ATTRACT
- The symbol for CHARGE is “q”
- The unit is the COULOMB(C), named after Charles Coulomb

Some important constants

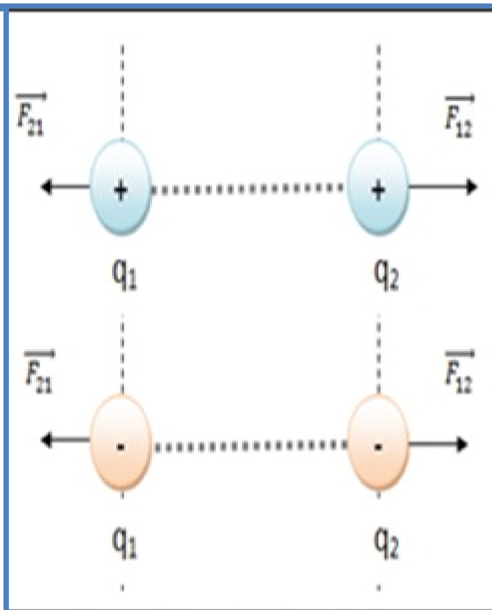
Particle	Charge	Mass
Proton	$1.6x10^{-19}$ C	$1.67 x10^{-27}$ kg
Electron	$-1.6x10^{-19}$ C	$9.11 x10^{-31}$ kg

Electric force

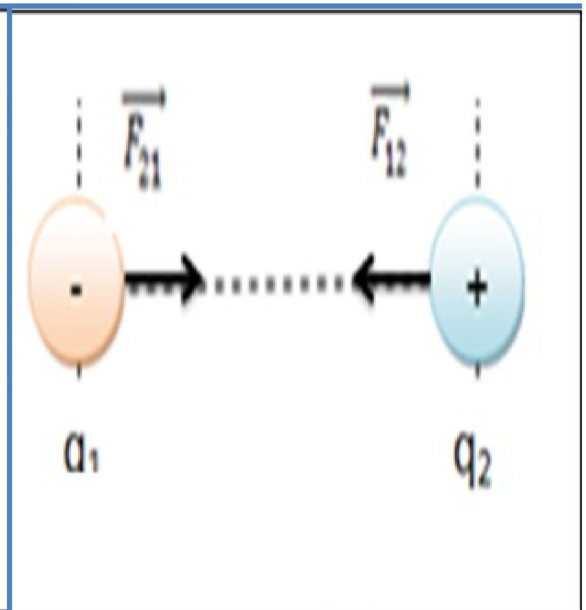
Electric force between electric charges, and this force is called the **Coulomb force**.

•The force between two electric charges of the same kind (i.e. both positive or both negative) is **repulsive**

•the force between electric charges of different kinds (i.e. one positive and one negative) is **attractive** as shown in Fig.

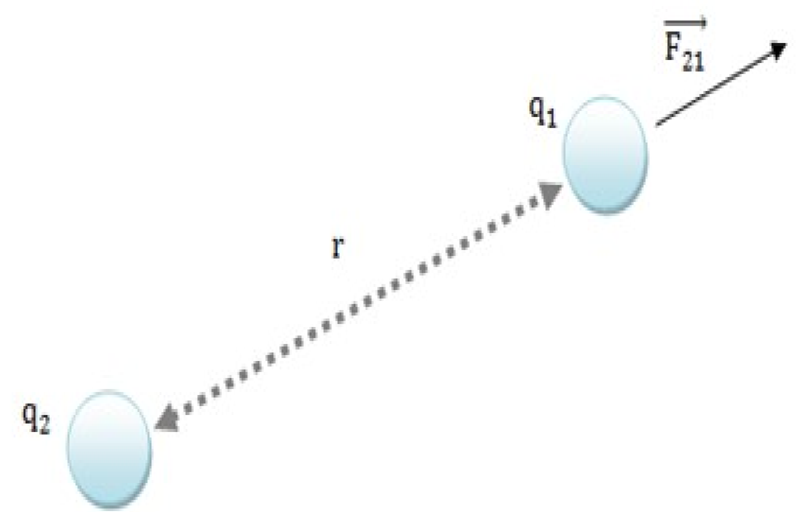


Repulsive Coulomb force



Attractive Coulomb force

We denote the direction vector of point charge q_1 measured from the position of q_2 as $\vec{r}$, as shown in Fig. Then, its magnitude is $r = \|\overrightarrow{r}\|$ and the unit vector pointing from q_2 to q_1 is $\vec{u} = \frac{\vec{r}}{r}$.



Electric force

The force between two point charges , q_1 and q_2 in vacuum, separated by distance r is expressed as

$\vec{F}_{21} = K\frac{q_2 q_1}{r^2}\vec{u}$ and $K = \frac{1}{4\pi\epsilon_0}$ .Where

K is the **Coulomb constant** and it in SI units has the value $\mathbf{K = 9 \times 10^9 Nm^2 / C^2}$.

ϵ_0 is the **permittivity of vacuum** given by

$\mathbf{\epsilon_0 = 8.8542 \times 10^{-12} C/Nm^2}$.

The magnitude of the force is proportional to the product of the two electric charges and is inversely proportional to the square of the distance. The magnitude of the force is

$$F_{2/1} = \frac{K|q_2||q_1|}{r^2}$$

The SI unit of the Coulomb force $\vec{F}$ is newton (N).

Coulomb's Law for a system of charges:

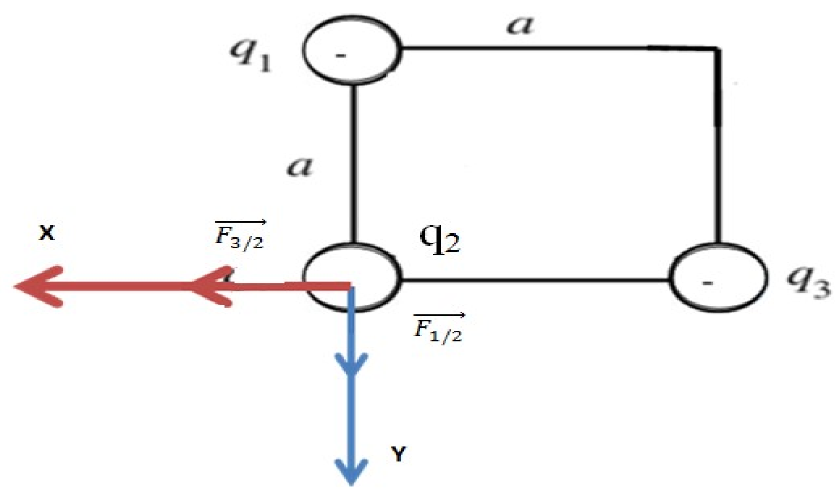
Consider a system of N charges (positive and negative), $q_1, q_2, \ldots, q_N$. The force exerted on any charge i from all other charges is

$$\overrightarrow{F_1} = \sum_{J=1\neq I}^{N} \overrightarrow{F_{J/1}} = \sum_{J=1\neq I}^{N} K\frac{q_J q_I}{r_{J/I}^2}\overrightarrow{U_{J1}}$$

where $\mathbf{r_{ji}}$ is a unit vector directed from q_j to q_i.

Example :

Consider the three point charges $q_1 = q_2 = q_3 = -q$, that are shown in Fig. Find the resultant force exerted on the charge q_2 by the two charges q_1 and q_3.



Solution :

•Finding the resultant force vector exerted on q_2.

Using the Coulomb's law, $\vec{F_2} = \overrightarrow{F_{1/2}} + \overrightarrow{F_{3/2}}$, and $\vec{F_2} = F_{2x}\vec{i} + F_{2y}\vec{j}$

Where $\vec{F_2}\begin{pmatrix} F_{2_x} = F_{1/2_x} + F_{3/2_x} \\ F_{2_y} = F_{1/2_y} + F_{3/2_y} \end{pmatrix}$

- **Calculating the force vector $\overrightarrow{F_{3/2}}$**

$$\overrightarrow{F_{3/2}}\begin{pmatrix} F_{3/2_x} = F_{3/2} \\ F_{3/2_Y} = 0 \end{pmatrix}$$

From Coulomb's law, we calculate the magnitude of the force $\overrightarrow{F_{3/2}}$ as follows:

$$F_{3/2} = k\frac{|q_3||q_2|}{r_{3/2}^2} = \frac{kq^2}{a^2} \quad \text{Thus} \quad \overrightarrow{F_{2/3}}\begin{pmatrix} F_{2/3_x} = \frac{kq^2}{a^2} \\ F_{2/3_y} = 0 \end{pmatrix}$$

- **Calculating the force vector $\overrightarrow{F_{1/2}}$**

$$\overrightarrow{F_{1/3}}\begin{pmatrix} F_{1/2_x} = 0 \\ F_{1/2_y} = F_{1/2} \end{pmatrix}$$

From Coulomb's law, we calculate the magnitude of the force $\overrightarrow{F_{1/2}}$ as follows:

$$F_{1/2} = k\frac{|q_1||q_2|}{r_{1/2}^2} = \frac{kq^2}{a^2} \quad \text{Thus} \quad \overrightarrow{F_{1/2}}\begin{pmatrix} F_{1/2_x} = 0 \\ F_{1/2_y} = \frac{kq^2}{a^2} \end{pmatrix}$$

$$\text{Then, } \vec{F_2}\begin{pmatrix} F_{2_x} = F_{1/2_x} + F_{3/2_x} \\ F_{2_y} = F_{1/2_y} + F_{3/2_y} \end{pmatrix} \Rightarrow \vec{F_2}\begin{pmatrix} F_{2_x} = \frac{kq^2}{a^2} + 0 \\ F_{2_y} = 0 + \frac{kq^2}{a^2} \end{pmatrix}$$

$$\Rightarrow \vec{F_2}\begin{pmatrix} F_{2_x} = \frac{kq^2}{a^2} \\ F_{2_y} = \frac{kq^2}{a^2} \end{pmatrix} \Rightarrow \vec{F_2} = \frac{kq^2}{a^2}\vec{i} + \frac{kq^2}{a^2}\vec{j}$$

The magnitude of the force

$$F_2 = \sqrt{F_{2_x}{}^2 + F_{2_y}{}^2} = \sqrt{2F_{2_x}{}^2} = \sqrt{2}\,\frac{kq^2}{a^2}$$

The second method

Using the Coulomb's law, $\overrightarrow{F_2} = \overrightarrow{F_{1/2}} + \overrightarrow{F_{3/2}}$

Thus: $F_2 = \sqrt{F_{1/2}{}^2 + F_{3/2}{}^2 + 2F_{1/2}F_{3/2}\cos 90^\circ}$

$$\Rightarrow F_2 = \sqrt{F_{1/2}{}^2 + F_{3/2}{}^2}$$

Where $\begin{cases} F_{3/2} = \dfrac{kq^2}{a^2} \\ F_{1/2} = \dfrac{kq^2}{a^2} \end{cases} \Rightarrow F_{1/2} = F_{3/2}$

Then

$$F_2 = \sqrt{F_{1/2}{}^2 + F_{3/2}{}^2} \Rightarrow F_2 = \sqrt{2F_{1/2}{}^2} = \sqrt{2}\,\frac{kq^2}{a^2}$$

Electric Field due to a Point Charge q_2

Definition:

The electric field vector $\vec{E}$ at a point P in space is defined as the electric force vector $\vec{F}$acting on test charge q_1 located at that point divided by the test charge:

$$\vec{E_p} = \frac{\vec{F_{2/1}}}{q_1}$$

Using the definition of electric field vector at point p and the Coulomb's law, the electric field vector at a distance r from a point charge q_2 is given by

$$\vec{E}_{2P} = \frac{K\frac{q_1 q_2}{r^2}}{q_1}\vec{u} \Rightarrow \vec{E}_{2P} = K\frac{q_2}{r^2}\vec{u}$$

Where $\vec{u}$ is unit vector pointing from q_2 to P.

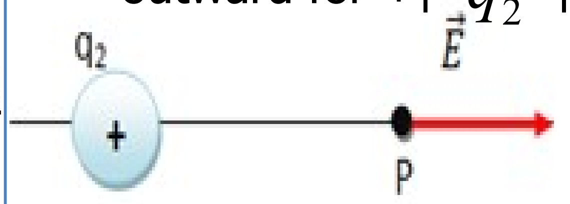
The magnitude of the Electric field vector is

$$E_{2P} = \frac{K|q_2|}{{r_{2/p}}^2}$$

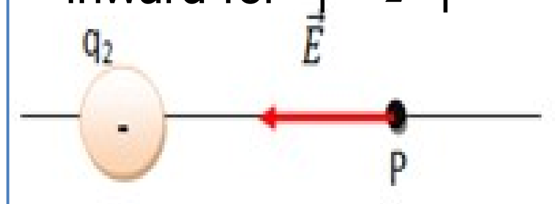
The SI unit of the electric field $\vec{E}$ is (N/C).This is also expressed as [V/m]

The direction of the electric field $\vec{E}$.

outward for + $|q_2|$



inward for - $|q_2|$



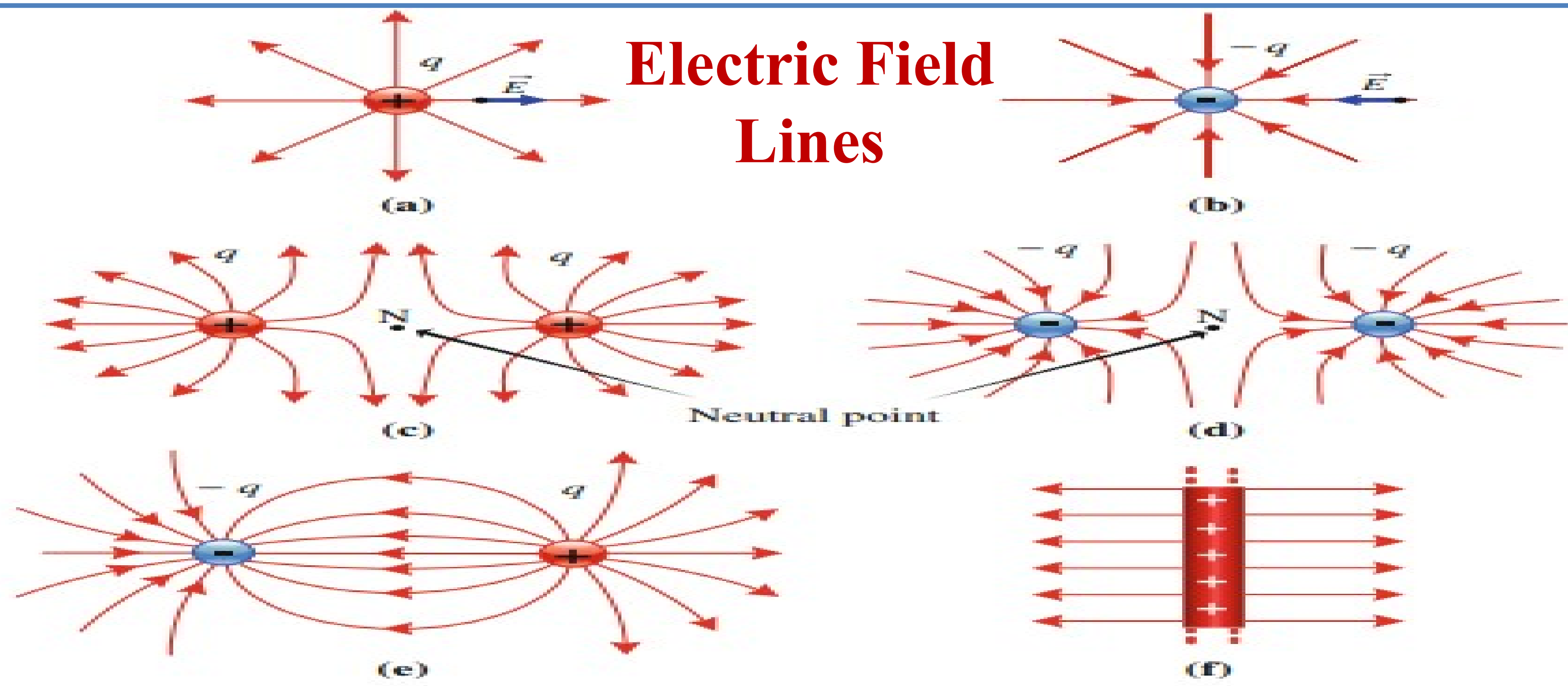
Electric Field due to a group of individual charge

$$\vec{F}_i = \vec{F}_{i1} + \vec{F}_{i2} + \ldots + \vec{F}_{in}$$

$$\vec{E}_i = \frac{\vec{F}_i}{q_i} = \frac{\vec{F}_{i1}}{q_i} + \frac{\vec{F}_{i2}}{q_i} + \ldots + \frac{\vec{F}_{in}}{q_i}$$
$$= \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n$$

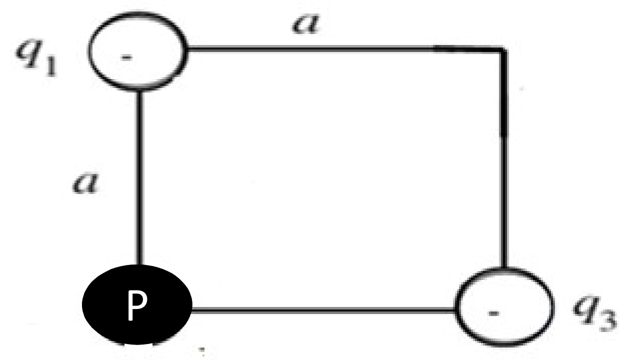
$$\vec{E}_i = \frac{1}{4\pi\varepsilon_0} \sum_j \frac{q_j}{r_{ij}^{\,2}} \overrightarrow{Ur}_{ij}$$

Electric Field Lines



Example :

Two point charges $q_1 = q_3 = -q$, are placed at the two corners of a square of side a see Fig. Find the electric field vector at the point P.



Solution :

- Finding the electric field vector at the point P

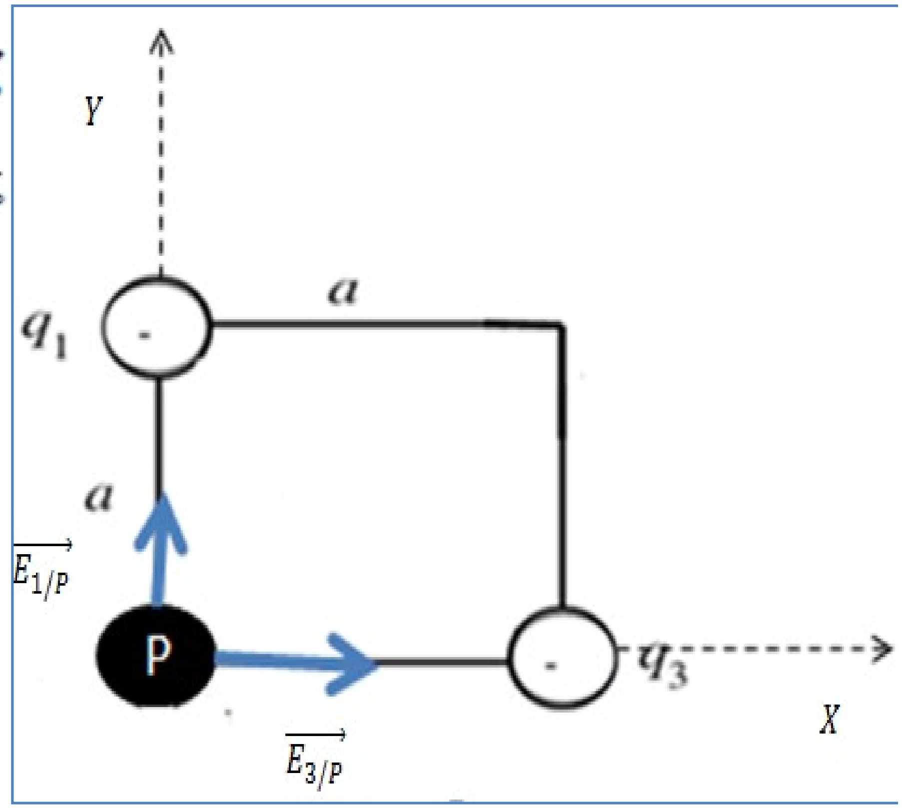
The first method

Using superposition principle, $\overrightarrow{E_P} = \overrightarrow{E_{1/P}} + \overrightarrow{E_{3/P}}$

Thus: $E_P = \sqrt{{E_{1/P}}^2 + {E_{3/P}}^2 + 2E_{1/P}E_{3/P}\cos 90^\circ}$

$\Rightarrow E_P = \sqrt{{E_{1/P}}^2 + {E_{3/P}}^2}$

Where $\begin{cases} E_{3/P} = k\dfrac{|q_3|}{{r_{3/P}}^2} = \dfrac{kq}{a^2} \\ E_{1/P} = k\dfrac{|q_1|}{{r_{1/P}}^2} = \dfrac{kq}{a^2} \end{cases}$ $\Rightarrow E_{1/P} = E_{3/P}$



Then

$E_P = \sqrt{{E_{1/P}}^2 + {E_{3/P}}^2} \Rightarrow E_P = \sqrt{2{E_{1/P}}^2} = \sqrt{2}\dfrac{kq}{a^2}$

The second method

Using superposition principle,

$\overrightarrow{E_P} = \overrightarrow{E_{1/P}} + \overrightarrow{E_{3/P}}$ Where

$$\overrightarrow{E_P}\begin{pmatrix} E_{P_x} = F_{1/P_x} + F_{3/P_x} \\ E_{P_y} = F_{1/P_y} + F_{3/P_y} \end{pmatrix}$$

- **Calculating the electric field vector $\overrightarrow{E_{3/P}}$**

$$\overrightarrow{E_{3/P}}\begin{pmatrix} E_{3/P_x} = E_{3/P} \\ E_{3/P_Y} = 0 \end{pmatrix}$$

$E_{3/P} = k\frac{|q_3|}{r_{3/P}{}^2} = \frac{kq}{a^2}$ Thus $\overrightarrow{E_{3/P}}\begin{pmatrix} E_{3/P_x} = \frac{kq}{a^2} \\ E_{3/P_y} = 0 \end{pmatrix}$

- **Calculating the electric field vector $\overrightarrow{E_{1/P}}$**

$$\overrightarrow{E_{1/P}}\begin{pmatrix} E_{1/P_x} = 0 \\ E_{1/P_y} = E_{1/P} \end{pmatrix}$$

$E_{1/P} = k\frac{|q_1|}{r_{1/P}{}^2} = \frac{kq^2}{a^2}$ Thus $\overrightarrow{E_{1/P}}\begin{pmatrix} E_{1/P_x} = 0 \\ E_{1/P_y} = \frac{kq}{a^2} \end{pmatrix}$

Then, $\overrightarrow{E_P}\begin{pmatrix} E_{P_x} = E_{\frac{1}{P}x} + E_{\frac{3}{P}x} \\ E_{P_y} = E_{\frac{1}{P}y} + E_{\frac{3}{P}y} \end{pmatrix}$

$$\Rightarrow \overrightarrow{E_P}\begin{pmatrix} E_{P_x} = \frac{kq}{a^2} + 0 \\ E_{P_y} = 0 + \frac{kq}{a^2} \end{pmatrix}$$

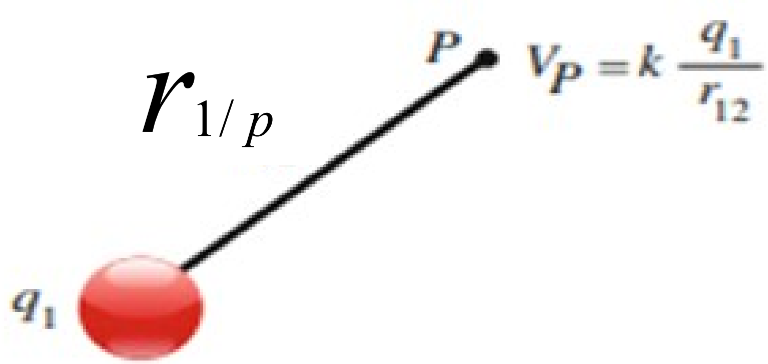
$$\Rightarrow \overrightarrow{E_P}\begin{pmatrix} E_{P_x} = \frac{kq}{a^2} \\ E_{P_y} = \frac{kq}{a^2} \end{pmatrix} \Rightarrow \overrightarrow{E_P} = \frac{kq}{a^2}\vec{i} + \frac{kq}{a^2}\vec{j}$$

- **Finding the magnitude of the electric field vector $\overrightarrow{E_P}$**

$$E_P = \sqrt{E_{P_x}{}^2 + E_{P_y}{}^2} = \sqrt{2E_{P_x}{}^2} = \sqrt{2}\frac{kq}{a^2}$$

Electric Potential of a Point Charge

$$V_{1/p} = k\frac{q_1}{r_{1/p}}$$



Potential from a system of N point charges:

$$V_p = \sum_{i=1}^{N} k\frac{q_i}{r_{i/p}}$$

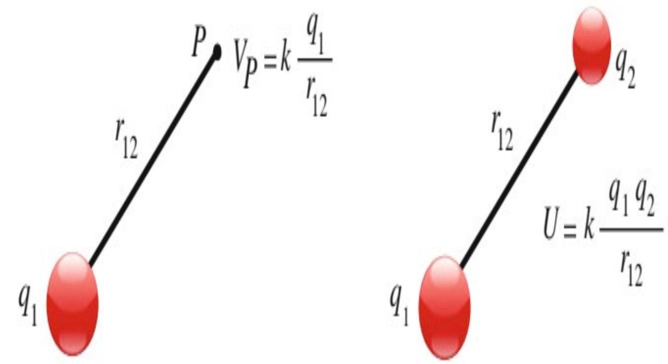
- Voltage, unlike Electric Field, is NOT a vector! So if you have MORE than one charge you don't need to use vectors. Simply add up all the voltages that each charge contributes since voltage is a SCALAR.

- WARNING! You must use the "sign" of the charge in this case.

- The unit of electric potential is [Nm/C] and is defined as [V] (volt).

Electric Potential Energy of a System of Point Charges

$$\Delta U = U_f - U_i = -W = \vec{F} \cdot \vec{r} = q_2 \vec{E}_p \cdot \vec{r} = q_2 E_p \cdot r$$



Where

$$V_p = E_p \cdot r = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

- Start with (set U_i=0 at ∞ and U_f=U at r)

- We have

$$U = q_2 V_p = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_1}{r}$$

- N charges:

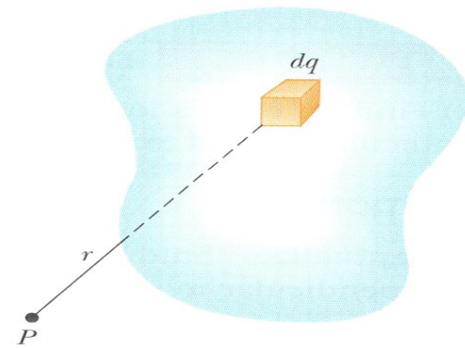
$$U = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1 and i \neq j}^{n}\frac{q_i q_j}{r_{ij}}$$

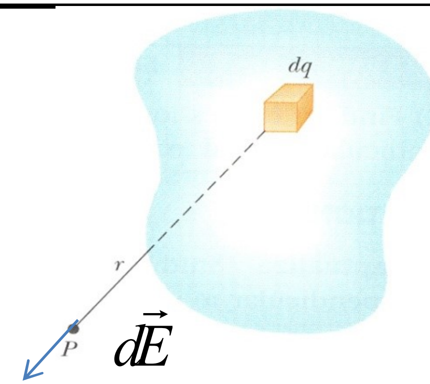
Electric Field and Electric Potential of a Continuous Charge Distribution

In the case of a continuous distribution of charge we first divide the distribution up into small pieces, and then we sum the contribution, to the field, from each piece:

In the limit of very small pieces, the sum is an *integral*

- **Find an expression for dq:**
  - **dq = λdl for a line distribution**
  - **dq = σdA for a surface distribution**
  - **dq = ρdV for a volume distribution**





$d\vec{E}$

Each dq : $dV = \frac{dq}{4\pi\varepsilon_0 r}$

Then : $V = \sum dV$

For very small pieces :

$V = \int dV$

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

Each dq : $d\vec{E} = \frac{dq}{4\pi\varepsilon_0 r^2}\overrightarrow{Ur}$

Then : $\vec{E} = \sum d\vec{E}$

For very small pieces :

$\vec{E} = \int d\vec{E}$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\overrightarrow{Ur}$$

The Electric Field of a Uniformly Charged Disk

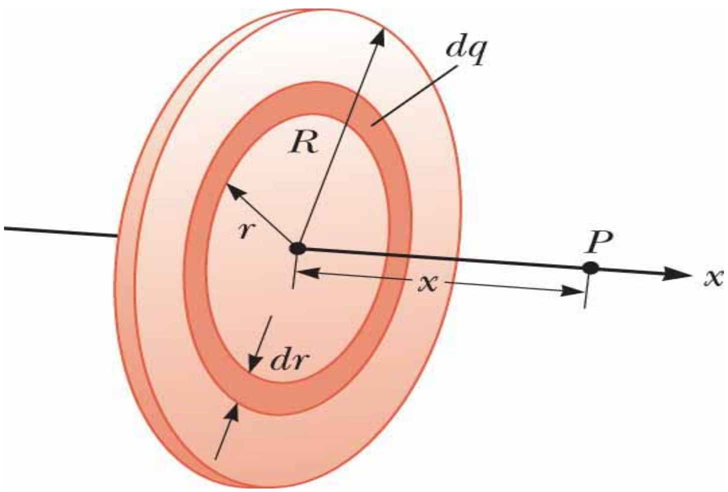
A disk of radius R has a uniform surface charge density σ. Calculate the electric field at a point *P* that lies along the central perpendicular axis of the disk and a distance *x* from the center of the disk.

Each dq $(2\pi r dr)$: $dE_x = dE\cos(\theta) = \frac{kdq}{r^2}\cos(\theta)$

where $r_{d/p} = \sqrt{(x^2 + r^2)}$

$\cos(\theta) = \frac{x}{r_{d/p}} = \frac{x}{\sqrt{(x^2 + r^2)}}$

$$dE_x = \frac{kx}{(x^2 + r^2)^{3/2}} dq = \frac{kx\sigma 2\pi r dr}{(x^2 + r^2)^{3/2}}$$



$$E_x = k\,x\pi\sigma\int_0^R \frac{2r\,dr}{(r^2 + x^2)^{3/2}}$$

$$= k\,x\pi\sigma\int_0^R (r^2 + x^2)^{-3/2} d(r^2)$$

$$= k\,x\pi\sigma\left[\frac{(r^2 + x^2)^{-1/2}}{-1/2}\right]_0^R = \boxed{2\pi k\,\sigma\left[1 - \frac{x}{(R^2 + x^2)^{1/2}}\right]}$$

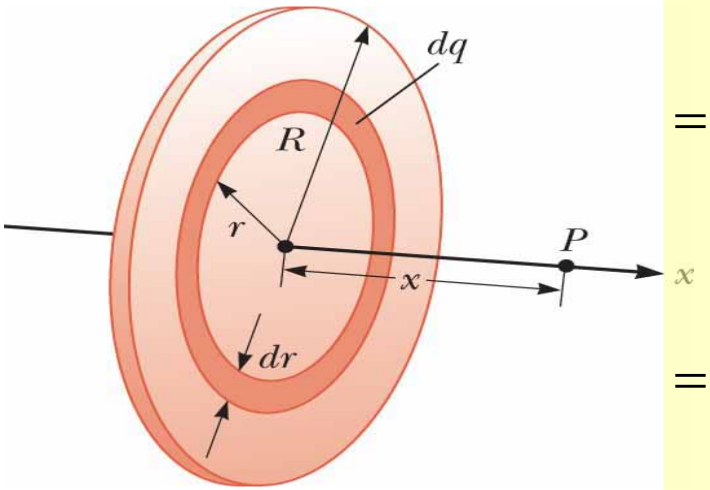
$$\text{for } x \ll R: E = 2\pi k\,\sigma = \frac{\sigma}{2\varepsilon_0}$$

What if we let the radius of the disk grow so that the disk becomes an infinite plane of charge

$$E = 2\pi k\,\sigma = \frac{\sigma}{2\varepsilon_0}$$

The Electric potential of a Uniformly Charged Disk

A disk of radius R has a uniform surface charge density σ. Calculate the electric potential at a point *P* that lies along the central perpendicular axis of the disk and a distance *x* from the center of the disk.



Each dq $(2\pi r dr)$:

$$dV_{d/p} = \frac{kdq}{r_{d/p}} = \frac{kdq}{\sqrt{(x^2 + r^2)}} = \frac{k\sigma 2\pi r dr}{(x^2 + r^2)^{1/2}}$$

$$V = \int_0^R \frac{k\sigma 2\pi r dr}{(x^2 + r^2)^{1/2}} = k\sigma\pi \int_0^R \frac{2 r dr}{(x^2 + r^2)^{1/2}}$$

$$= k\sigma\pi \int_0^R (x^2 + r^2)^{-1/2} d(r^2)$$

$$= k\sigma\pi \int_0^R (x^2 + r^2)^{-1/2} d(r^2)$$

$$= kx\sigma\pi \left[\frac{(x^2 + r^2)^{1/2}}{\frac{1}{2}} \right]_0^R = \frac{2\sigma\pi}{4\pi\varepsilon_0} \left[(x^2 + R^2)^{1/2} - x \right]$$

$$V = \frac{\sigma}{2\varepsilon_0} \left[(x^2 + R^2)^{1/2} - x \right]$$

Electric Dipole

A system of equal and opposite charges, separated by a finite distance is called as an electric dipole.

As shown in figure, the two electric charges of electric dipole are +q and –q and distance between them is 2a.

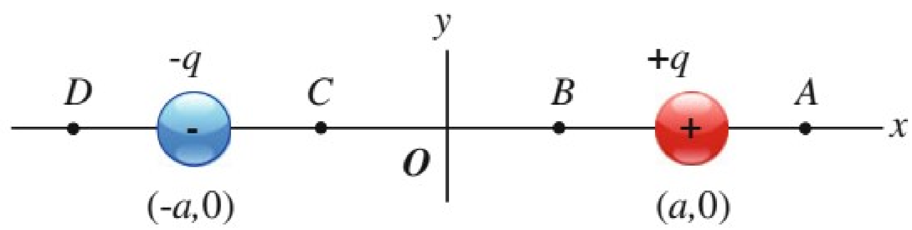
Electric dipole moment ($\vec{P}$) of the system can be defined as follows: $\vec{P} = 2q\vec{a}$

The SI unit of electric dipole is coulomb meter (C. m)

Electric dipole is a vector quantity, and its direction is from negative charge (-q) to positive charge (+q)

The net electric charge on an electric dipole is zero but its electric field is not zero, since the position of the two charges is different.

Find the electric potential along the axis of the electric dipole at the four points *A, B, C,* and *D* in Fig



r+ and *r*- as the distance from each point to the positive and negative charges, respectively:

(1) For point A in Fig. we have $x > a$. Therefore, $r_+ = x - a$ and $r_- = a + x$. The electric potential V_A is:

$$V_A = kq\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = kq\left(\frac{1}{x-a} - \frac{1}{a+x}\right) = \frac{2k\,qa}{x^2 - a^2} \quad (V_A \text{ positive})$$
$$\simeq \frac{2k\,qa}{x^2} \quad (x \gg a)$$

(2) For point B in Fig.. , we have $0 < x < a$. Therefore $r_+ = a - x$ and $r_- = a + x$. The electric potential V_B is:

$$V_B = kq\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = kq\left(\frac{1}{a-x} - \frac{1}{a+x}\right) = \frac{2kq}{a^2 - x^2}x \quad (V_B \text{ positive})$$

(3) For point C in Fig. , we have $-a < x < 0$. Therefore $r_+ = a - x$ and $r_- = a + x$. The electric potential V_C is:

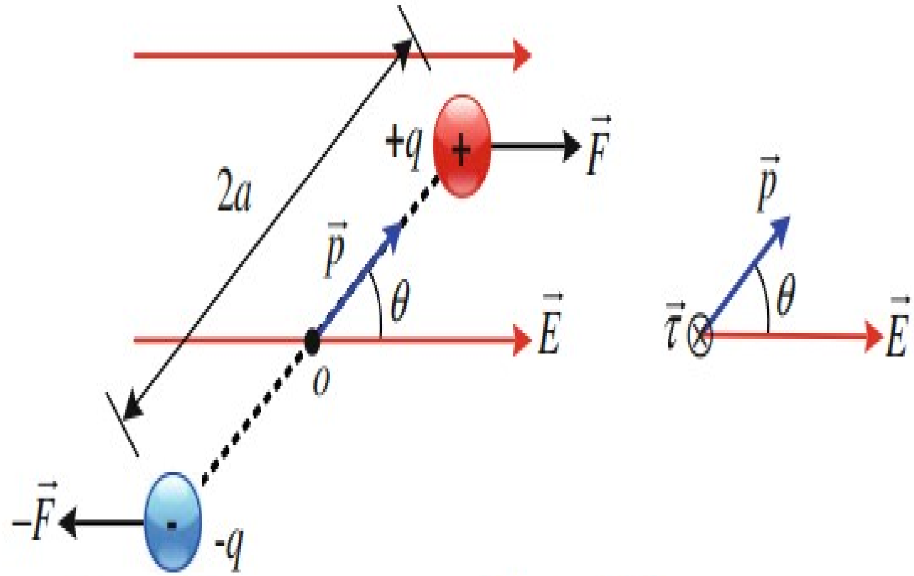
$$V_C = kq\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = kq\left(\frac{1}{a-x} - \frac{1}{a+x}\right) = \frac{2kq}{a^2 - x^2}x \quad (V_C \text{ negative})$$

(4) For point D in Fig. , we have $x < -a$. Therefore $r_+ = a - x$ and $r_- = -x - a$. The electric potential V_D is:

$$V_D = kq\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = kq\left(\frac{1}{a-x} + \frac{1}{a+x}\right) = -\frac{2kqa}{x^2 - a^2} \quad (V_D \text{ negative})$$
$$\simeq -\frac{2kqa}{x^2} \quad (x \ll -a)$$

Electric Dipole in an External Electric Field

Consider an electric dipole of electric dipole moment $\vec{p}$ is placed in a uniform external electric field $\vec{E}$, as shown in Fig.



The vector torque $\vec{\tau}$ on the dipole is therefore the cross product of the vectors $\vec{p}$ and $\vec{E}$. Thus:

$$\vec{\tau} = \vec{p} \times \vec{E}$$

The effect of this torque is to rotate the dipole until the dipole moment $\vec{p}$ is aligned with the electric field $\vec{E}$.

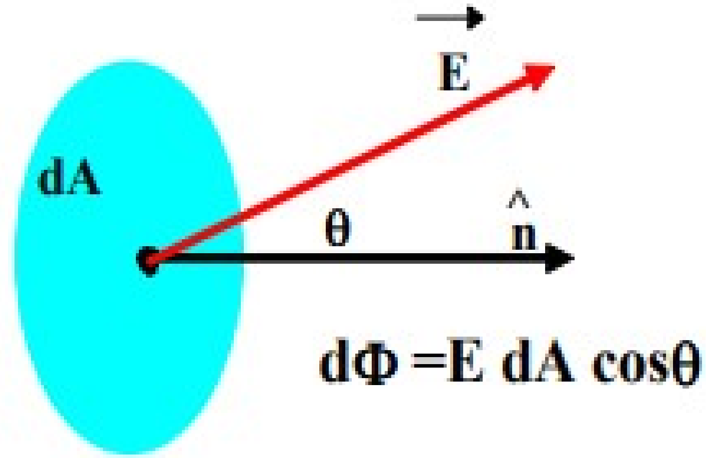
Electric Flux

Electric flux through the infinitesimal area dA is equal to

$$d\Phi = \vec{E} \cdot d\vec{A}$$

where

$$d\vec{A} = A\hat{n}$$



Total Electric Flux through a Closed Surface:

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A}$$

Gauss's Law

The total flux within a closed surface …

… is proportional to the enclosed charge.

$$\Phi = \oint\!\!\oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

Gauss's Law is always true, but is only useful for certain very simple problems with great symmetry.

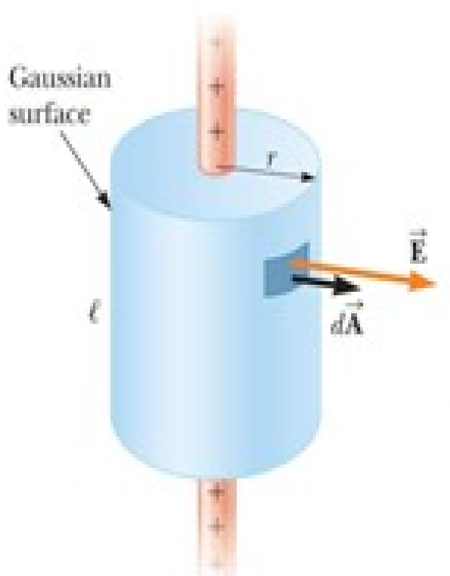
Applying Gauss's Law

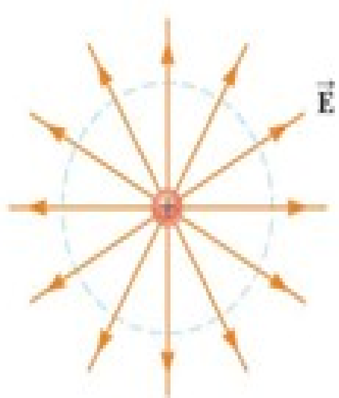
A Cylindrically Symmetric Charge Distribution

Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ.

$$\Phi_E = \oiint \vec{E} \bullet d\vec{A} = \vec{E} \oiint d\vec{A} = \vec{E} \bullet \vec{A} = \mathbf{E}\mathbf{A} = \frac{Q_{enclosed}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

$$\mathbf{E}(2\pi r l) = \frac{\lambda l}{\varepsilon_0} \rightarrow \mathbf{E} = \frac{2\lambda}{4\pi\varepsilon_0 r} \rightarrow \mathbf{E} = \frac{\lambda}{2\pi\varepsilon_0 r} = 2K\frac{\lambda}{r}$$

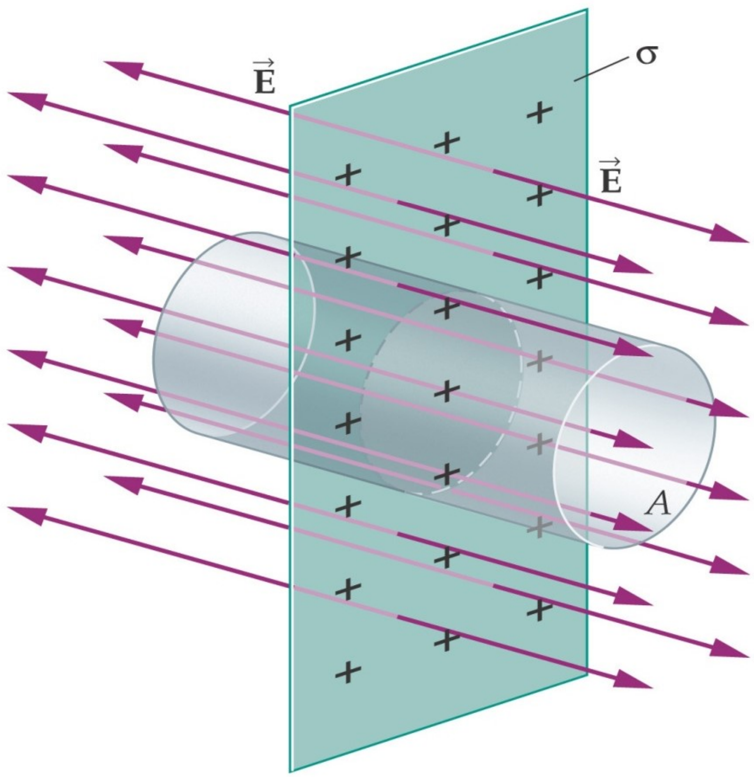




What if the line segment in this example were not infinitely long?

$$\mathbf{E} \neq 2K\frac{\lambda}{r}$$

Infinite sheet or Plane of charges



1. Select Gauss surface
In this case a cylindrical

2. Calculate the flux of the electric field through the Gauss surface
$\Phi = 2 E A$

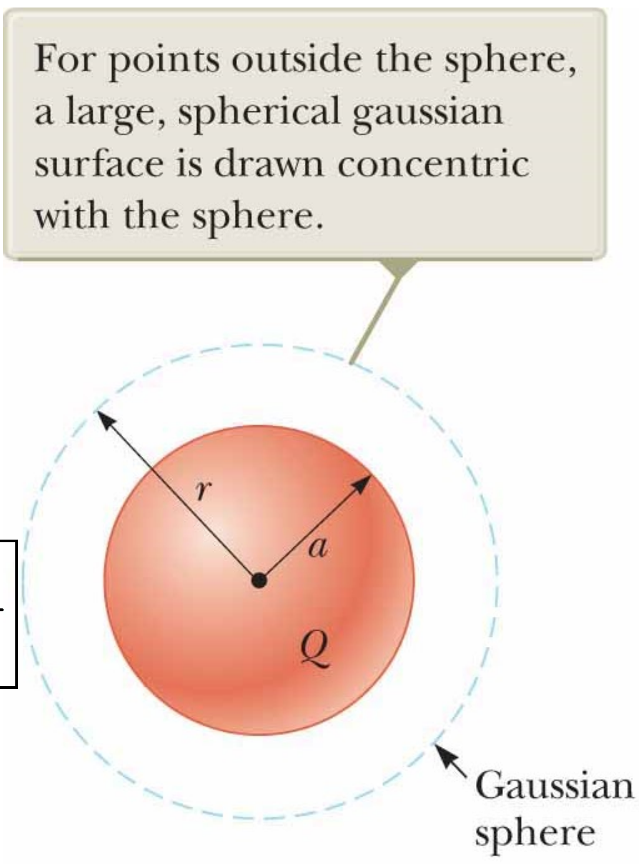
3. Equate $\Phi = q_{encl}/\varepsilon_0$
$2EA = q_{encl}/\varepsilon_0$

4. Solve for E
$E = q_{encl} / 2 A \varepsilon_0 = \sigma A / 2 A\varepsilon_0$
(with $\sigma = q_{encl} / A$)
$E = \sigma / 2 \varepsilon_0$

A Spherically Symmetric Charge Distribution

An conducting solid sphere of radius a has a uniform volume charge density ρ and carries a total positive charge Q.

(A) Calculate the magnitude of the electric field at a point outside the sphere.



$$\Phi_E = \oint\!\!\oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}} = \oint\!\!\oint \mathbf{E} \bullet dA = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\oint\!\!\oint \mathbf{E} \bullet dA = \mathbf{E} \oint\!\!\oint dA = E \times A = \mathbf{E}\left(4\pi \mathbf{r}^2\right) = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\mathbf{E} = \frac{Q_{enclosed}}{\left(4\pi \mathbf{r}^2\right)\varepsilon_0} = K\frac{Q_{enclosed}}{\mathbf{r}^2} = K\frac{Q}{\mathbf{r}^2} \left(\text{For} \quad r \succ a\right)$$

A Spherically Symmetric Charge Distribution

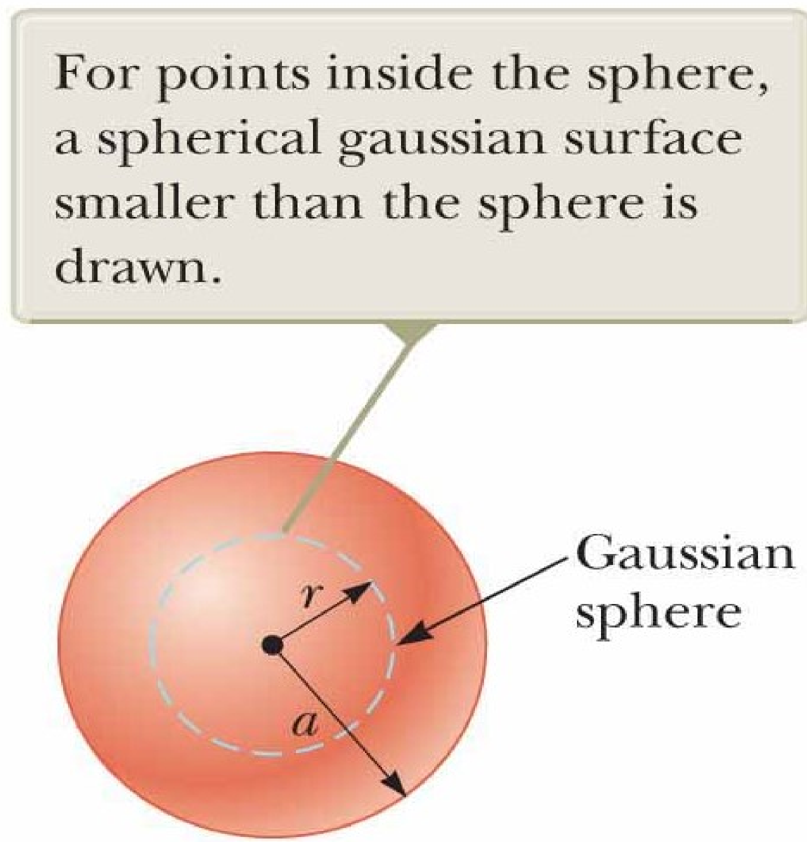
(B) Find the magnitude of the electric field at a point inside the sphere.

$$Q_{enclosed} = \rho V' = \rho\left(\frac{4}{3}\pi r^3\right)$$

$$\oiint \mathbf{E} \bullet dA = \mathbf{E} \oiint dA = E \times A = \mathbf{E}\left(4\pi \mathbf{r}^2\right) = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\mathbf{E} = \frac{Q_{enclosed}}{\left(4\pi \mathbf{r}^2\right)\varepsilon_0} = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{\left(4\pi \mathbf{r}^2\right)\varepsilon_0} = \frac{\rho}{3\varepsilon_0} r$$

$$\mathbf{E} = \frac{Q \Big/ \left(\frac{4}{3}\pi a^3\right)}{3\left(1 \Big/ 4\pi K\right)} r = K\frac{Q}{a^3}\mathrm{r} \quad (\mathrm{For} \prec \mathrm{a})$$



A Spherically Symmetric Charge Distribution

Suppose the radial position r = a is approached from inside the sphere and from outside. Do we obtain the same value of the electric field from both directions?

$$E = \lim_{r \to a}\left(k\,\frac{Q}{r^2} \right) = k\,\frac{Q}{a^2}$$

$$E = \lim_{r \to a}\left(k\,\frac{Q}{a^3}\,r \right) = k\,\frac{Q}{a^3}\,a = k\,\frac{Q}{a^2}$$