

## Problems Set 2

### Exercise 1: ( Operations on languages)

- 1- Give all the prefixes, infixes and suffixes of the word abbba for  $V = \{a, b\}$
- 2- Lets  $L1 = \{a\}V^*$  and  $L2 = \{b\}V^*$  such as  $V = \{a, b\}$   
Give the definitions of the following languages:  $L1 \cup L2$  and  $L1 \cap L2$
- 3- Give the formal definition of languages on the vocabulary  $V$  of mirror words (palindromes) of length 4. Give this language for  $V = \{a, b\}$
- 4- Give the formal definition of a language  $L$  generated by a grammar  $G$
- 5- Which language is produced by the following algorithm?

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For n from 0 to infinity do
  For i from 0 to n do
    w = epsilon
    For j from 0 to i do
      w = w.a
    For j from i+1 to n do
      w = w.b
  Display w

```

6- Lets the following formal languages:

$L1 = \{a^i b^j \mid i \geq j \geq 1\}$      $L2 = \{\epsilon, a, aa\}$      $L3 = \{b, ba\}$

$L4 = \{\epsilon\}$      $L5 = \{a^i b^k c^k \mid i \geq 0, k \geq 0\}$

$L6 = \{a^i b^i \mid i \geq 1\}$

Find the following languages:

$L2.L3$  ,  $L2.L1$  ,  $L1.L3$ ,  $L5 \cap L1$ ,  $L6 \cup L5$ ,  
 $L1.(L2 \cap L4)$ ,  $L1(L2 \cap L3)$ ,  $(L1.L2)^R$ ,  $(L1)^R.(L2)^R$

### Exercise 2: (generated languages)

Find the languages generated by the following grammars:  $G = (V_t, V_n, S, R)$

a)  $V_t = \{a, b\}$      $V_n = \{S\}$

$R = (S \rightarrow a S a \mid b S b \mid \epsilon)$

b)  $V_t = \{a, b, c\}$      $V_n = \{S, A, B\}$

$R = (S \rightarrow a S b \mid b A c$

$A \rightarrow b A c \mid bc \mid a B$

$B \rightarrow a B \mid \epsilon)$

c)  $V_t = \{a, b\}$      $V_n = \{S, A, B\}$

$R = (S \rightarrow A b B a$

$A \rightarrow a A \mid a$

$B \rightarrow b B a \mid b a$

d)  $V_t = \{a, b, c\}$      $V_n = \{S, B\}$

$R = (S \rightarrow a S B \mid abc \quad cB \rightarrow Bc \quad bBc \rightarrow bbcc)$

e)  $V_t = \{a, b, c\}$      $V_n = \{S, A, B, C, D, F\}$

$R = (S \rightarrow DAF$

$A \rightarrow aAC$

$CF \rightarrow FBc$

$cB \rightarrow Bc$

$aF \rightarrow Fa$

$DF \rightarrow \epsilon$

$aB \rightarrow ab$

$bB b \rightarrow bb)$

### **Exercise 3 : (grammars )**

Give a grammar that generates the following languages and specifying the type of the grammar:

- 1-  $L(G) = \{ w \in Vt^* / w = a^n b^n c^p \quad n > 0 \quad \text{and} \quad p > 0 \}$  with  $Vt = \{a, b, c\}$
- 2-  $L(G) = \{ w \in Vt^* / w = a^n c a^p \quad n \geq 0 \quad \text{and} \quad p \geq 0 \}$  with  $Vt = \{a, c\}$
- 3-  $L(G) = \{ w \in Vt^* / w \text{ contains as many } a \text{ as } b \}$  with  $Vt = \{a, b\}$
- 4-  $L(G) = \{ w \in Vt^* / w \text{ contains neither consecutive } a \text{ nor consecutive } b \}$  with  $Vt = \{a, b\}$
- 5-  $L(G) = \{ w \in Vt^* / |w| \text{ is even or zero } \}$  with  $Vt = \{a, b\}$
- 6-  $L(G) = \{ w \in Vt^* / w = a^n b^m a^{n+m} \quad n \geq 0 \quad \text{and} \quad m \geq 0 \}$  with  $Vt = \{a, b\}$
- 7-  $L(G) = \{ w \in Vt^* / w = a^n b^m c^p \quad \text{and} \quad n > m + p \}$  with  $Vt = \{a, b, c\}$
- 8-  $L(G) = \{ w \in Vt^* / w = a^n b^m \quad n \geq 0, m \geq 0 \text{ and } n \neq m \}$  with  $Vt = \{a, b\}$
- 9-  $L(G) = \{ a^n b^p c^q d^r / n, p, q, r > 0 \} \cup \{ a^p b^n c^q d^n / n, p, q > 0 \}$  ( do it alone )

### **Exercise 4: ( identifier )**

Find a grammar that generates identifiers for a programming language similar to FORTRAN.

The identifier is defined as follows:

- The length is between 1 and 7
- It must start with a letter
- It is made up of letters (a,b,...,z, A,B,...,Z) and numbers (0,1,...,9)

### **Exercise 5: ( specific languages )** ( do it alone )

- 1- We consider the alphabet  $Vt = \{ |, +, = \}$ . Define a grammar that generates the button addition language for strictly positive integers.

**Example:**  $|| + || = |||$

- 2- Try to enrich this grammar so that it also performs addition for zero integers

**Example :**  $| + = |$  or  $|| + || = ||$