Chapter IV

Kinematic analysis of planar mechanisms

IV Kinematic analysis of planar mechanisms

IV.1 Introduction

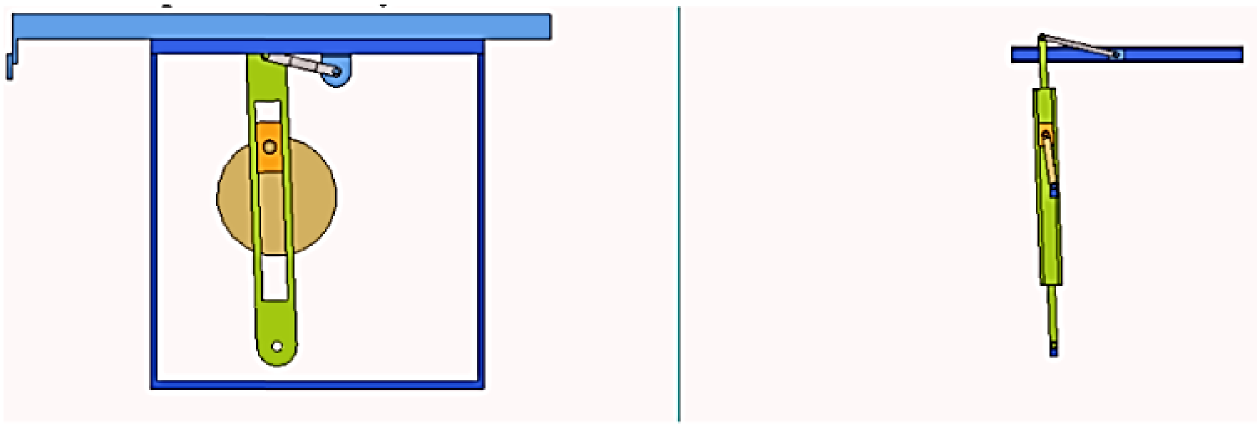
Every machine has moving members to perform a designed job. There are Prime Movers or Drives that use energy from some source to provide the necessary power to a Driven Machine or Load. The Prime Movers can be reciprocating steam engines (almost extinct now) which use steam from external combustion of coal in a boiler, reciprocating diesel or petrol engines which contain internal combustion in a cylinder, steam turbines using steam from external combustion of coal, gas turbines that use oil or gas in external combustion to produce hot gases or electrical machines that use electric power produced in power plants and transmitted to the location where it is needed. These prime movers drive a load that could be a propeller of a ship, an automobile for road transportation, a reciprocating or rotating compressor in oil and gas industries, an aircraft for air transportation, a generator to produce electricity or a machine tool that removes metal in a manufacturing process, etc.

As an example, in Figure IV.1-a, we show a Shaping Machine. The power from an electric motor is transferred through the crank, sliding block, rocker arm and connecting link to the cutting tool. These are rigid bodies to which the cutting tool is connected in a suitable manner to give constrained motion and remove the metal by shaping action. Theory of Machines is concerned with the layout of such members in a given machine, their motion and understanding of how the kinematic quantities, displacement, velocity and acceleration vary with time and kinetics of these members, viz., what forces and moments cause these motions. From these forces and moments, we can determine the stresses and size the members of the machine accordingly. That will be subject of Machine Design, which is not a part of Theory of Machines.

Before we proceed to formally develop the subject in a systematic manner, let us look at Figure IV.1-b that depicts schematically the shaping machine of Figure IV.1-a.

The crank, sliding block, rocker arm, connecting link and the ram here are represented simply by lines with joints connecting them and provide a transfer of motion from the crank to the ram. Comparing Figures IV.1-a and IV.1-b we find the same motions are achieved by different bodies, e.g., watch the ram and the tool attached to it.

Therefore we recognize that we can schematically represent each machine member by a rigid body. If all the bodies of the machine move in one plane as in this case, the rigid bodies are simply represented by straight line between two joints where the neighboring bodies of the machine are attached. The student should quickly learn to imagine what would be the actual shape of a machine component which might be schematically shown as a straight line (or as a triangle if there are three attachments to this body). Now let us define these machine components in a systematic manner to develop the subject further.



(a) (b)

Figure IV. 1 : Shaping Machine schematically represented.

IV.2 Definition of a planar mechanism

If all particles of a given system undergo motion in parallel planes, we say that the system experiences planar motion. We shall consider only planar mechanisms, in which all links undergo planar motion.

IV.3 Kinematic pairs

For planar mechanisms, only four types of joints or kinematic pairs arise; they are called :

1. Hinge (also called turning, revolute, or pinned) pairs
2. Sliding (also called prismatic) pairs
3. Rolling (or gear) pairs
4. Cam pairs

IV.3.1 Hinge pairs

All of the joints shown in Figure IV.2 and IV.3 are hinge pairs. If the link 4 of the four-bar mechanism shown in Figure IV.2-b is very long, it may be desirable to use the more compact, *kinematically equivalent* arrangement shown in Figure IV.5-a. When the radius R approaches infinity, the connection between members 4 and 1 is called a *sliding* or *prismatic pair*.

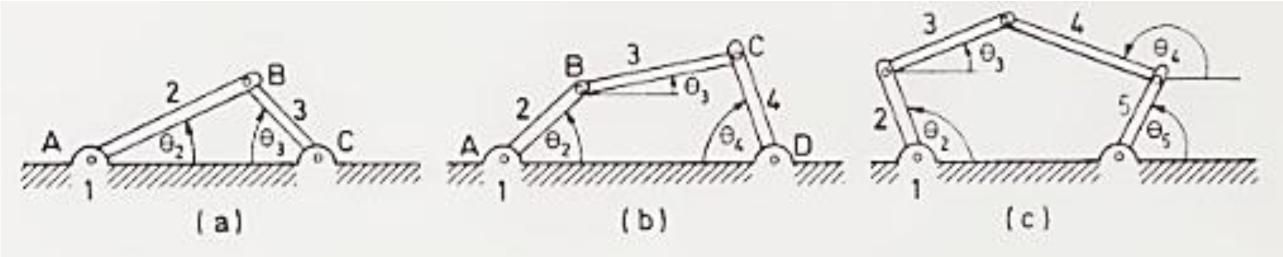


Figure IV. 2 : (a) 3-bar chain or 3 hinged truss with 0 DOF, (b) 4-bar mechanism with 1 DOF, (c) 5-bar mechanism with 2 DOFs.

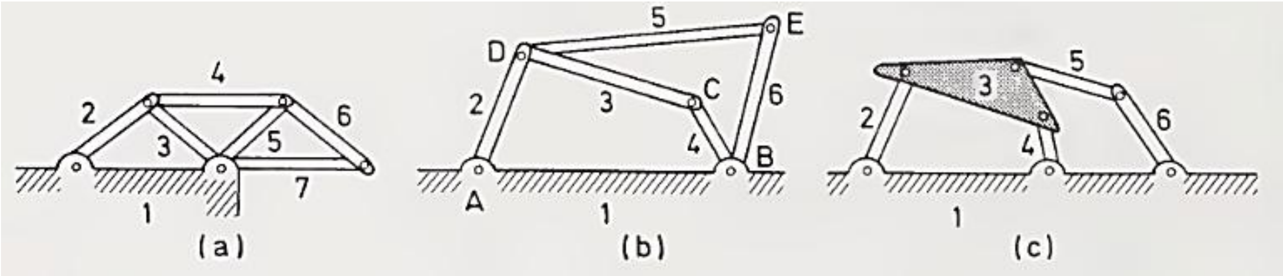


Figure IV. 3 : (a) Rigid truss, (b) A 6-bar mechanism with 1 DOF, (c) A 6-link mechanism with 1 DOF.

IV.3.2 Sliding or prismatic pair

The sliding joint is also called a piston or prismatic joint. It allows linear sliding between the links that it connects (Figure IV.4). The mechanism shown in Figure IV.5-b is called an *offset slider-crank mechanism*, and the distance ***h*** is called the offset.

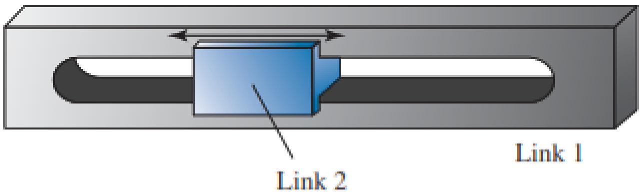


Figure IV. 4 : Representation of a Sliding pair.

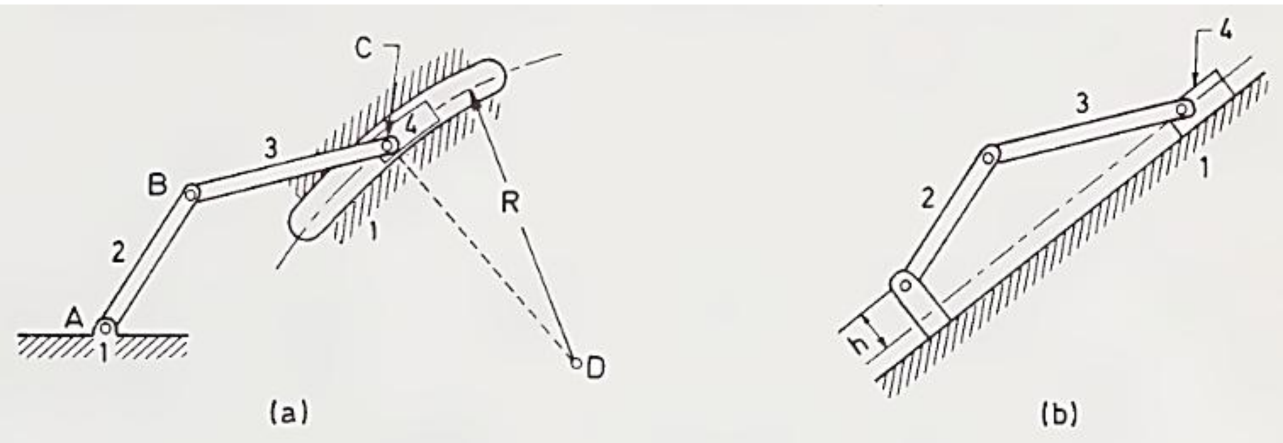


Figure IV. 5 : Kinematic equivalent of 4-bar mechanism, (b) Slider-crank mechanism.

IV.3.3 Rolling pair

When two friction wheels roll about fixed centers A and B, as shown in Figure IV.6, and no slip occurs at points of contact, the kinematic pair is said to be a rolling pair. If teeth are cut into the wheels, to provide a driving force independent of friction, the wheels become *gears*, and the equivalent rolling circles are called the *pitch circles* of the gears, which touch at the *pitch point P*; thus the name *gear pair* is equivalent to rolling pair.

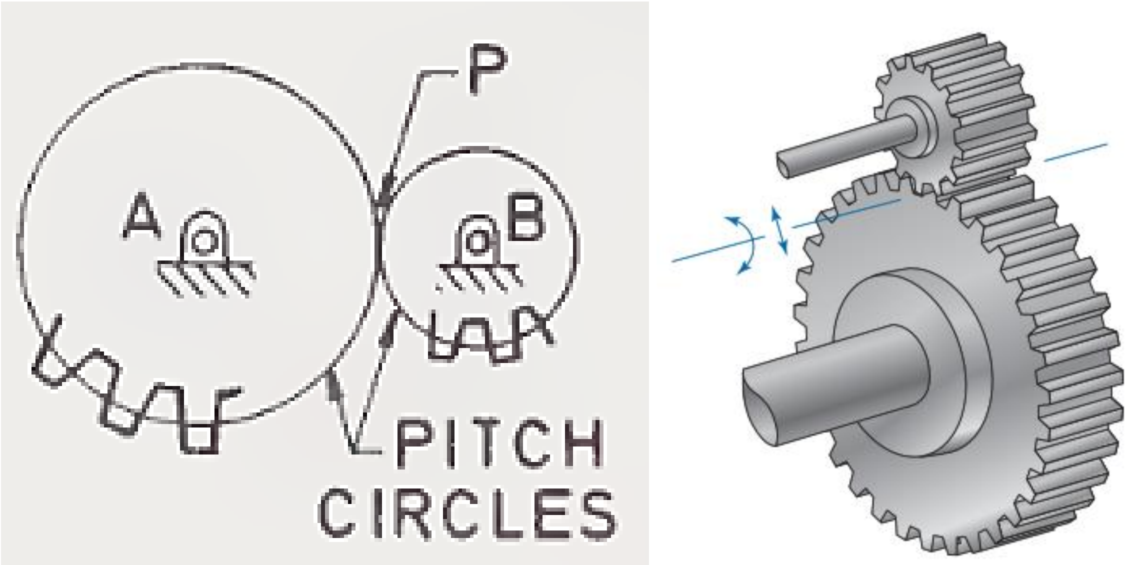


Figure IV. 6 : Rolling pair.

It is possible for noncircular pitch curves to roll together without slipping. When meshing teeth are cut into the pitch curves we speak of the members as noncircular gears. Unless indicated otherwise, we shall henceforth assume that the gears we deal with have circular pitch curves.

IV.3.4 Cam pair

Figure IV.7 shows two members with curved outlines which touch at a common point C, where sliding action occurs at the so-called cam pair. The distinction between the cam pair and the prismatic pair is that the former has a line contact and the latter has surface contact. Reuleaux classified joints with surface contact as lower pairs and those with point or line contact as higher pairs. Other writers have redefined the terms so that a lower pair refers to the case where the paired members have one DOF relative to each other. For simplicity, we accept Reuleaux's definition whereby hinges and sliding pairs are classified as lower pairs and gear and cam pairs are considered higher pairs.

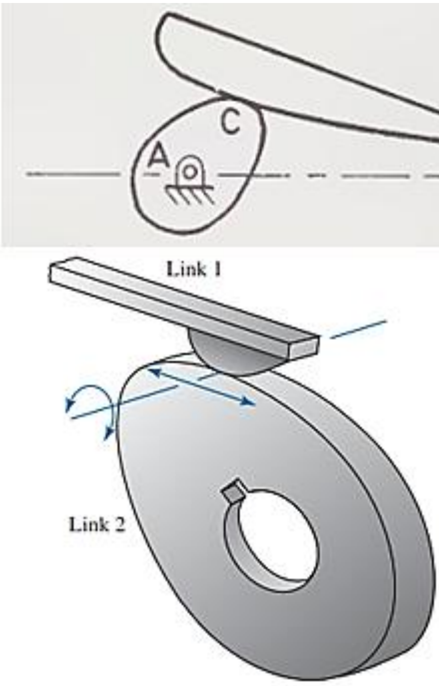


Figure IV. 7 : Representation of Cam pair.

IV.4 Parameters of a planar mechanism

IV.4.1 Plan links

The links that can exist in a plan are (see figure IV.8) :

- Pivot with axis perpendicular to the plane (1ddl=Rotation)
- Slide with axis parallel to the plane (1ddl=Translation)
- Point with axis perpendicular to the plane (3ddl=2Translations+Rotation)
- Rectilinear linear with axis parallel to the plane (3ddl=2Translations+Rotation)

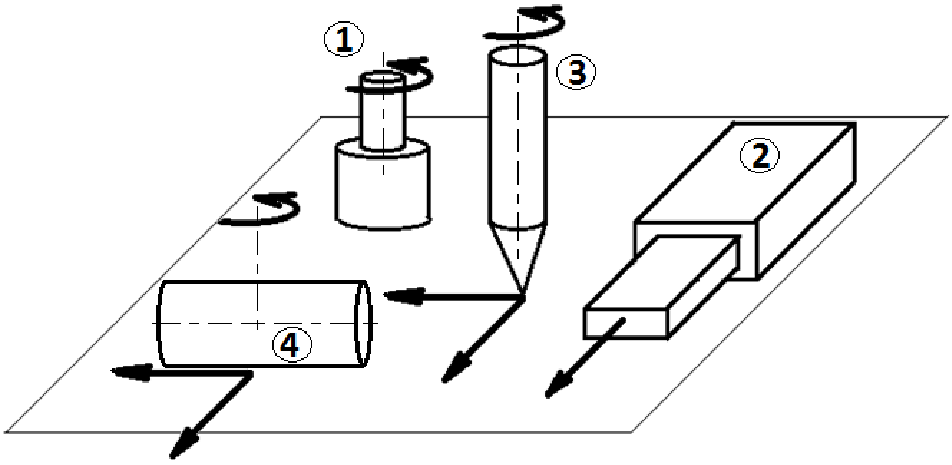


Figure IV. 8: Different plane links

IV.4.2 Parameters of a mechanism

Knowledge of the parameters of an mechanism is necessary for the analysis of the mechanism, we distinguish between constant and variable parameters.

The mechanism represented in figure IV.9 admits the following parameters :

a) ***Fixed parameters***

l_1= AB , l_2= BC , l_3= CD and l_4= AD

b) ***Variable parameters***

$\alpha = \alpha(t)$, $\beta = \beta(t)$ *and* $\gamma = \gamma(t)$

With β *and* γ *are variables are dependent on* α *:* $\beta = \beta(\alpha)$ *and* $\gamma = \gamma(\alpha)$

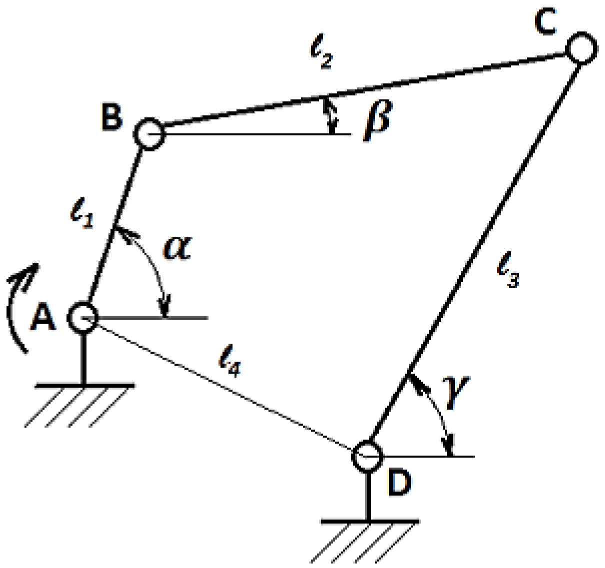


Figure IV. 9 : Four bar mechanism parameters

IV.4.3 Geometric characteristics of a mechanism element

Except for the open chain where we can come across an element with one link to another, most of the elements of the mechanisms have at least two links; we say that they are binary. For binary elements, we distinguish 3 cases according to the nature of the links:

- An element with two pivot links is characterized by the perpendicular distance between the two axes of its links (Figure IV.10-a).
- An element with a pivot link and a slide link is characterized by the distance from the center of the pivot link to the axis of the slide (Figure IV.10-b).
- An element with two sliding links is characterized by the angle between the two axes of its links (Figure IV.10-c).

For ternary elements (with 3 links), we can distinguish 3 cases:

- An element with three pivot links is characterized by three perpendicular distances between the axes two by two (Figure IV.10-d).
- An element with two pivot links and a slide is characterized by a perpendicular distance between the pivot axes and two distances between the pivot centers and the slide axis (Figure IV.10-e).
- An element with two slide links and a pivot is characterized by two distances between the center of the pivot and the axes of the slides and an angle between the axes of the two slides (Figure IV.10-f).

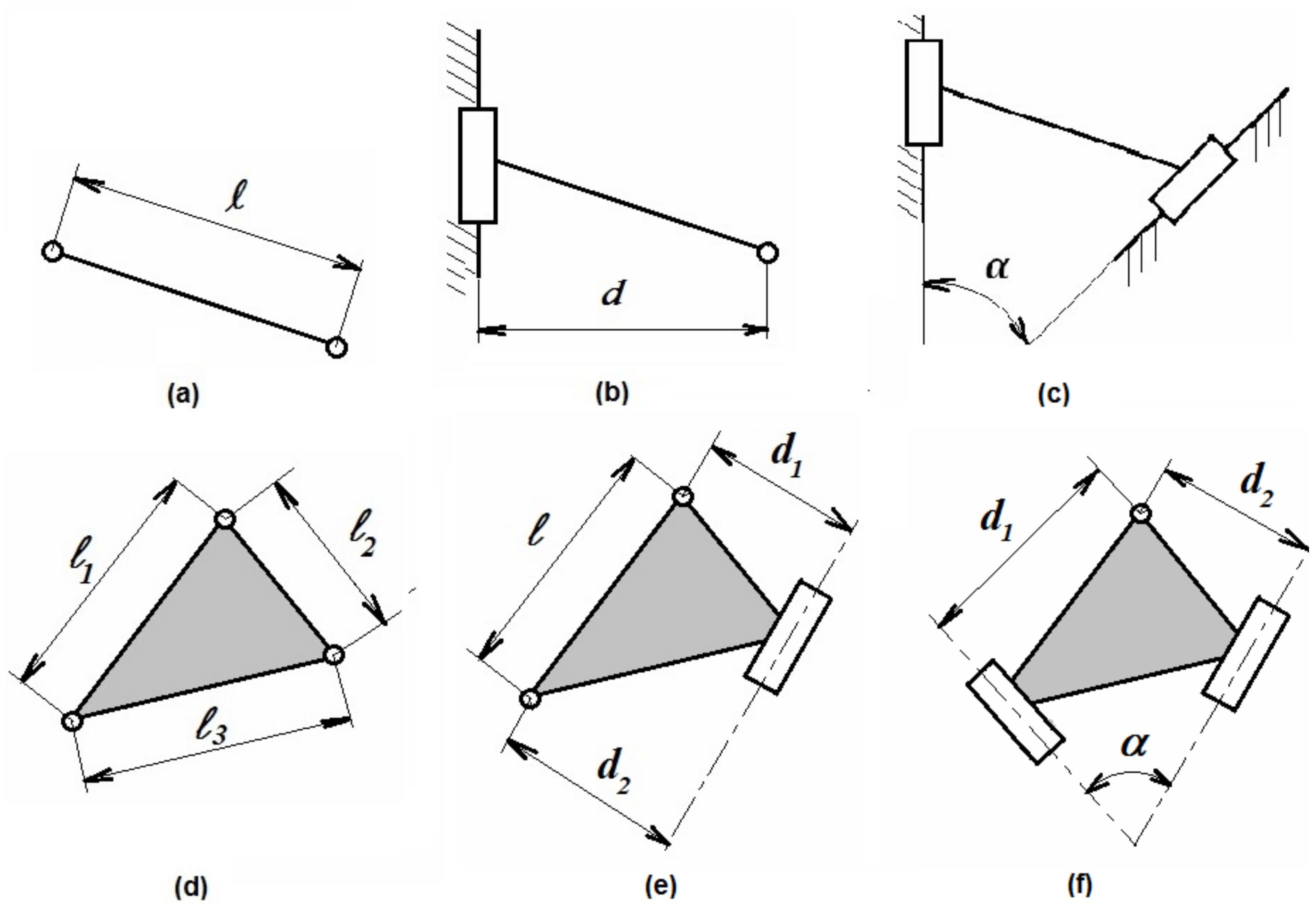


Figure IV. 10 : Geometric characteristics of a mechanism element.

IV.5 The 4-bar mechanism

The simplest and most common linkage is the four-bar linkage (Example in figure IV.11) . It is a combination of four links, one being designated as the frame and connected by four pin joints (links).

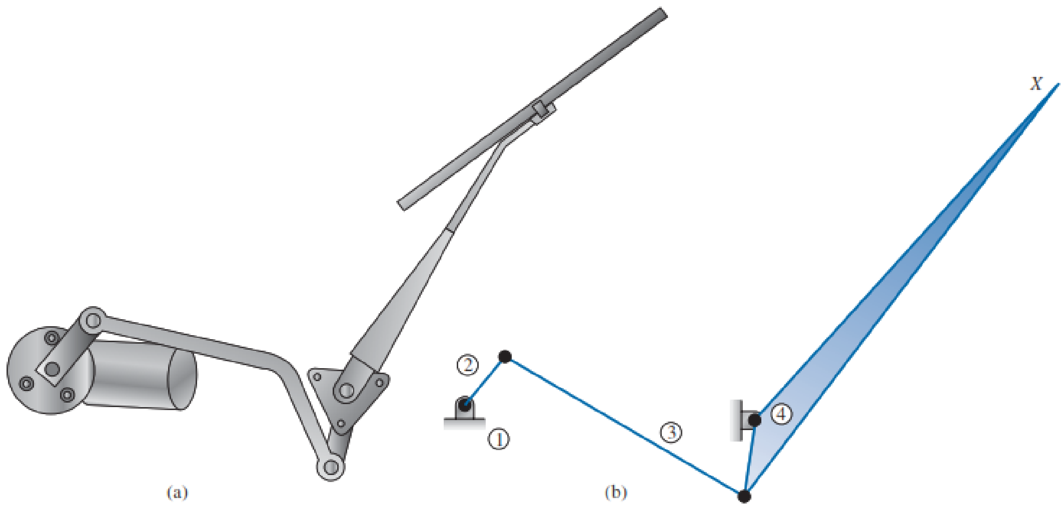


Figure IV. 11 : The wiper system as a 4-bar mechanism

Because it is comprised of four links connected by four pin joints and one link is unable to move. The mobility of a four-bar mechanism consists of the following:

N = 4, J_p=4 pins, J_h=0

$M=3.(n-1) - 2. J_p - J_h = 3.(4-1) - 2.4 - 0 = 1$

Because the four-bar mechanism has one degree of freedom, it is constrained or fully operated with one driver. The wiper system in figure IV.11 is activated by a single DC electric motor.

Of course, the link that is unable to move is referred to as the frame. Typically, the pivoted link that is connected to the driver or power source is called the input link. The other pivoted link that is attached to the frame is designated the output link or follower. The coupler or connecting arm "couples" the motion of the input link to the output link.

IV.5.1 Inversion

A mechanism is formed by fixing one of the links of a chain. Clearly, when different links of the same chain are chosen to become frame-link, different mechanisms will result. The process of choosing different links of a kinematic chain for becoming frame is known as ***kinematic inversion***.

a) ***Properties of inversion***

1- Number of inversion possible for a kinematic chain equals the number of links in the parent kinematic chain.
2- Relative motion (displacement, velocity and acceleration) between any two links does not change with inversion. This is simply beacause relative motion between different links is a property of parent kinematic chain.
3- Absolute motion of points on various links (measured with respect to the frame-link) may, however, change drastically from one inversion to the other, even in direct inversion.

Unlike lower paired mechanism, a higher paired mechanism cannot be inverted. This is simply because the two elements of a higher pair cannot be interchanged with each other without affecting overall motion of the mechanism.

b) ***Important of Inversion***

Important aspect of the concept of inversion can be summarised as under:

1- The concept of inversion enables us to categorise a group of mechanisms arising out of inversions of a parent kinematic chain as a family of mechanisms. Members of this family have a common characteristic in respect of relative motion.
2- In case of direct inversion, as relative velocity and relative acceleration between two links remain the same, it follows that complex problem of Velocity/Accelaration analisys may often be simplified by considering a kinematically simpler direct inversion of the original mechanism. Such procedure is the basis of Goodman's ingenious method of indirect acceleration analysis. The concept is also useful in converting motion analysis problem of an epi-cyclic gear-train to that of a simpler gear train by fixing arm and freeing the fixed member.
3- In many cases of inversions, by changing proportions of length of links, desirable features of the inversion may be accentuated and many useful mechanisms may be developed.

IV.5.2 Grashoff's laws for 4-bar articulated mechanisms

The following nomenclature is used to describe the length of the four links (see figure IV.9).

S : length of the shortest link (l_1)

L : length of the longest link (l_3)

p : length of one of the intermediate length links (l_2)

q : length of the other intermediate length links (l_3)

Grashof 's theorem states that a four-bar mechanism has at least one revolving link if:

$$s + l \leq p + q$$

Conversely, the three nonfixed links will merely rock if:

$$s + l > p + q$$

All four-bar mechanisms fall into one of the five categories listed in Table IV.1. The different categories are illustrated in Figure IV.12.

Table IV. 1 : Categories of Four-Bar Mechanisms

Case	Criteria	Shortest link	Category
1	$s + l < p + q$	Frame	Double crank
2	$s + l < p + q$	Slide	Crank-rocker
3	$s + l < p + q$	Coupler	Double rocker
4	$s + l = p + q$	Any	Change point
5	$s + l > p + q$	Any	Triple rocker

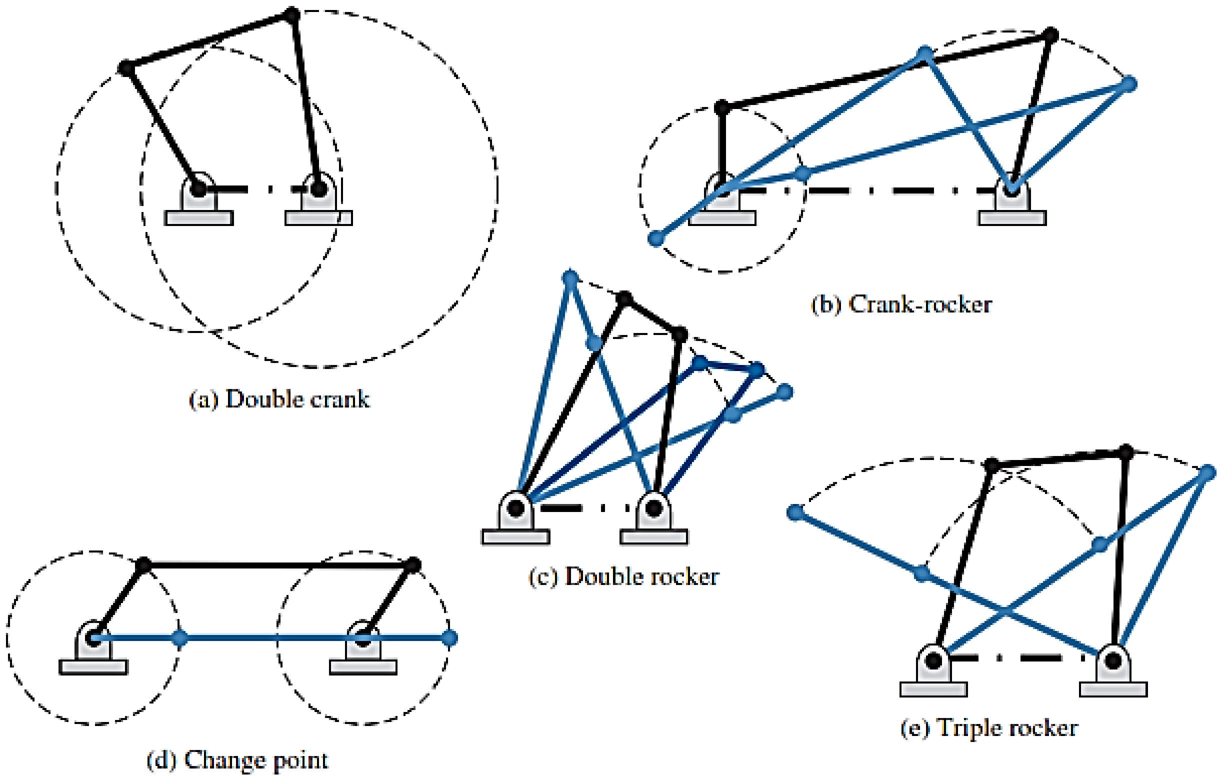


Figure IV. 12 : Categories of Four-Bar Mechanisms.

IV.5.3 Categories of Four-Bar Mechanisms.

a) ***Double Crank***

A double crank, or crank-crank, is shown in Figure IV.12-a. As specified in the criteria of Case 1 of Table IV.1, it has the shortest link of the four-bar mechanism configured as the frame. If one of the pivoted links is rotated continuously, the other pivoted link will also rotate continuously. Thus, the two pivoted links, 2 and 4, are both able to rotate through a full revolution. The double crank mechanism is also called a drag link mechanism.

b) ***Crank-rocker***

A crank-rocker is shown in Figure IV.12-b. As specified in the criteria of Case 2 of Table IV.1, it has the shortest link of the four-bar mechanism configured adjacent to the frame. If this shortest link is continuously rotated, the output link will oscillate between limits. Thus, the shortest link is called the crank, and the output link is called the rocker. The wiper system in Figure IV.11 is designed to be a crank-rocker. As the motor continuously rotates the input link, the output link oscillates, or “rocks.” The wiper arm and blade are firmly attached to the output link, oscillating the wiper across a windshield.

c) ***Double rocker***

The double rocker, or rocker-rocker, is shown in Figure IV.12-c. As specified in the criteria of Case 3 of Table IV.1, it has the link opposite the shortest link of the four-bar mechanism configured as the frame. In this configuration, neither link connected to the frame will be able to complete a full revolution. Thus, both input and output links are constrained to oscillate between limits, and are called rockers. However, the coupler is able to complete a full revolution.

d) ***Change point mechanism***

A change point mechanism is shown in Figure IV.12-d. As specified in the criteria of Case 4 of Table IV-1, the sum of two sides is the same as the sum of the other two. Having this equality, the change point mechanism can be positioned such that all the links become collinear. The most familiar type of change point mechanism is a parallelogram linkage.

The frame and coupler are the same length, and so are the two pivoting links. Thus, the four links will overlap each other. In that collinear configuration, the motion becomes indeterminate. The motion may remain in a parallelogram arrangement, or cross into an antiparallelogram, or butterfly, arrangement. For this reason, the change point is called a singularity configuration.

e) ***Triple rocker linkage***

A triple rocker linkage is shown in Figure IV.12-e. Exhibiting the criteria in Case 5 of Table IV-1, the triple rocker has no links that are able to complete a full revolution. Thus, all three moving links rock.

Example IV.1 :

A nosewheel assembly for a small aircraft is shown in Figure IV.13. Classify the motion of this four-bar mechanism based on the configuration of the links.

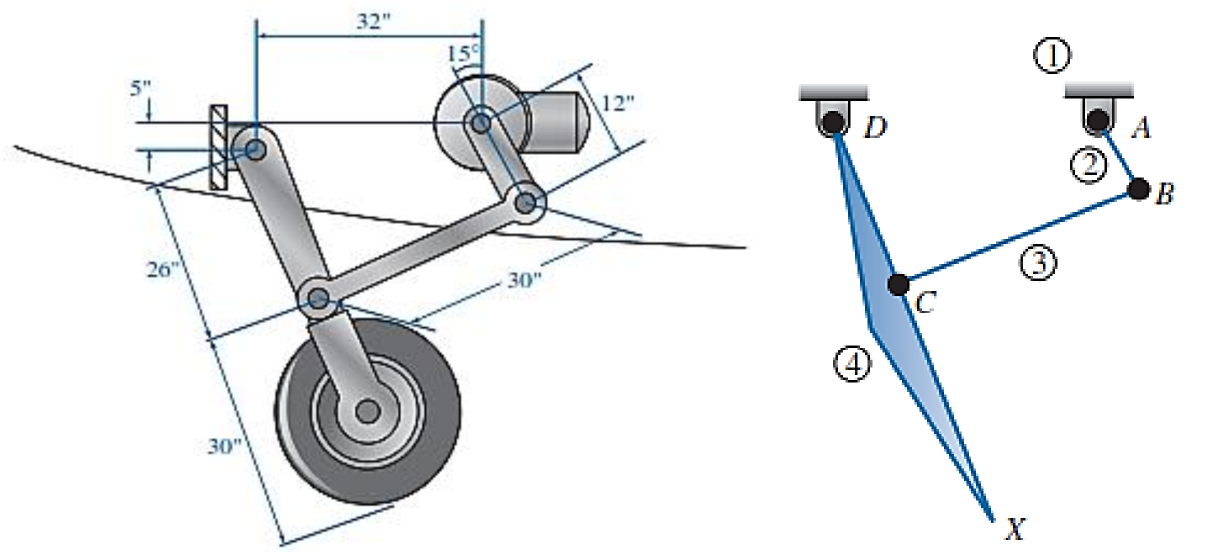


Figure IV. 13 : Nosewheel assembly.

Solution IV.1 :

1- Distinguish the Links Based on Length

In an analysis that focuses on the landing gear, the motion of the wheel assembly would be determined relative to the body of the aircraft. Therefore, the aircraft body will be designated as the frame. The figure IV.13 shows the kinematic diagram for the wheel assembly, numbering and labelling the links. The tip of the wheel was designated as point of interest X.

The lengths of the links are: s = 12 in.; l = 32 in.; p = 30 in.; q = 26 in.

2- Compare to Criteria

The shortest link is a side, or adjacent to the frame. According to the criteria in Table IV.1, this mechanism can be either a crank-rocker, change point, or a triple rocker. The criteria for the different categories of four-bar mechanisms should be reviewed.

3- Check the Crank-Rocker (Case 2) Criteria

The criteria Is: $\boldsymbol{s + l < p + q}$

$\mathbf{12 + 32 < 30 + 26}$ $\mathbf{44 < 56}$ **Verfied**

Because the criteria for a crank-rocker are valid, the nosewheel assembly is a crank-rocker mechanism.

f) ***Slider-crank mechanism***

Another mechanism that is commonly encountered is a slider-crank. This mechanism also consists of a combination of four links, with one being designated as the frame. This mechanism, however, is connected by three pin joints and one sliding joint.

Example IV.2

A mechanism that drives a manual water pump is shown in figure IV.14-a. The corresponding kinematic diagram is given in Figure IV.14-b.

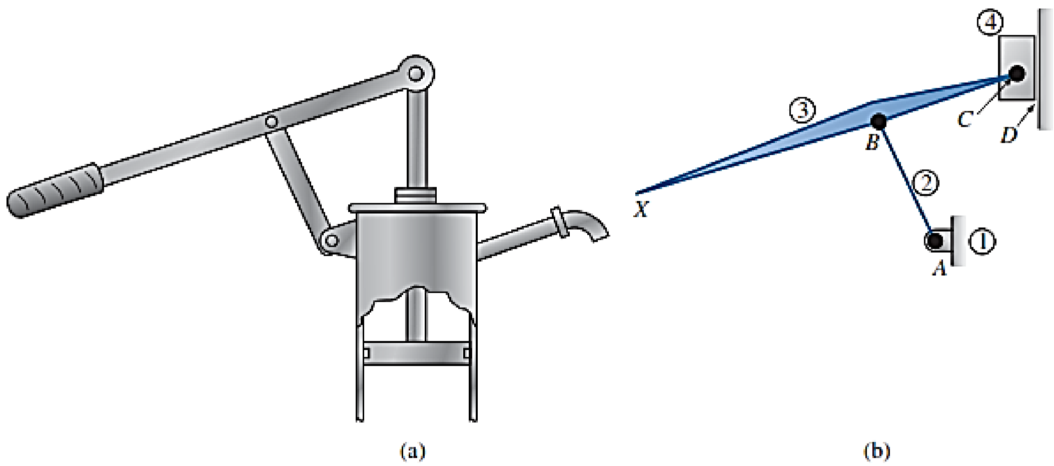


Figure IV. 14 : Pump mechanism for a manual water pump: (a) Mechanism and (b) Kinematic diagram.

The mobility of a slider-crank mechanism is represented by the following:

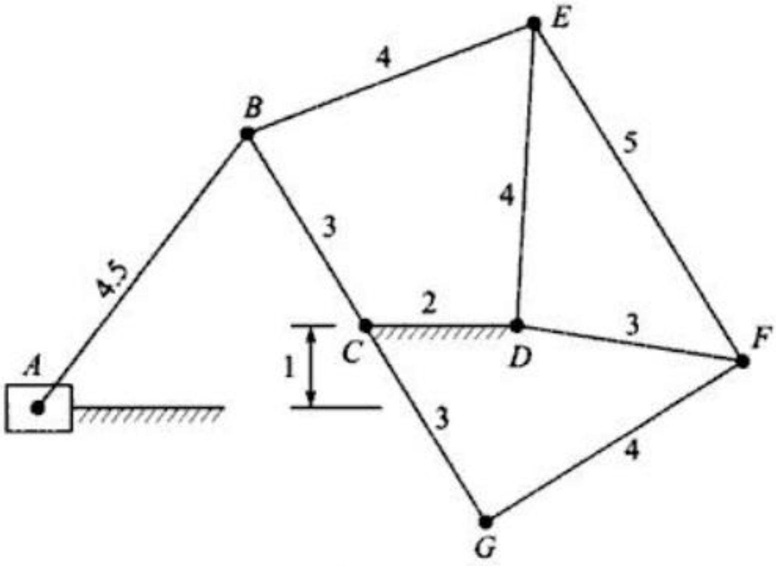
$n = 4$, $jp = (3 \text{ pins} + 1 \text{ sliding}) = 4$, $j_h = 0$ and

$M = 3(n - 1) - 2j_p - j_h = 3(4 - 1) - 2(4) - 0 = 1$

Because the slider-crank mechanism has one degree of freedom, it is constrained or fully operated with one driver. The pump in Figure IV.14 is activated manually by pushing on the handle (link 3). In general, the pivoted link connected to the frame is called the crank. This link is not always capable of completing a full revolution. The link that translates is called the slider. This link is the piston/rod of the pump. The coupler or connecting rod “couples” the motion of the crank to the slider.

Example IV.3 (Inversions of Grashof’s chain) :

The followin scheme shows a planar mechanism with link-lenghts given in some unit. If slider ***A*** is the driver, will link ***CG*** revolve or oscillate. Justify your answer.



Solution IV.3:

The loop formed by three links ***DE, EF*** and ***FD*** represents a structure. Thus the loop can be taken to represent a ternary link.

Now in the 4-link loop ***CDEB***, s=2, l =4 and p+q=7. Tus the 4-link loop portion ***CDEB*** satisfies Grashoff’s criterion. And as the shortest link CD is fixed, link CB is capable of complete revolution. Also, 4-link loop ***CDFG*** satisfies Grashoff’s criterion (***l+s=p+q***)and the shortest link ***CD*** is fixed. Thus whether considered a part of 4-link loop ***CDEB*** or that of ***CDFG***, link ***BCG*** is capable of full revolution.

Example IV.4

In a 4-bar mechanism, the lengths of driver crank, coupler and follower link are 150 mm, 250 mm, and 300 mm. The fixed link-length is L_0. Find the range of values for L_0 so as to make it a :

- Crank-rocker mechanism
- Crank-crank mechanism

Solution IV.4

a- For a crank-rocker mechanism the conditions to be satisfied are :

- Link adjacent to fixed link must be the smallest link and

- $s + l \leq p + q$

We have to consider both the possibilities, namely, (i) ***L_0*** is the longest link and (ii) when ***L_0*** is not the longest link.

When ***L_0*** is the longest link, from Grashoff's criterion,

$$l_0 + 150 \leq 250 + 300 \quad \text{so} \quad l_0 \leq 400\ mm$$

Or when l_0 is not the longest link, from Grashoff's criterion,

$$300 + 150 \leq l_0 + 250 \quad \text{so} \quad l_0 \geq 200\ mm$$

Or Thus, for crank-rocker mechanism, range of values for l_0 is

$$200 \leq l_0 \leq 400$$

b- For crank-crank mechanism, the conditions to be satisfied are :

- (i) Shortest link must be the frame link, and
- (ii) $s + l \leq p + q$

Thus, $l_0 + 300 \leq 150 + 250 \quad$ so $\quad l_0 \leq 100\ mm$

IV.5.4 Special purpose mechanisms

a) ***Straight-Line Mechanisms***

Straight-line mechanisms cause a point to travel in a straight line without being guided by a flat surface. Historically, quality prismatic joints that permit straight, smooth motion without backlash have been difficult to manufacture. Several mechanisms have been conceived that create straight-line (or nearly straight-line) motion with revolute joints and rotational actuation. Figure IV.15-a shows a Watt linkage and Figure. IV.15-b shows a Peaucellier-Lipkin linkage.

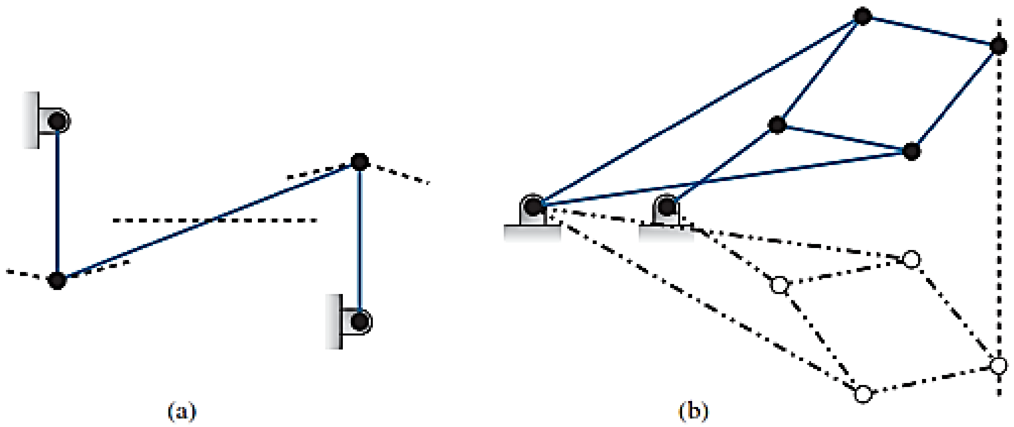


Figure IV. 15 : Straight-line mechanism

b) ***Parallelograms mechanism***

Mechanisms are often comprised of links that form parallelograms to move an object without altering its pitch. These mechanisms create parallel motion for applications such as balance scales, glider swings, and jalousie windows. Two types of parallelogram linkages are given in Figure IV.16-a which shows a scissor linkage and Figure IV.16-b which shows a drafting machine linkage.

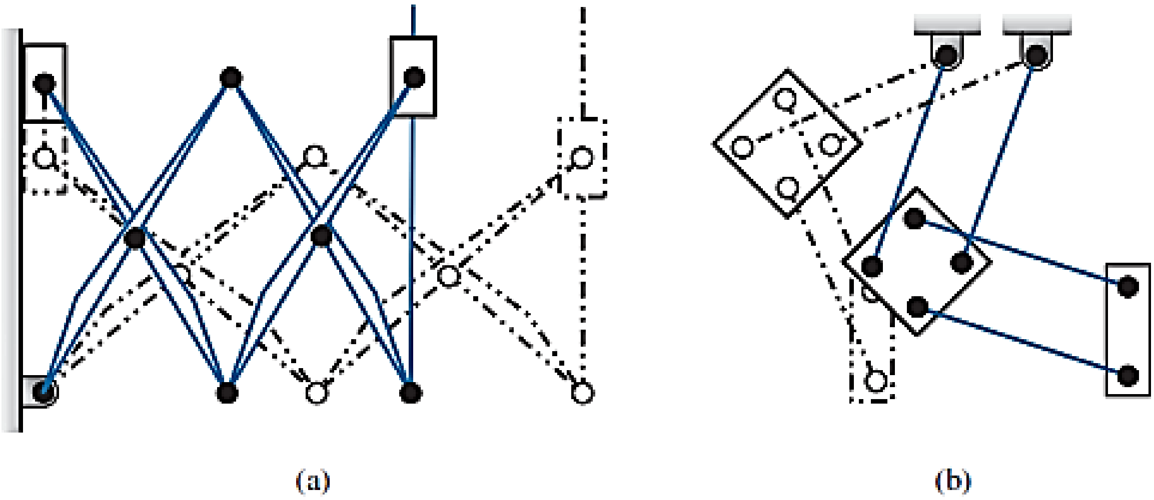


Figure IV. 16 : Parallelograms mechanism

c) ***Quick-return mechanisms***

Quick-return mechanisms exhibit a faster stroke in one direction than the other when driven at constant speed with a rotational actuator. They are commonly used on machine tools that require a slow cutting stroke and a fast return stroke. The kinematic diagrams of two different quick-return mechanisms are given in Figure IV.17-a which shows an offset slider-crank linkage and Figure IV.17-b which shows a crank-shaper linkage.

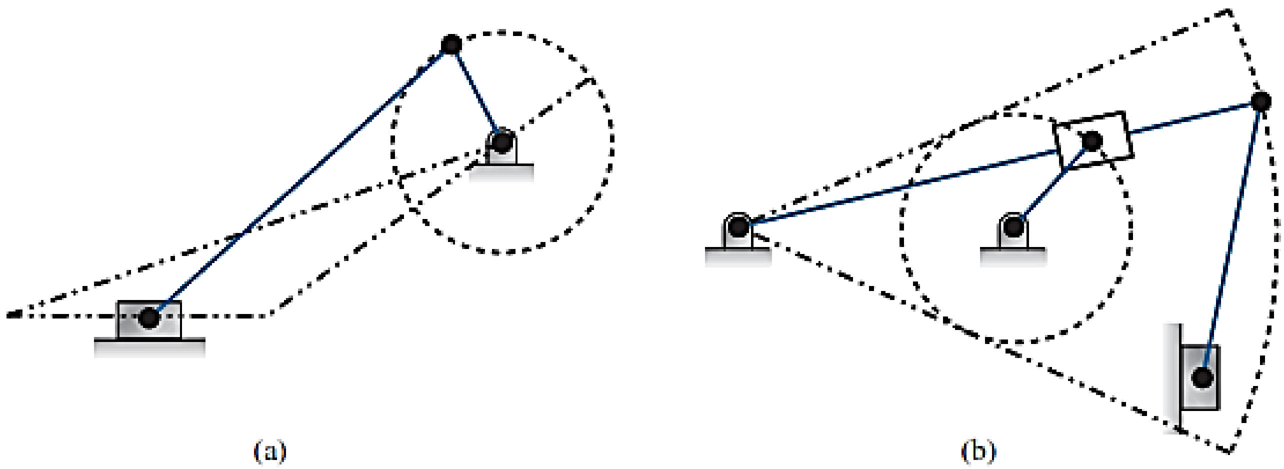


Figure IV. 17 : Quick-return mechanism

d) ***Scotch yoke mechanism***

A scotch yoke mechanism is a common mechanism that converts rotational motion to linear sliding motion, or vice versa. As shown in Figure IV.18, a pin on a rotating link is engaged in the slot of a sliding yoke. With regards to the input and output motion, the scotch yoke is similar to a slider-crank, but the linear sliding motion is pure sinusoidal. In comparison to the slider-crank, the scotch yoke has the advantage of smaller size and fewer moving parts, but can experience rapid wear in the slot.

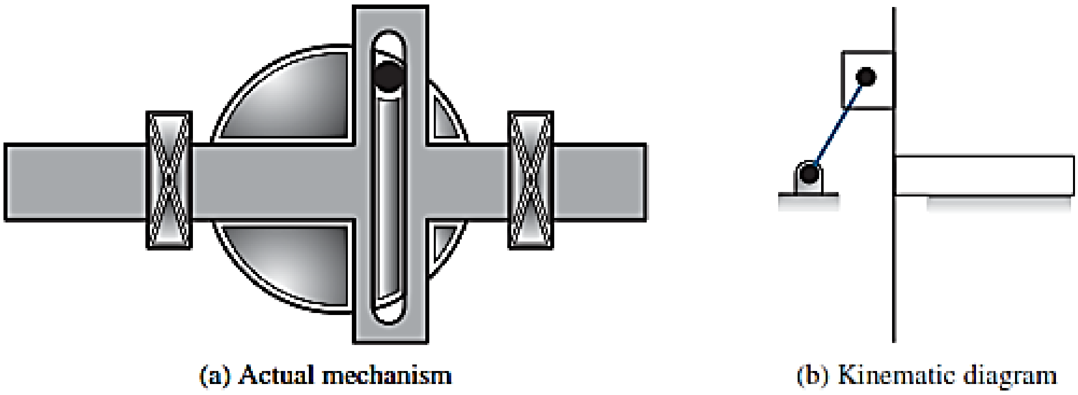


Figure IV. 18 : Scotch yoke mechanism

IV.6 Analysis of the displacement of a planar mechanism

In analyzing the motion of a planar linkage, the most basic issue encountered is defining the concept of position and displacement. Because motion can be thought of as a time series of displacements between successive positions, it is important to understand the meaning of the term position.

Position is a term that tells where an item is located if the item is a point. Its position can be specified by its distance from a predefined origin and its direction with respect to a set reference axes. If we choose to work with a Cartesian coordinate system, we can specify the position of a point by giving its X and Y coordinates. If we choose to work in a polar coordinate system, then we need to specify the distance from the origin and the angle relative to one of its reference axes. In any case, the position of a point in two-dimensional (2D) space is a vector quantity.

If we want to specify the position of a rigid body, it is necessary to specify more than just its (x, y) coordinates. It is necessary to specify enough information that the location of every point on the rigid body is uniquely determined. A rigid body in 2D space can be defined by two points, A and B, on the rigid body. The position of point B with respect to point A is equal to the position of point B minus the position of point A. Another way to say this is the position of point B can be defined by defining the position of point A and the position of point B relative to point A. Since the object is rigid, all of its points are defined relative to these two points.

Since the purpose of a planar linkage is to move one of its links through a specified motion, or have a point on one of its links move through a specified motion, it is important to be able to verify that the linkage performs its desired function. Thus, position analysis will be covered first.

Successive positions of a moving point define a curve. The curve has no thickness; however, the curve has length because it occupies different positions at different times. This curve is called a path or locus of moving points relative to a predefined coordinate system.

IV.6.1 Graphical method

Graphical position analysis can be used quickly to check the location of a point or the orientation of a link for a given input position. All that is needed is a straight edge, a scale, and a protractor. If a parametric CAD system, Solidworks, AutoCAD…. is used, then the user simply needs to sketch the linkage, adjust the sizes and orientation of the known links, then request the position and orientation of the unknown links and/or points. Graphical position analysis is also very useful in checking the analytical position analysis solution at several points to verify that the analytical solution is valid. The graphical solution procedures for 4-bar and slider-crank linkages are outlined below. A similar procedure can be used to draw other linkages such as 6-bars.

a) ***Graphical Analysis for a 4-Bar***

Example IV.5: Assume we want to determine the proper orientation for the 4-bar linkage shown in Figure IV.19 when the input, link 2, is at 40 . Note that the input link, L2, is on the right in this figure.

Given:

The origin is located at bearing A_0. Bearing B_0 is 3.50 in. left of bearing A_0. The input, link 2, is 1.25 in. long. Link 3 is 4.00 in. long. Link 4 is 2.00 in. long. Distance from point C to point P is 3.00 in. Distance from point D to point P is 2.00 in.

- *What is the angular orientation of links 3 and 4?*
- *What is the (x, y) location of point P relative to the origin?*

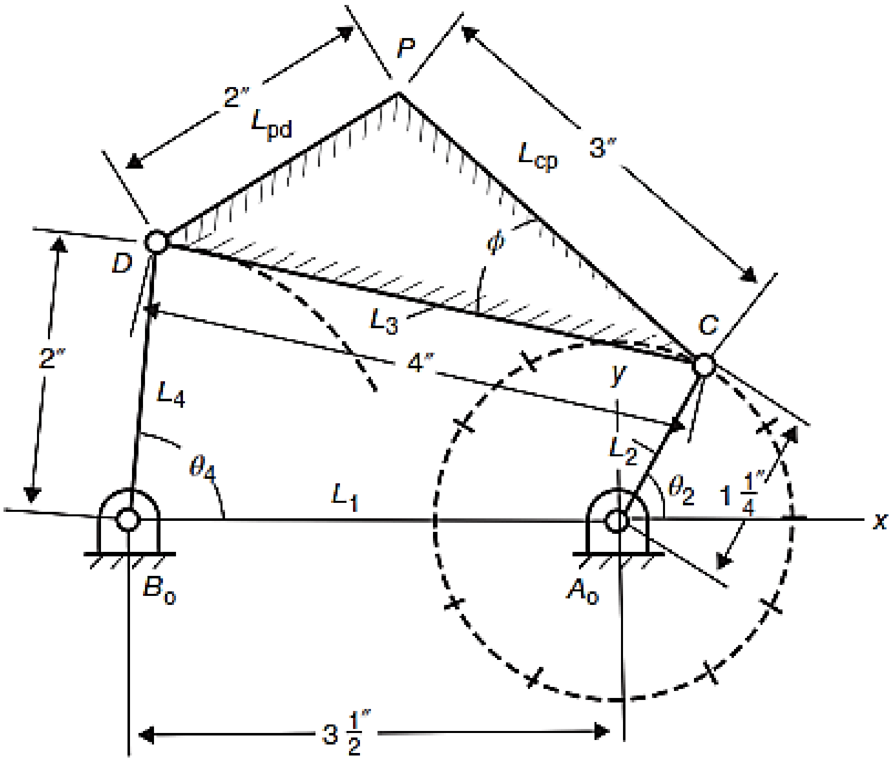


Figure IV. 19 : 4-Bar linkage sketch.

Procedure:

1- Locate the origin and label it $\boldsymbol{A_0}$. Select an appropriate scale, in this example, actual size is 1 model unit = 2 in.

2- Locate bearing $\boldsymbol{B_0}$ relative to $\boldsymbol{A_0}$.

3- Draw a line starting at $\boldsymbol{A_0}$ at a 40 angle above the x-axis, then mark off it length of 1.25 model units. Mark this point ***C***.

4- Set your compass at the length of link 4, 2.00 model units. Draw an arc centered at bearing $\boldsymbol{B_0}$.

5- Set your compass at the length of link 3, 4.00 model units. Draw an arc centered at point ***C*** so that it crosses the previously drawn arc. The intersection of these two arcs is point ***D***. (Note there are two intersection points; one above the x-axis for the uncrossed linkage and one below the x-axis for the crossed linkage. Choose the intersection point above the x-axis.)

6- Draw in links 3 and 4.

7- Set your compass at the length of $\boldsymbol{L_{cp}}$, 3.00 model units. Draw an arc centered at point ***C***.

8- Set your compass at the length of $\boldsymbol{L_{pd}}$, 2.00 model units. Draw an arc centered at point ***D*** so it intersects the previously drawn arc. The intersection of these two arcs is point ***P***. (Note there are two intersection points; choose the proper intersection point.)

9- Draw in the sides, $\boldsymbol{L_{cp}}$ and $\boldsymbol{L_{pd}}$, to complete the 4-bar linkage.

10- Using a protractor, measure the angular orientation of links 3 and 4.

11- Using your scale, measure the horizontal and vertical distances from the origin to point ***P***.

12- Box in your answers with the appropriate units.

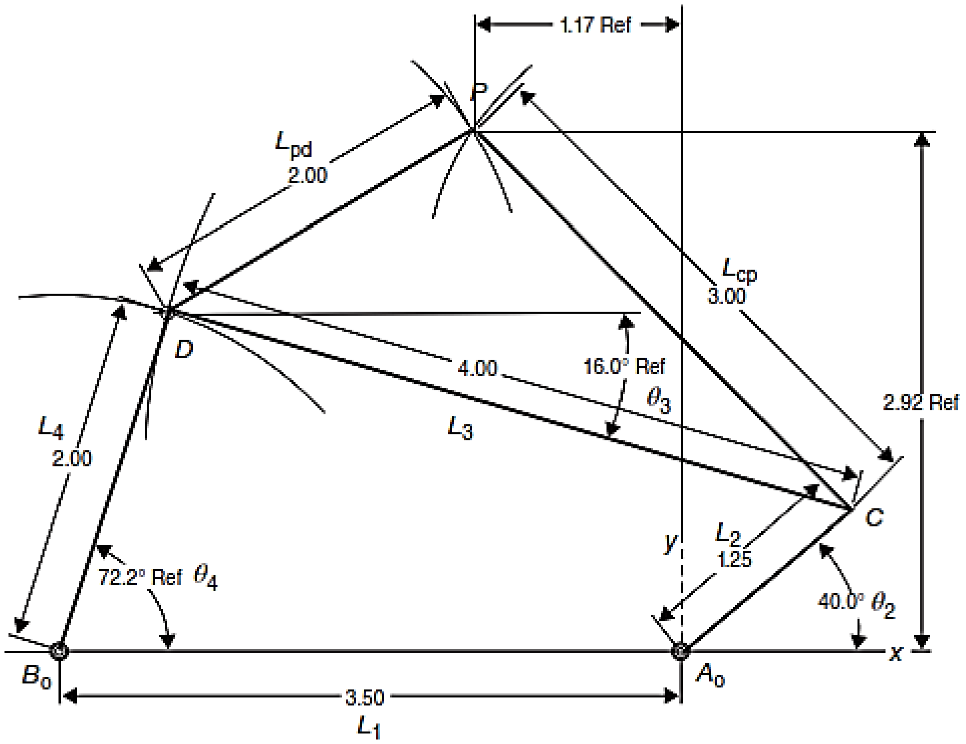


Figure IV. 20 : Graphical solution of 4-bar.

From the graphical solution shown in Figure IV.20, it can be seen that $\theta_3 = -16.0$, $\theta_4 = 72.2$, and point P =(−1.17, 2.92) in. relative to the origin located at bearing $\boldsymbol{A_0}$. The construction arcs were left on the sketch to help clarify the construction procedure.

b) ***Graphical Analysis for a Slider-Crank Linkage***

Example IV.6 : Assume we want to determine the proper orientation for the slider-crank linkage shown in Figure IV.21 when the input, link 2, is at 115 .

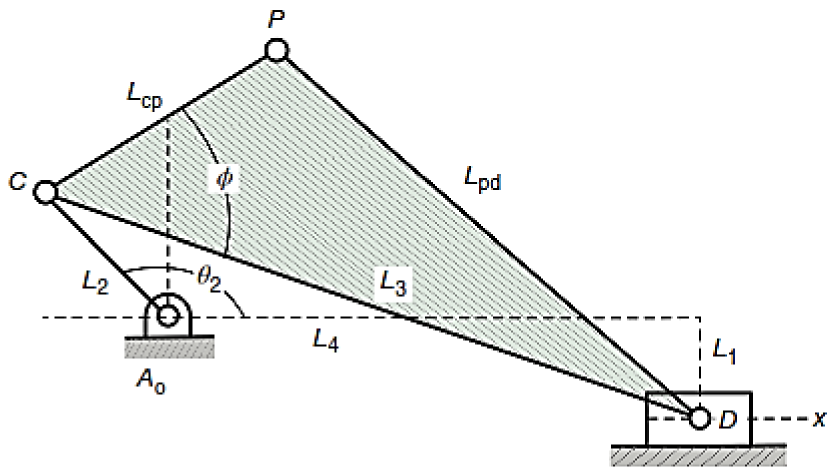


Figure IV. 21 : Slider-crank sketch.

Given:

The origin is located at bearing $\boldsymbol{A_0}$. Point ***D*** is 30 mm below bearing $\boldsymbol{A_0}$. The input, link 2, is 50 mm long. Link 3 is 185 mm long. Distance from point ***C*** to point ***P*** is 75 mm.

Distance from point ***D*** to point ***P*** is 145 mm.

- What is the angular orientation of link 3?
- What is the horizontal distance from the origin to point ***D***?
- What is the (x, y) location of point ***P*** relative to the origin?

Procedure:

1- Locate the origin and label it $\boldsymbol{A_0}$. Select an appropriate scale such as 1 model unit = 1 mm (actual size).
2- Since point ***D*** is below the origin and travels along a horizontal line, draw a vertical line from point $\boldsymbol{A_0}$ the length of $\boldsymbol{L_1}$, then a horizontal line that will indicate the possible locations of point ***D***.
3- Draw a line starting at $\boldsymbol{A_0}$ at a 115 angle above the x-axis, then mark off it length of 50 model units. Mark this point ***C***.
4- Set your compass at the length of link 3, 185 model units. Draw an arc centered at point ***C*** so that it crosses the horizontal line which indicates possible locations for point ***D***. The intersection is point ***D***.
5- Draw in link 3, and then draw a rectangle centered at point ***D*** to represent the slider.
6- Set your compass at the length of $\boldsymbol{L_{cp}}$, 75 model units. Draw an arc centered at point ***C***.
7- Set your compass at the length of $\boldsymbol{L_{pd}}$, 145 model units. Draw an arc centered at point ***D*** so it intersects the previously drawn arc. The intersection of these two arcs is point ***P***. (Note there are two intersection points; choose the proper intersection point.)
8- Draw in the sides, $\boldsymbol{L_{cp}}$ and $\boldsymbol{L_{pd}}$, to complete the slider-crank linkage.
9- Using a protractor, measure the angular orientation of link 3.
10- Using your scale, measure the horizontal distance from the origin at point $\boldsymbol{A_0}$ to point ***D***.
11- Measure the horizontal and vertical distances from the origin to point ***P***.
12- Box in your answers with the appropriate units.

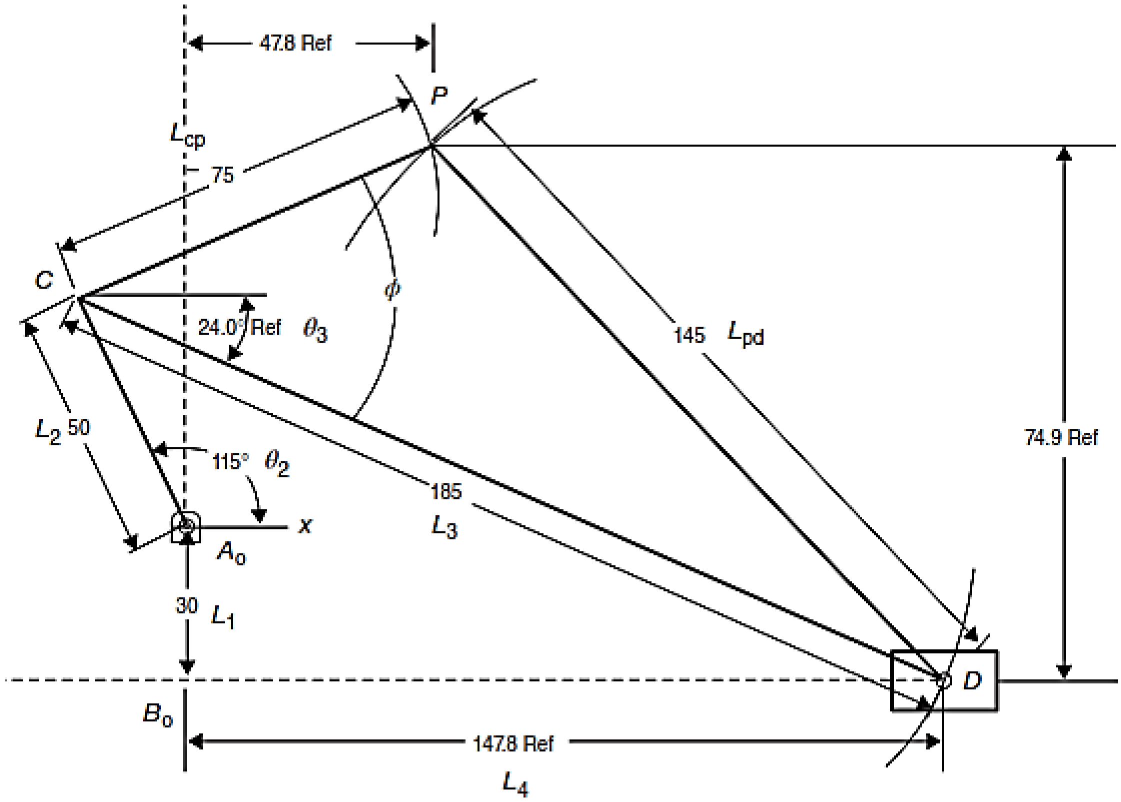


Figure IV. 22 : Slider-crank graphical position.

From the graphical solution shown in Figure IV.22, it can be seen that $\theta_3 = -24.0$, $L_4 = 147.8$ mm, and point ***P*** = (47.8 mm, 74.9 mm) relative to the origin located at bearing $\boldsymbol{A_0}$.

The construction arcs were left on the sketch to help clarify the construction procedure. If you wanted to determine the angular position of link 3 and the linear position of the slider relative to the origin at $\boldsymbol{A_0}$, you would need to redraw the figure starting at step 3. For each position of link 2 that you want to analyze, you would need to redraw the figure. A better way to analyze the positions of the slider-crank linkage for varying angular positions of link 2 would be to design equations that define its position as a function of link 2's angular position.

IV.6.2 Analytical method

a) ***Vector loop position analysis***

A Euclidean vector is a geometric entity having a magnitude and a direction. In engineering, Euclidean vectors are used to represent physical quantities that have both magnitude and direction, such as force or velocity. In contrast, scalar quantities, such as mass or volume, have a magnitude but no direction.

A position vector is a vector representing the position of a point in a finite space in relation to a reference point and a coordinate system (Figure IV.23). A displacement vector is a vector that specifies the change in position of a point relative to its previous position.

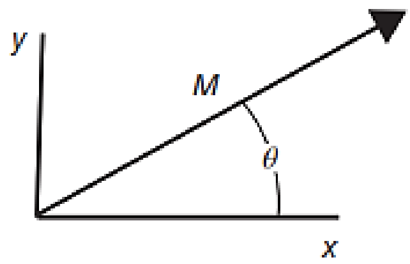


Figure IV. 23 : Representation of vector in plan.

For a 2D vector you need its magnitude, M, and its angle relative to the positive x-axis, θ. In this part, all 2D vectors will be defined from the positive x-axis with the angle being positive when measured in the counterclockwise (c.c.w.) direction. A negative angle is defined as the angle from the positive x-axis measured in the clockwise (c.w.) direction. When you are looking for (X, Y) components of a vector, the following is always true.

$$X = \text{magnitude of x} - \text{axis component vector} = \mathrm{M}.\cos(\theta)$$

$$Y = \text{magnitude of y} - \text{axis component vector} = \mathrm{M}.\sin(\theta)$$

When adding vectors, you first need to find the (X, Y) components of the vectors to be added, and then add the X-components and add the Y-components as shown below. Finally, combine the components back into a magnitude and an angle.

$$\vec{V} = X.\vec{\imath} + Y.\vec{\jmath} \quad \text{and} \quad \vec{V} = \overrightarrow{V_1} + \overrightarrow{V_2}$$

With :

$$\overrightarrow{V_1} = \{X_1, Y_1\} = \{M_1.\cos(\theta_1)\,, M_1.\sin\,(\theta_1)\} \qquad \text{and} \qquad \overrightarrow{V_2} = \{X_2, Y_2\} = \{M_2.\cos(\theta_2)\,, M_2.\sin\,(\theta_2)\}$$

We have : $X = X_1 + X_2$ and $Y = Y_1 + Y_2$

$\vec{V} = (X_1 + X_2).\vec{\imath} + (Y_1 + Y_2).\vec{\jmath}$ and the magnitude M of the vector $\vec{V}$ equal :

$$M = \sqrt{X^2 + Y^2}$$

b) ***Position Analysis of 4-Bar Linkage***

This part describes the method that uses algebra and trigonometry to create several equations that are easy to solve using your calculator.

Example IV.7 : Using Vector Loop approach for position analysis of a 4-bar linkage (Figure IV.24) leads to:

$$\overrightarrow{L_2} + \overrightarrow{L_3} - \overrightarrow{L_1} - \overrightarrow{L_4} = \vec{0}$$

Or : $$\overrightarrow{L_2} + \overrightarrow{L_3} = \overrightarrow{L_1} + \overrightarrow{L_4}$$

Writing the equations for the y-component of each vector and then the x-component of each vector, we get:

Ox axe : $L_2.\cos(\theta_2) + L_3.\cos(\theta_3) = L_1.\cos(\theta_1) + L_4.\cos(\theta_4)$

Oy axe : $L_2.\sin(\theta_2) + L_3.\sin(\theta_3) = L_1.\sin(\theta_1) + L_4.\sin(\theta_4)$

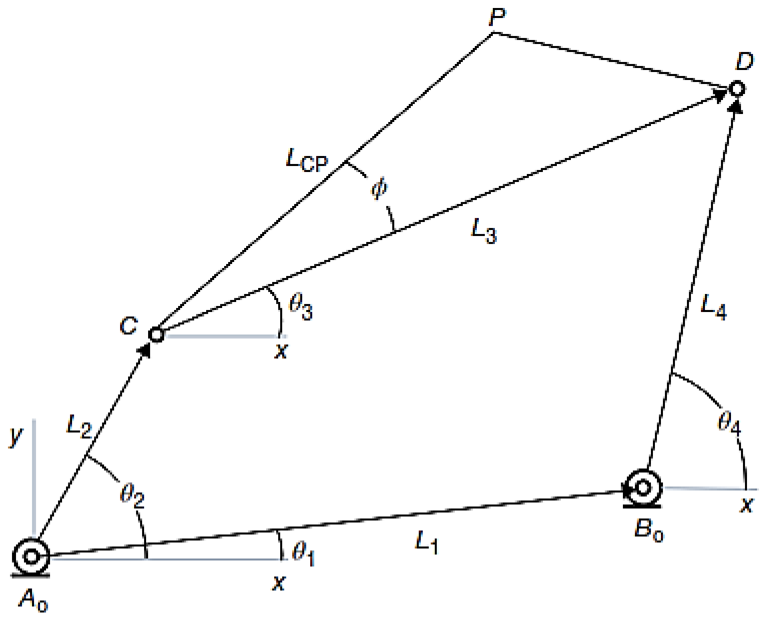


Figure IV. 24 : Vector Loop approach for position analysis of a 4-bar linkage.

Based on the assumption that the location of the bearings at ***A₀*** and ***B₀*** are defined along with the size of links 2, 3, and 4, the two unknowns θ_3 and θ_4 can be determined. Since link 2 is assumed the input link, its angular position is known, θ_2. Rearranging the two equations so that the unknowns are on the left and the two known values are on the right, we get:

Ox axe : $L_3.\cos(\theta_3) - L_4.\cos(\theta_4) = L_1.\cos(\theta_1) - L_2.\cos(\theta_2)$

Oy axe : $L_3.\sin(\theta_3) - L_4.\sin(\theta_4) = L_1.\sin(\theta_1) - L_2.\sin(\theta_2)$

c) ***Position Analysis of Slider-Crank Linkage***

This section describes the technique for determining the angular positions of link 3 and the linear position of the slider, link 4. Link 4 is in the direction of the slider motion. Link 1 is perpendicular to link 4.

Using Vector Loop approach for position analysis of a slider-crank linkage (Figure IV.25) leads to:

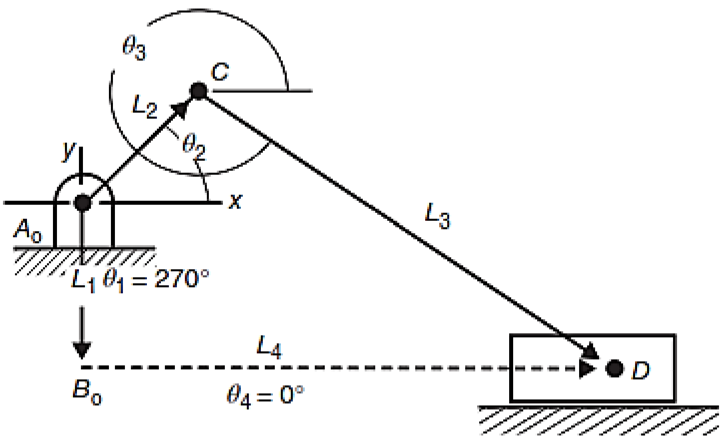


Figure IV. 25: 4-Bar linkage in crossed orientation.

Example IV.8

Given a slider-crank linkage with link lengths of L_1 = 2.10 in., L_2 = 2.00 in., and L_3 = 6.50 in. Bearing A_o is located at the origin. For the current position of θ_2 =43 , determine the angle θ_3 and the length L_4 as shown in Figure IV.25. Note θ_1 = 270 and θ_4 =0.

$$\overrightarrow{L_2} + \overrightarrow{L_3} - \overrightarrow{L_4} - \overrightarrow{L_1} = \vec{0}$$

0r : $$\overrightarrow{L_2} + \overrightarrow{L_3} = \overrightarrow{L_1} + \overrightarrow{L_4}$$

Writing the equations for the y-component of each vector and then the x-component of each vector, we get:

Ox axe : $L_2.\cos(\theta_2) + L_3.\cos(\theta_3) = L_1.\cos(\theta_1) + L_4.\cos(\theta_4)$

Oy axe : $L_2.\sin(\theta_2) + L_3.\sin(\theta_3) = L_1.\sin(\theta_1) + L_4.\sin(\theta_4)$

Based on the assumption that the location of the bearings at A_0 is defined along with the size of links 1, 2 and 3, the two unknowns θ_3 and L_4 can be determined. Since link 2 is assumed the input link, its angular position is known, θ_2. Rearranging the two equations so that the unknowns are on the left and the two known values are on the right, we get:

Ox axe : $L_3.\cos(\theta_3) - L_4.\cos(\theta_4) = L_1.\cos(\theta_1) - L_2.\cos(\theta_2)$

Oy axe : $L_3.\sin(\theta_3) - L_4.\sin(\theta_4) = L_1.\sin(\theta_1) - L_2.\sin(\theta_2)$

With : $\theta_1 = 270$, $\theta_4 = 0$, $\theta_2 = 43°$, L_1 = 2.10 in., L_2 = 2.00 in., and L_3 = 6.50 we obtain :

Ox axe : $L_3.\cos(\theta_3) - L_4.\cos(\theta_4) = L_1.\cos(\theta_1) - L_2.\cos(\theta_2)$

Oy axe : $L_3.\sin(\theta_3) = L_1.\sin(\theta_1) - L_2.\sin(\theta_2)$

$$\sin(\theta_3) = \frac{L_1.\sin(\theta_1) - L_2.\sin(\theta_2)}{L_3}$$

So : $\theta_3 = 327{,}79°$

Ox axe : $L_3.\cos(\theta_3) - L_4.\cos(\theta_4) = L_1.\cos(\theta_1) - L_2.\cos(\theta_2)$

$$L_4 = \frac{L_3.\cos(\theta_3) + L_2.\cos(\theta_2) - L_1.\cos(\theta_1)}{\cos(\theta_4)}$$

$$L_4 = 6{,}96 \text{ in}$$