ChapterIII:
Kinematics of material points

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III.1. Definition

Kinematic of material point is the study without mentioning the cause (forces).

A material point is every particle whose dimension can be considered negligible.

Kinematics is the study of the motion of a solid, determining its position, velocity, and acceleration.

Kinematics is the branch of mechanics that examines and describes the motion of an object considered infinitesimally small, referred to as a point particle, denoted as M.

III.2.Movement and statics

Movement and statics are relative concepts; a person in the car is stationary in relation to it but he is moving in relation to a person in the road.

Therefore, to study the movement of an object a reference must be set to analyze this movement.

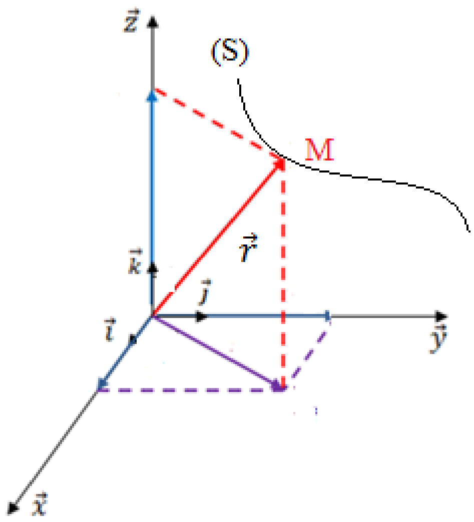
This study takes in one of two forms:

Vector shape: using Position vector $\overrightarrow{OM}$, velocity vector $\vec{v}$ and acceleration vector $\vec{a}$.

Scalar shape: to find the equation of the movement for the path (trajectory).

III.2.1.Position vector

The position of material point (M) in the reference R($\vec{x}, \vec{y}, \vec{z}$) is defined as position vector $\overrightarrow{OM}$.



With: $\overrightarrow{OM} = \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

Material point is stationary if its coordinates are constant, that are independent of a time and we call this point is in the movement if its coordinates are dependent of the time

$$\overrightarrow{OM} = \vec{r}(t) = \begin{cases} x = f(t) \\ y = g(t) \\ z = h(t) \end{cases}$$

With: f(t), g(t) and h(t) are called parametric equations of motion.

Trajectory (Path)

(S) is the path (trajectory) of the material point and we express it as the sum of the successive positions that this point occupies during successive times.

The study of plane motion is done in the reference where the position becomes defined by:

$$\vec{r}(t) = \begin{cases} x(t) \\ y(t) \end{cases}$$

The relation $x \to y(x)$ is called the Cartesian equation of the path (trajectory).

We find it, by eliminating the time (t) between the two equations $\begin{cases} x(t) \\ y(t) \end{cases}$.

<u>Example</u>

Let (M) be material point defined by its parametric equations of motion.

$$\vec{r}(t) = \begin{cases} x = 2t \\ y = -5t^2 + 4 \end{cases}$$

- Find the trajectory (Path) of (M) and what its shape is?

<u>Solution</u>

We have: $$\vec{r}(t) = \begin{cases} x = 2t & (1) \\ y = -5t^2 + 4 & (2) \end{cases}$$

from (1): $$t = \frac{x}{2}$$

$$\Longrightarrow y = -5\left(\frac{x}{2}\right)^2 + 4\left(\frac{x}{2}\right)$$

$$\Longrightarrow y = -\frac{5}{4}x^2 + 2x$$

The path of (M) is parabola because its quadratic equation.

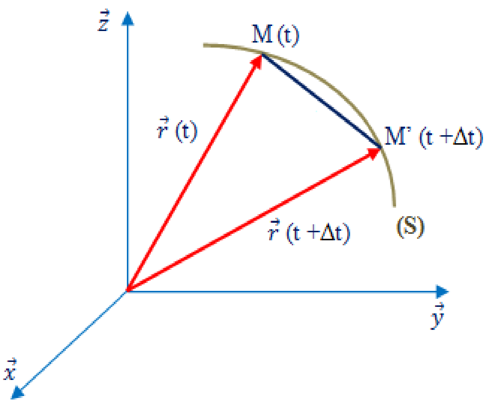
III.2.1.Velocity vector

Velocity is a vector quantity that provides information about the change in position of a point with respect to time. It is the rate of change of position with respect to time with units of m/s (meters per second).

By analyzing the velocity vector, you can determine the direction and sense of motion, while the displacement vector provides information about the magnitude of the change in position. These concepts are fundamental in understanding the kinematics of a particle's motion**.**

Velocity is a vector quantity; its direction is tangent to the trajectory.

Average velocity:



Let material point (M) in the reference $R(\vec{x}, \vec{y}, \vec{z})$

With:

M(t):is a position of M at time (t).

M'(t+Δt): is the position of point M at time (t+Δt).

$\overrightarrow{MM'}$: is a vector ofdisplacement ofpoint M.

(S): is the path of M.

Average velocity ($\overrightarrow{V_m}$) is the displacement of an object divided by the time:

It is the ratio of the displacement to the time it takes to cover that displacement.

$$\overrightarrow{V_m} = \frac{\overrightarrow{MM'}}{\Delta t} = \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t} = \frac{\Delta \vec{r}}{\Delta t}$$

Instantaneous Velocity

It is the velocity at a specific moment t. It can be defined as the average velocity between the positions (M) of the point at time t and the position (M') of the same point at time (t+Δt), where Δ t represents a very small duration

Instantaneous velocity represents the first derivative with respect to time of the position vector. It is defined by:

$$\vec{V} = \frac{\partial \vec{r}}{dt}$$

This vector always remains tangent to the trajectory and in the same direction of movement.

We have

$$\overrightarrow{OM} = \vec{r}(t) = \begin{cases} x = f(t) \\ y = g(t) \\ z = h(t) \end{cases}$$

$$\vec{v} = \begin{cases} v_x = \dfrac{\partial x}{dt} = \dot{x} \\ v_y = \dfrac{\partial y}{dt} = \dot{y} \\ v_z = \dfrac{\partial z}{dt} = \dot{z} \end{cases}$$

And the module of the velocity is defined by:

$$v = \sqrt{{v_x}^2 + {v_y}^2 + {v_z}^2}$$

III.2.2.Acceleration vector

Just as the velocity vector informs us about the change in the position vector over time, the acceleration vector informs us about the changes in the velocity vector over time

Average acceleration

The average acceleration between two instants (t and t+Δt) is defined by:

$$\overrightarrow{a_m} = \frac{\vec{v}(t+\Delta t) - \vec{v}(t)}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$$

Instantaneous acceleration

The instantaneous acceleration represents the first derivative with respect to time of the velocity vector or the second derivative of the position vector

$$\vec{a} = \frac{\partial \vec{v}}{dt} = \frac{\partial^2 \vec{r}}{dt}$$

We have

$$\overrightarrow{OM} = \vec{r}(t) = \begin{cases} x = f(t) \\ y = g(t) \\ z = h(t) \end{cases}$$

$$\overrightarrow{v} = \begin{cases} v_x = \dfrac{\partial x}{dt} = \dot{x} \\ v_y = \dfrac{\partial y}{dt} = \dot{y} \\ v_z = \dfrac{\partial z}{dt} = \dot{z} \end{cases}$$

$$\overrightarrow{a} = \begin{cases} a_x = \dfrac{\partial v_x}{dt} = \ddot{x} \\ a_y = \dfrac{\partial v_y}{dt} = \ddot{y} \\ a_z = \dfrac{\partial v_z}{dt} = \ddot{z} \end{cases}$$

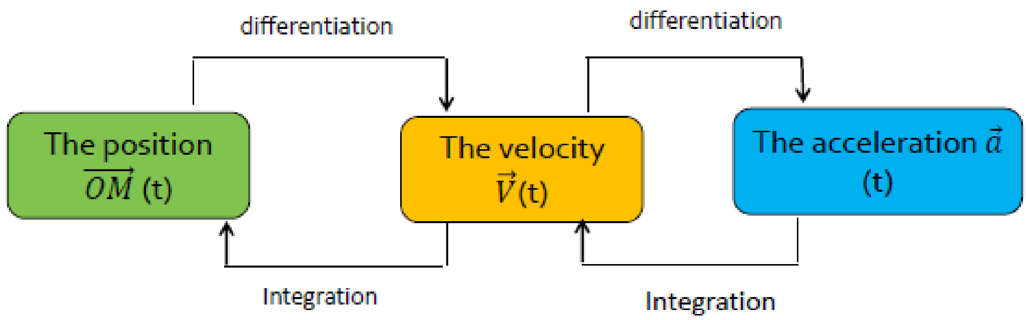
And the module of the acceleration is defined by:

$$a = \sqrt{{a_x}^2 + {a_y}^2 + {a_z}^2}$$

Noticed

- The acceleration vector is always directed towards the concavity of the path in curved motion.
- The motion is accelerating: $\vec{a}.\vec{v} > 0$
- The motion is decelerating: $\vec{a}.\vec{v} < 0$

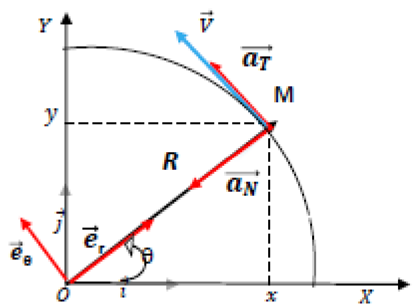
The three kinematic equations, which are the $\overrightarrow{0M}(t)$,the velocity $\vec{v}(t)$ and the acceleration $\vec{a}(t)$, are mathematical functions that can be derived from each other through differentiation and integration.



III.3.Circular motion

In circular motion, the trajectory of point M is a circle with center O and radius r. It is logical to choose the origin of the coordinate system as the center O of the circle.

With *R* = constant and $\theta = f(t)$



$\vec{e}_r$: is the unit vector tangent to the path.

$\vec{e}_\theta$: is the unit vector perpendicular to the $\vec{e}_r$.

The position vector can be written as:

$$\overrightarrow{OM} = R\vec{e}_r$$

And we have

$$\vec{e}_r = cos\theta\,\vec{i} + sin\theta\,\vec{j}$$

$$\vec{e}_\theta = -sin\theta\,\vec{i} + cos\theta\,\vec{j}$$

The velocity vector can be written as:

$$\vec{v} = \frac{\partial\overrightarrow{OM}}{dt} = \frac{\partial(R\vec{e}_r)}{dt} = R\frac{\partial\vec{e}_r}{dt}$$

$$\frac{\partial\vec{e}_r}{dt} = -\dot{\theta}sin\theta\,\vec{i} + \dot{\theta}cos\theta\,\vec{j}$$

$$= \dot{\theta}[-sin\theta\,\vec{i} + cos\theta\,\vec{j}]$$

$$= \dot{\theta}\vec{e}_\theta$$

$$\vec{v} = R\dot{\theta}\vec{e}_\theta$$

$\dot{\theta}$*:* is the angular velocity

Hence, the speed of point (M) is its tangent to the circle at point (M) and its direction is the direction of motion$(\overrightarrow{e_\theta})$.

The acceleration vector can be written as:

$$\vec{a} = \frac{\partial\vec{v}}{dt} = \frac{\partial(R\dot{\theta}\vec{e}_\theta)}{dt} = R\frac{\partial\dot{\theta}}{dt}\,\vec{e}_\theta + R\dot{\theta}\frac{\partial\vec{e}_\theta}{dt}$$

$$\frac{\partial\vec{e}_\theta}{dt} = -\dot{\theta}cos\theta\,\vec{i} - \dot{\theta}sin\theta\,\vec{j}$$

$$= -\dot{\theta}[cos\theta\,\vec{i} + sin\theta\,\vec{j}]$$

$$=-\dot{\theta}\vec{e}_r$$

$$\vec{a} = R\ddot{\theta}\vec{e}_\theta - R\dot{\theta}^2\vec{e}_r$$

$$\vec{a} = \vec{a}_T + \vec{a}_N$$

The module of *(a)*

$$a = \sqrt{a_N^2 + a_T^2}$$

With:

a_T is Tangential acceleration defined by: $\vec{a}_T = \frac{\partial \vec{v}}{dt}$

And

a_N is Normal acceleration defined by: $\frac{v^2}{R}$