ChapterII:
Forces and Moments of the forces

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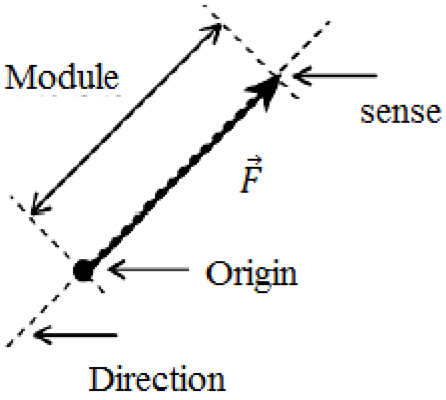
II.1 Definition of force

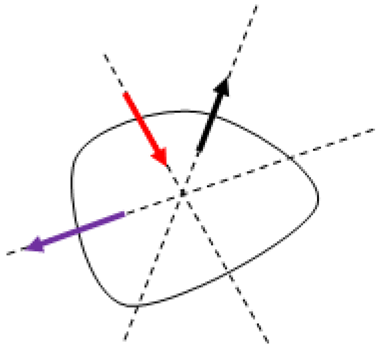
Force is any cause capable of modifying the shape or movement of an object to which it is applied. It can be considered as a special form of radiation to which the same properties apply.



II.2 Force Systems in space

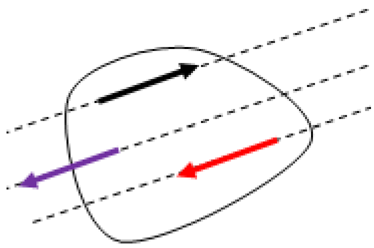
Force systems in space are classified into three categories:

- **Concurrent**: The directions of all forces in the system pass through the same point. This is called concurrent forces at a point.



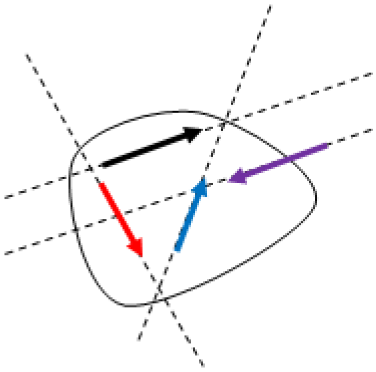
Concurrent forces

- **Parallels**: the directions of the forces are all parallel, we also say Parallel forces.



Parallel forces

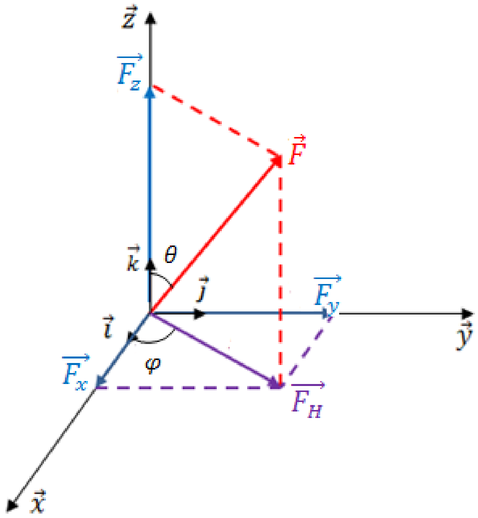
- **Non-concurrent and non-parallel**: the forces are not all concurrent and not all parallel.



Non-concurrent and non-parallel

II.3 Components of a force

Let a force $\vec{F}$ be applied to the origin O of an orthonormal frame R $(O,\vec{\imath}, \vec{\jmath}, \vec{k})$. The components of this force are defined by:



$$\vec{F} = \vec{F}_H + \vec{F}_z$$

$$\vec{F} = \vec{F}_H + F\cos\theta\,\vec{k}$$

$$\vec{F}_H = F_x\,\vec{\imath} + F_y\,\vec{\jmath}$$

$$= F\sin\theta\cos\varphi\,\vec{\imath} + F\cos\theta\,\sin\varphi\,\vec{\jmath}$$

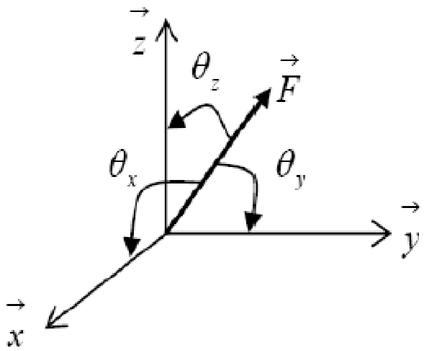
$$\vec{F} = F\sin\theta\cos\varphi\,\vec{\imath} + F\cos\theta\,\sin\varphi\,\vec{\jmath} + F\cos\theta$$

with: $\vec{r}$

$$\overrightarrow{F_x} = F\sin\theta\cos\varphi, \quad \overrightarrow{F_y} = F\cos\theta\,\sin\varphi \quad \text{et} \quad F_z = \vec{F}\cos\theta$$

II.4 Direction cosines

The projections of the force F on the three axes ox, oy, oz respectively give the angles $\theta x, \theta y\ et\ \theta z$, we will then have:



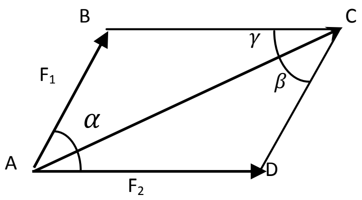
$$F_x = F.\cos\theta_x, \quad F_y = F.\cos\theta_y$$

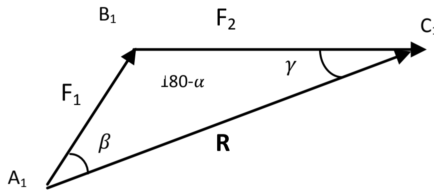
$$F_z = F.\cos\theta_z$$

II.5 Resultant of a set of concurrent forces

II.5.1 Resultant of two forces

We can determine the geometric sum R of the two forces F1 and F2 either by using the parallelogram method or by constructing the triangle of forces





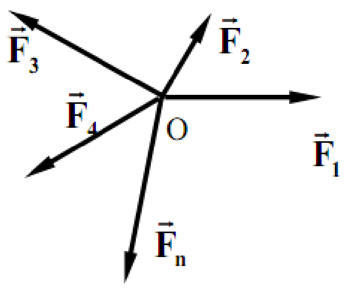
with : $$R^2 = {F_1}^2 + {F_2}^2 - 2F_1F_2\cos(180-\alpha)$$

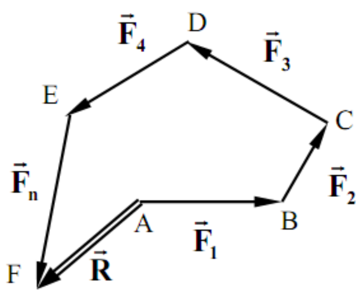
$$R = \sqrt{{F_1}^2 + {F_2}^2 + 2F_1F_2\cos\alpha}$$

II.5.2.Resultant of several forces

a. Graphical solution "Rule of the polygon of forces

To construct the polygon of forces, we respect the sense and direction of each force. First, we place the origin of the vector $\overrightarrow{F_2}$ at the end of the vector $\overrightarrow{F_1}$, then place the origin of $\overrightarrow{F_3}$ at the end of $\overrightarrow{F_2}$, ...etc. by joining the point of application of the forces and the end of $\overrightarrow{F_n}$, we obtain the resultant R $^{\rightarrow}$. The polygon ABCDEF formed by the forces is called the polygon of forces, and the vector $\vec{R}$ closing the polygon, is called the resultant of the forces.





The result is represented by the sum:

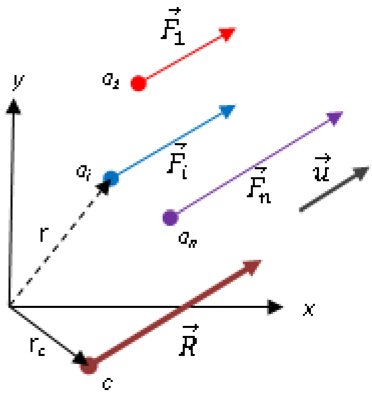
$$\vec{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \cdots + \overrightarrow{F_n}$$

$$\vec{R} = \sum_{i=1}^{n} \vec{F_i}$$

II.6 Resultant of a set of Parallel forces

Let be a set of forces whose modulus are $F_1, F_i, \dots \dots \dots, F_n$ applied to points $a_1, a_i, \dots \dots \dots, a_n$

The resultant $\vec{R}$ written as:



$$\vec{R} = \sum_{i=1}^{n} \vec{F_i} = \sum_{i=1}^{n} F_i \vec{u}$$

and we have $$\vec{R} = R\vec{u}$$

$$\Rightarrow\ R = \sum_{i=1}^{n} F_i$$

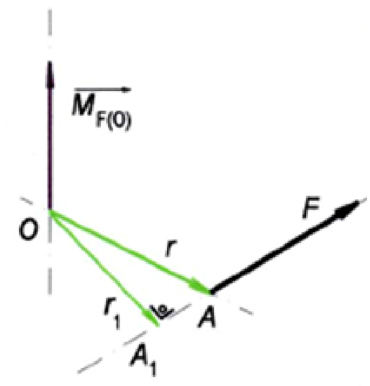
Point (C) is the bigning of the resultant (R) and is called thef center of parallel forces. can be calculated by:

$$\vec{r_c} = \frac{\sum_{i=1}^{n} \vec{F_i} \cdot \vec{r_i}}{\sum_{i=1}^{n} F_i}$$

$$\vec{r_c} = \begin{cases} x_c = \dfrac{\sum_{i=1}^{n} F_{yi} \cdot x_i}{\sum_{i=1}^{n} F_{yi}} \\ y_c = \dfrac{\sum_{i=1}^{n} F_{xi} \cdot y_i}{\sum_{i=1}^{n} F_{xi}} \end{cases}$$

III.6.Moment of a force relative to a point

The moment of a force $\vec{F}$ about a point O is equal to the vector product of the radius vector r = OA ; joining the point O to the origin A of the force, by the force $\vec{F}$ it self



The moment of a force $\vec{F}$ about a point O can be written by:

$$\vec{M}_{/0}(\overrightarrow{F}) = \vec{r} \wedge \vec{F}$$

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$$\vec{M}_{/0}(\overrightarrow{F}) = \vec{r} \wedge \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\vec{M}_{/0}(\overrightarrow{F}) = (y.F_z - F_y.z).\vec{\imath} + (F_x.z - x.F_z).\vec{\jmath} + (x.F_y - F_x.y).\vec{k}$$

The module of the moment of the force about a point (O) defined by:

$$M_{/0}(\overrightarrow{F}) = r.F.sin\theta$$

$$M_{/0}(\overrightarrow{F}) = F.r.sin\theta$$

$$M_{/0}(\overrightarrow{F}) = F.d$$

(d) is the perpendicular distance between the direction of the force ($\vec{F}$) and the point point (0).