Chapter I: Mathematical Reviews

Chapter I:
Mathematical Reviews

I.1.Vector Calculus (الحساب الشعاعي)

Before we dive into vectors, it's important to understand the distinction between scalars and vectors:

- **Scalars** are quantities that have magnitude (size) only, such as distance or speed.
- **Vectors** are quantities that have both magnitude and direction, such as displacement, velocity or force.

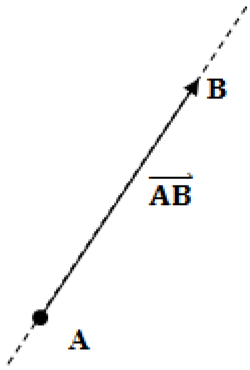
A **scalar quantity** is expressed by a numerical value followed by the corresponding unit.

Exemple: the length, mass, volume, temperature, time, work, voltage, density, resistance, etc….

A **vector quantity** that has **magnitude** as well as **direction** is called a **vector**.

A directed line segment AB, having an origin A and an end B, defined by:

- Its origin (point A).
- Its direction (its inclination relative to the axis).
- Its magnitude (length).



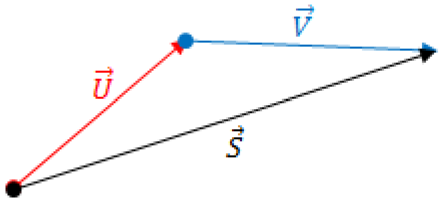
Vectors are represented graphically as arrows. The length of the arrow represents the vector's magnitude (modulus), and the direction of the arrow indicates its direction.

We can mention several types of vectors that we will discuss in the upcoming lessons, displacement, velocity, acceleration, force, weight and moment of the forces etc…..

We symbolize the vector by: $\overrightarrow{AB}$ or $\vec{V}$ and we symbolize the magnitude (modulus) by $\overrightarrow{\|AB\|}$ or $\overrightarrow{\|V\|}$ and by: AB or V.

I.2. Operations on vectors (عمليات على الاشعة)

I.2.1. Addition of vectors (جمع اشعة)

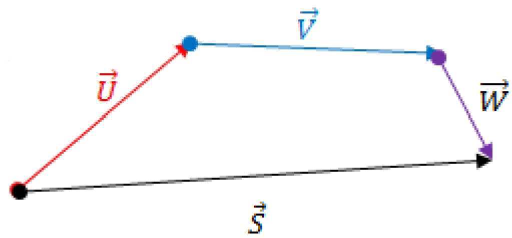


The sum of two vectors $\vec{U}$ and $\vec{V}$is another vector $\vec{S}$ defined by:

$\vec{S} = \vec{U} + \vec{V}$ So it is a commutative operation.

The Sum of multiple vectors is another vector defined by:

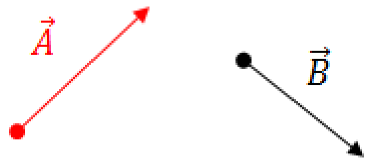
$$\vec{S} = \vec{U} + \vec{V} + \vec{W}$$



Thus, we can know that the vector $\vec{S}$ begins with the beginning of the first vectoe $\vec{U}$ and ends with the end of the last vector $\vec{W}$.

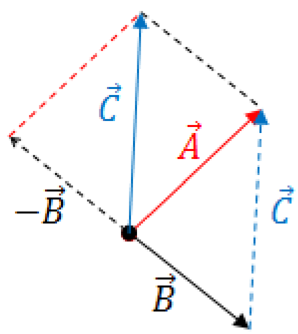
I.2.2. Subtraction of vectors (طرح اشعة)

Let, $\vec{A}$ and $\vec{B}$ two vectors



The subtraction between two vectors $\vec{A}$ and $\vec{B}$ in this order is another vector $\vec{C}$ that must be added to the second vectors $\vec{B}$ to get the first vectors $\vec{A}$and we write:

$$\vec{C} = \vec{A} - \vec{B}$$



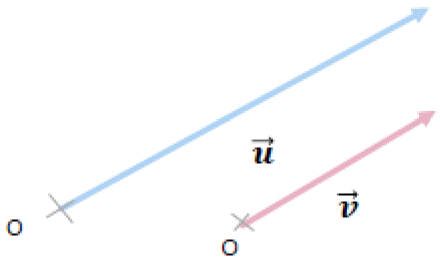
I.2.3. Scalar Multiplication (الضرب العددي)

Vectors can be multiplied by scalars to change their magnitude. Multiplying a vector by a positive scalar scales its magnitude, while multiplying by a negative scalar reverses its direction.

The Product of a vector, $\vec{v}$ by a scalar α is a vector noted $\alpha\vec{v}$.

Note that, $\alpha\vec{v}$ is also a vector, collinear to the vector $\vec{v}$.

$$\vec{u} = \alpha\vec{v}$$



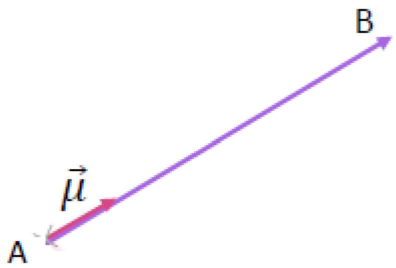
I.3.Unit vector (شعاع الوحدة)

The unit $\vec{\mu}$ vector of the vector $\overrightarrow{AB}$ is obtained by dividing this vector by its magnitude (modulus) AB.

$$\vec{\mu} = \frac{\overrightarrow{AB}}{AB}$$

Hence, the vector can be written by:

$$\overrightarrow{AB} = AB.\vec{\mu}$$



Unit vectors are vectors with a magnitude of 1 and are often used to specify direction. In three dimensions, the unit vectors along the x, y, and z axes are denoted as $\vec{i}$, $\vec{j}$, and $\vec{k}$, respectively.

I.4. Vector componentes (مركبات الشعاع)

Let A and B be two points in a Cartesian coordinate system

$A\ (x_a, y_a)$ and $B\ (x_b, y_b)$ So the vector components $\overrightarrow{AB}$:

$\overrightarrow{AB}$ can be written by:

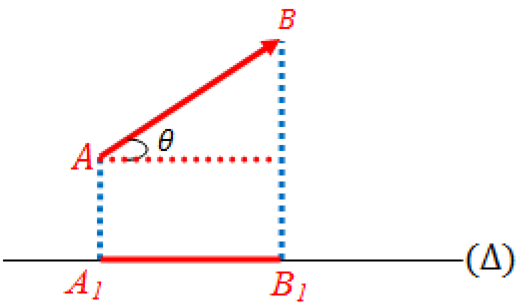
$$\overrightarrow{AB}\,(x_b - x_a; y_b - y_a) \text{ or } \overrightarrow{AB}\begin{pmatrix} x_b - x_a \\ y_b - y_a \end{pmatrix}$$

The module of the vector $\overrightarrow{AB}$ is written:

$$\|\overrightarrow{AB}\| = AB = \sqrt{(x_b - x_a)^2 - (y_b - y_a)^2}$$

I.4.1 perpendicular projecting of a vector into axis

The projection ***(A_1B_2)*** is difined by two perpendiculars from the bignning and the end of the vector $\overrightarrow{AB}$ on the axis (Δ).



And we say:

$Proj_{/(\Delta)}(\overrightarrow{AB}) = A_1B_2 = AB.\ Cos\theta$

Were (***θ***) is the angle between the vector $\overrightarrow{AB}$ and the axis (Δ).

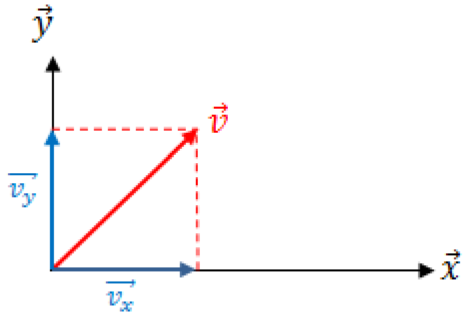
I.4.2 Decomposing the vectors

Any vector can be divided into two vectors in the plane or into three vectors in space.

In the plan

Let the vector $\vec{v}$ in the plan R(o, $\vec{x}$, $\vec{y}$)

To find the components of the vector $\vec{v}$, we must project the vector perpendicularly onto the two axes ($\vec{x}$, $\vec{y}$) as shown in the figure.



And we say:

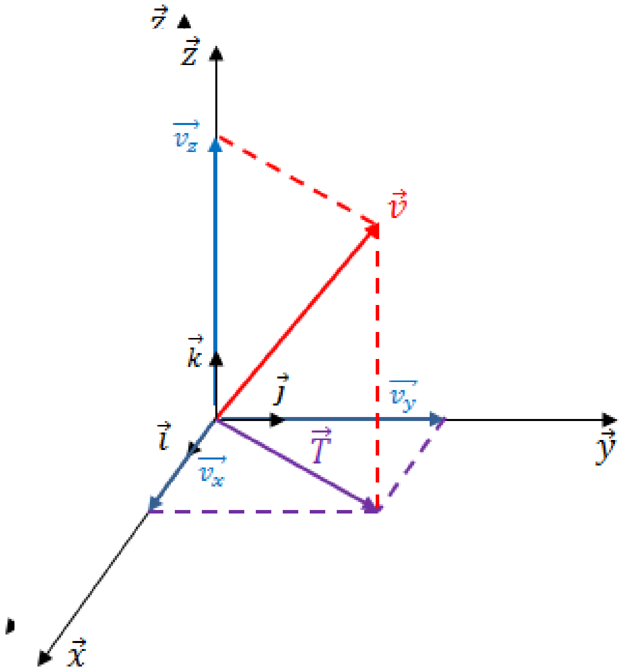
($\overrightarrow{v_x}$, $\overrightarrow{v_y}$) are the componentes of the vector $\vec{v}$ and we writte $\vec{v}\begin{pmatrix}\overrightarrow{v_x}\\ \overrightarrow{v_y}\end{pmatrix}$

With: $\vec{v} = \overrightarrow{v_x} + \overrightarrow{v_y}$

In the space

Let the vector $\vec{v}$ in the plan R(o, $\vec{x}$, $\vec{y}$, $\vec{z}$).

To find the components of the vector $\vec{v}$, we felow the felowing stups:



- we must project the vector

perpendicularly onto the plan $(\vec{x}, \vec{y})$ to find the vector $\vec{T}$ and into the axis $\vec{z}$ to find the vector $\overrightarrow{v_z}$, Then project the vector $\vec{T}$ onto the plane $(\vec{x}, \vec{y})$ to find the components $(\overrightarrow{v_x}, \overrightarrow{v_y})$ and thus

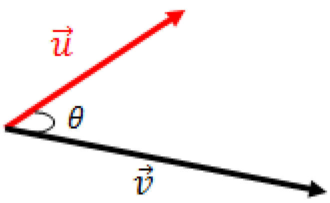
$(\overrightarrow{v_x}, \overrightarrow{v_y}, \overrightarrow{v_z})$ are the componentes of the vector $\vec{v}$ and we writte $\vec{v}\begin{pmatrix}\overrightarrow{v_x}\\ \overrightarrow{v_y}\\ \overrightarrow{v_z}\end{pmatrix}$

With: $\vec{v} = \overrightarrow{v_x} + \overrightarrow{v_y} + \overrightarrow{v_z}$

I.5. Scalar product (الجداء السلمي)

I.5.1. Geometrical interpretation of scalar product

Let $\vec{u}$ and $\vec{v}$ be two vectors forming a geometric angle θ, the real (scalar) number is called the scalar product and is denoted as $\vec{\boldsymbol{u}}.\vec{\boldsymbol{v}}$:



$$\vec{u}.\vec{v} = \|\vec{u}\|.\|\vec{v}\| . \cos\theta$$

It is the product of the module of $\overrightarrow{\|\vec{v}\|}$ by projecting $\vec{\boldsymbol{u}}$ on to the direction of $\vec{v}$: $\|\vec{u}\| . \cos\theta$.

a- Properties: Let $\vec{u}$ and $\vec{v}$ be two vectors.

- If the two vectors $\vec{u}$ and $\vec{v}$ are parallel, then:

$$\vec{u}.\vec{v} = \|\vec{u}\|.\|\vec{v}\|$$

- If the two vectors $\vec{u}$ and $\vec{v}$ are perpendicular, then:

$$\vec{u}.\vec{v} = 0$$

I.5.2. Analytical expression of scalar product

Let $\vec{u}$ and $\vec{v}$ be two vectors defined by: $\vec{u}\ (x_u, y_u, z_u)$ and $\vec{v}\ (x_v, y_v, z_v)$

So that:

$$\vec{u} = x_u.\vec{\imath} + y_u.\vec{\jmath} + z_u.\vec{k}$$

And

$$\vec{v} = x_v.\vec{\imath} + y_v.\vec{\jmath} + z_v.\vec{k}$$

The scalar product can be writte by:

$$\vec{u}.\vec{v} = x_u.x_v + y_u.y_v + z_u.z_v$$

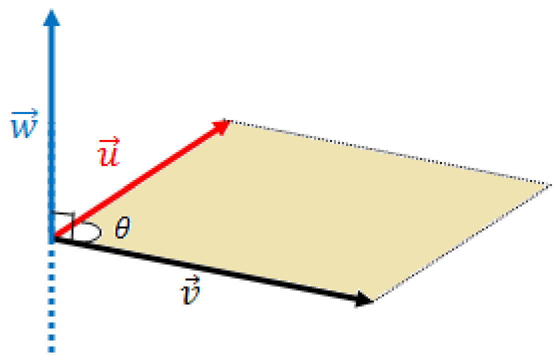
The angle (θ) between the two vectors $\vec{u}$ and $\vec{v}$ shown in the previous figure, can be calculated by the relation

$$\cos\theta = \frac{\vec{u}.\vec{v}}{\|\vec{u}\|.\|\vec{v}\|}$$

Hence, the condition of the perpendicularity of two vectors $\vec{u}$ and $\vec{v}$ is expressed by the following relationship:

$$x_u.x_v + y_u.y_v + z_u.z_v = 0$$

I.6. Vectorial product (الجداء الشعاعي)



The vectorial product of two vectors $\vec{u}$ and $\vec{v}$ noted $\vec{\boldsymbol{u}} \wedge \vec{\boldsymbol{v}}$.defined by:

- Its derection is perpendicular (orthogonal) to the plan $(\vec{u}, \vec{v})$

$$\vec{\boldsymbol{u}} \wedge \vec{\boldsymbol{v}} = \|\vec{u}\|.\|\vec{v}\|.sin\,\theta$$

- Its module is defined by:

$$\|u^{\to} \wedge v^{\to}\| = \|\vec{u}\|.\|\vec{v}\|.sin\,\theta$$

Module of vectorial product is the area of a parallelogram formed by the plane $(\vec{u}, \vec{v})$.

- Its sense forms the direct triad

The cross product is another mathematical operation involving vectors, which results in a vector as its output, unlike the dot product which results in a scalar.

I.5.2. Analytical expression of vectorial product

Let $\vec{u}$ and $\vec{v}$ be two vectors defined by: $\vec{u}\,(x_u, y_u, z_u)$ and $\vec{v}\,(x_v, y_v, z_v)$

So that:

$\vec{u} = x_u.\vec{\imath} + y_u.\vec{\jmath} + z_u.\vec{k}$ and $\vec{v} = x_v.\vec{\imath} + y_v.\vec{\jmath} + z_v.\vec{k}$

The vectorial product can be writte by:

$$\vec{u} \wedge \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_u & y_u & z_u \\ x_v & y_v & z_v \end{vmatrix} = \begin{vmatrix} y_u & z_u \\ y_v & z_v \end{vmatrix} . \vec{i} - \begin{vmatrix} x_u & z_u \\ x_v & z_v \end{vmatrix} . \vec{j} + \begin{vmatrix} x_u & y_u \\ x_v & y_v \end{vmatrix} . \vec{k}$$

$$\vec{u} \wedge \vec{v} = (y_u . z_v - y_v . z_u) . \vec{i} + (x_v . z_u - x_u . z_v) . \vec{j} + (x_u . y_v - x_v . y_u) . \vec{k}$$

I.6 Derivation of vector

Let a vector $\vec{w}$ be applied of an orthonormal frame R (O,$\vec{\imath}$, $\vec{\jmath}$, $\vec{k}$). The components of this vector are defined as a function of the time variable (t):

$$\vec{w}(t) = x(t)\vec{\imath} + y(t)\vec{\jmath} + z(t)\vec{k}$$

Derivation of this vector with respect to time:

$$\frac{\partial \vec{w}(t)}{dt} = \vec{\dot{w}}(t) = \frac{\partial x(t)}{dt}\vec{\imath} + \frac{\partial y(t)}{dt}\vec{\jmath} + \frac{\partial z(t)}{dt}\vec{k}$$

$$\frac{\partial \vec{w}(t)}{dt} = \vec{\dot{w}}(t) = \dot{x}(t)\vec{\imath} + \dot{y}(t)\vec{\jmath} + \dot{z}(t)\vec{k}$$

I.6.1 Derivative properties

- $\frac{\partial(\vec{u}+\vec{v})}{dt} = \frac{\partial \vec{u}}{dt} + \frac{\partial \vec{v}}{dt}$
- $\frac{\partial(f.\vec{v})}{dt}$; where ($f$) is a constant, $\frac{\partial(f.\vec{v})}{dt} = f . \frac{\partial \vec{v}}{dt}$
- $\frac{\partial(\vec{u}.\vec{v})}{dt} = \frac{\partial \vec{u}}{dt} . \vec{v} + \vec{u} . \frac{\partial \vec{v}}{dt}$
- $\frac{\partial(\vec{u}\wedge\vec{v})}{dt} = \frac{\partial \vec{u}}{dt} \wedge \vec{v} + \vec{u} \wedge \frac{\partial \vec{v}}{dt}$