Université Mohamed KHIDDER de Biskra

Département d’Informatique

Module : Programmation linéaire

Le 18/01/2025

**Corrigé de l’examen**

**Exercice N° I :**

(1.5p)

 X*≥ 0* YA *≥ C*

P AX *≤* B D Min(W) = YB

Max (CX) Y*≥ 0*

x\* solutions de P ===> Ax\* *≤* B ===> Y\*AX\* *≤* Y\*B (1) avec Y\* solutions de D

Y\*A ≥ C ===> Y\*AX\* ≥ CX\* (2)

1. Et (2) donne CX\* ≤ Y\*B CQFD
2. En plus de **1.** CX\* > Y\*B alors CX\* ≤ Y\*B on suppose que x\*\* > x\* donc CX\*\* > CX\* aussi CX\* ≤ Y\*B résultat de **1.** le Max pour P est atteint X\* et le Min de D est atteint pour Y\* aussi si un problème P admet une solution optimale alors son dual l’admet aussi et inversement. (1.5)
3. Exemple après calcul en appliquant le simple et le dual simplexe on obtient Z = 12+ 10 pour (X = 3 et Y = 2 ) pour le dual W= 8+14+0=22 et ( Y1= 1 et Y2 = 2 Y3=0 ). (2p)

**Exercice N° II :**

Xi ≥ 0: nombre de jet effectué par chaque machine i. (1p)

X5 = 5 X4

(2p)

X1 = 3 X2

X1 ≥ 3X3+20

3X2 ≥ 5 X4+50

Max(Z) = 7X1 + 8\*X2 +9\*X3 +14\* X4 +15\*X5 (1p)

**Exercice N°III**

1. Montrer que X et Z dépendent de γ ? base optimale pour γ0 donné (γ0 = 0 ) AI XI+ AJ XJ = Ƃ + γ b’ ( J/ I ∩ J = Ø XI = (AI ) -1 Ƃ + (AI ) -1 γ b’ - (AI ) -1 AJ XJ Z =CI XI+ CJ XJ I base => XJ = 0 => XI = (AI ) -1 Ƃ + (AI ) -1 γ b’ XI = ẍI + γ XI’ => X = ẍ + γ X’ d’où X dépend de γ Z =CI (ẍI + γ XI’ )+ CJ XJ = CI Ẍi+ CI γXI’ = Zopt+ I γZ’ d’où Z dépend de γ.

(1.5p)

Donner la valeur de Ẍ et Zopt ? XI = (AI ) -1 Ƃ + (AI ) -1 γ b’ - (AI ) -1 AJ XJ • Ẍ =(AI ) -1 Ƃ et X’ = (AI ) -1 b’ • Zopt = CI (ẍI + γ XI’ )+ CJ XJ = Ci \*(AI ) -1 Ƃ + γ\* Cj\* (AI ) -1 b’ Montrer que γ = Min ( ẍ /- x’ ) ?

 La détermination des seuils limite de γ est réalisé par : XI = ẍI + γ XI’ ≥ 0 quel que soit I Comme ẍI ≥ 0 il reste la condition sur X’ > 0 ou x’ < 0

1er cas γ > 0 (1.5p)

* X’ > 0 : L a valeur limite de γ n’existe pas alors Arrêt.
* X’ < 0 : == > γ ≤ ( ẍI /- x’ ) ==> γ = Min ( ẍI /- x’ ) <==> γ = Min (Ƃ/- b’ ) tant que γ appartient à (γ, γ1) la solution est optimale XI = ẍs + γ Xs ’ ==> Xs = ẍs – (ẍs/- Xs ’), Xs = 0 Si γ > γ1 ==> Xs devient négatif donc il faut un changement de base et s l’indice de la ligne pivot I1 = I0 - (s) + (r) ; C0J = CJ- ( A(s,j) / A(s,r))Cr ≤ 0 il faut que cette condition soit toujours vérifier afin d’avoir une solution optimale réalisable, S quitte la base donc Cs ≤ 0 C0s = Cs- ( A(s,s) / A(s,r))CrA(s,s) = 1 , Cr < 0, Cs = 0 donc C0s =- Cr / A(s,r) ≤ 0 ==> il faut choisir r de manière que A(s,r) < 0 Si A(s,j) ≥ 0 quel que soit j appartenant à J ceci veut dire qu’il n’existe pas de solution optimale alors ARRET. S’il existe des (A(s,j) < 0 ) alors (Cj / A(s,j ) - Cr / A(s,r) ) ==> Cr / A(s,r) ≤ Cj / A(s,j ) donc r est l’indice tel que r = MIN(Cj / A(s,j ) avec A(s,j) < 0 )
1. (3p)

Changement de variable

**Exercice N° IV :**

1) The dual simplex method

**Problem is**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Min W(x) | = |  | 500 | *x*1 | + | 800 | *x*2 |

 |
|  |
|

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0.1 | *x*1 | + | 0.35 | *x*2 | ≥ | 100 |
|  | 0.2 | *x*1 | + | 0.2 | *x*2 | ≥ | 140 |
|  | 0.3 | *x*1 | + | 0.15 | *x*2 | ≥ | 150 |
|  | 0.4 | *x*1 | + | 0.3 | *x*2 | ≥ | 240 |

 |
|  and *x*1, *x2*≥0; |

In order to apply the dual simplex method, convert all ≥ constraint to ≤ constraint by multiply -1.

**Problem is**

0.25p

0.25p

 min

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | -max (-500 | *x*1 | - | 800 | *x*2) |

 |
| subject to |
|

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| - | 0.1 | *x*1 | - | 0.35 | *x*2 | ≤ | -100 |
| - | 0.2 | *x*1 | - | 0.2 | *x*2 | ≤ | -140 |
| - | 0.3 | *x*1 | - | 0.15 | *x*2 | ≤ | -150 |
| - | 0.4 | *x*1 | - | 0.3 | *x*2 | ≤ | -240 |

 |
| and *x*1,*x*2≥0;**After introducing slack variables**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| -Max  | ( | - | 500 | *x*1 | - | 800 | *x*2 | + | 0 | *e*1 | + | 0 | *e*2 | + | 0 | *e*3 | + | 0 | *e*4 ) |

 |
| subject to |
|

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| - | 0.1 | *x*1 | - | 0.35 | *x*2 | + |  | e1 |  |  |  |  |  |  |  |  |  | = | -100 |
| - | 0.2 | *x*1 | - | 0.2 | *x*2 |  |  |  | + |  | e2 |  |  |  |  |  |  | = | -140 |
| - | 0.3 | *x*1 | - | 0.15 | *x*2 |  |  |  |  |  |  | + |  | e3 |  |  |  | = | -150 |
| - | 0.4 | *x*1 | - | 0.3 | *x*2 |  |  |  |  |  |  |  |  |  | + |  | e4 | = | -240 |

 |
| and *x*1, *x*2, e1, e2, *e*3, e4≥0 |

 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | e1 | e2 | e3 | e4 | b |
| e1 | -0.1 | -0.35 | 1 | 0 | 0 | 0 | -100 |
| e2 | -0.2 | -0.2 | 0 | 1 | 0 | 0 | -140 |
| e3 | -0.3 | -0.15 | 0 | 0 | 1 | 0 | -150 |
| e4 | -0.4 | -0.3 | 0 | 0 | 0 | 1 | -240 |
|  | -500min | -800 | 0 | 0 | 0 | 0 | 0 |



0.5p

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | e1 | e2 | e3 | e4 | b |
| e1 | 0 | -0.27 | 1 | 0 | 0 | -0.25 | -40 |
| e2 | 0 | -0.05 | 0 | 1 | 0 | -0.5 | -20 |
| e3 | 0 | 0.075 | 0 | 0 | 1 | -0.75 | 30 |
| x1 | 1 | 0.75 | 0 | 0 | 0 | -2.5 | 600 |
|  | 0 | -425 | 0 | 0 | 0 | -1250 | 300000 |

0.5p

0.5p

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | e1 | e2 | e3 | e4 | b |
| x2 | 0 | 1 | -3.64 | 0 | 0 | 0.90 | 145.45 |
| e2 | 0 | 0 | -0.18 | 1 | 0 | -0.45 | -12.73 |
| e3 | 0 | 0 | 0.27 | 0 | 1 | -0.82 | 19.09 |
| x1 | 1 | 0 | 02.73 | 0 | 0 | -3.18 | 490.90 |
|  | 0 | 0 | -1545.45 | 0 | 0 | -863.64 | 361818.18 |

0.5p

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | e1 | e2 | e3 | e4 | b |
| e1 | 0 | 1 | -4 | 2 | 0 | 0 | 120 |
| e2 | 0 | 0 | 0.4 | -2.2 | 0 | 0 | 28 |
| e3 | 0 | 0 | 0.6 | -1.8 | 1 | 0 | 42 |
| e4 | 1 | 0 | 4 | -7 | 0 | 1 | 580 |
|  | 0 | 0 | -1200 | -1900 | 0 | 0 | 386000 |

Since all b*i*>=0, then the optimal solution is arrived with value of variables:

x1=120, x2=580, w(x)=386000. (0.25p)

2) The transposed method:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Max *Z* | = |  | 100 | *y1* | + | 140 | *y2* | + | 150 | *y3* | + | 240 | *y4* |

 |
|  |
|

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  0.10 | *y1* | + | 0.2 | *y2* | + | 0.30  | *y3* | + | 0.4 | *y4* | ≤ | 500 |
|  |  0.35 | *y1* | + | 0.2 | *y2* | + | 0.15 | *y3* | + | 0.3 | *y4* | ≤ | 800 |

 |
| and *y1*,*y2*,*y3*,*y4*≥0; |

**After introducing slack variables**

0.25p

0.5p

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Max *Z* | = |  | 100 | *y1* | + | 140 | *y2* | + | 150 | *y3* | + | 240 | *y4* | + | 0 | *e1* | + | 0 | *e2* |

 |
| 0.25p |
|

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0.1 | *y1* | + | 0.2 | *y2* | + | 0.3 | *y3* | + | 0.4 | *y4* | + |  | *e1* |  |  |  | = | 500 |
|  | 0.35 | *y1* | + | 0.2 | *y2* | + | 0.15 | *y3* | + | 0.3 | *y4* |  |  |  | + |  | *e2* | = | 800 |

 |
| and *y1*, *y2, y3, y4, e1, e2*≥0 |
| ***B*** | ***y1*** | ***y2*** | ***y3*** | ***y4*** | ***e1*** | ***e2*** | ***XB*** | **Min** |
| ***e1*** | 0.1 | 0.2 | 0.3 | **(0.4)** | 1 | 0 | 500 | 5000.4=1250**→** |
| *e2* | 0.35 | 0.2 | 0.15 | 0.3 | 0 | 1 | 800 | 8000.3=2666.6667 |
|  | 100 | 140 | 150 | 240↑ | 0 | 0 | *Z=0* |  |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ***B*** | ***y1*** | ***y2*** | ***y3*** | ***y4*** | ***e1*** | ***e2*** | ***XB*** |  |
| *y4* | 0.25 | 0.5 | 0.75 | 1 | 2.5 | 0 | 240 | 12500.25=5000 |
| ***e2*** | **(0.275)** | 0.05 | -0.075 | 0 | -0.75 | 1 | 0 | 4250.275=1545.4545**→** |
|  | 40↑ | 20 | -30 | 0 | -600 | 0 | ***Z*=300000** |  |

0.5p

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ***B*** | ***y1*** | ***y2*** | ***y3*** | ***y4*** | ***e1*** | ***e2*** | ***XB*** |  |
| ***y4*** | 0 | **(0.45)** | 0.82 | 1 | 3.18 | -0.90 | 863.64 | 863.63\*640.45=1900**→** |
| *y1* | 1 | 0.18 | -0.27 | 0 | -2.73 | 3.64 | 1545.45 | 1545.45450.18=8500 |
|  | 0 | 12.73↑ | -19.09 | 0 | -490.91 | -145.45 | ***Z*=361818.18** |  |

0.5p

0.5p

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ***B*** | ***y1*** | ***y2*** | ***y3*** | ***y4*** | ***e1*** | ***e2*** | XB |
| *y2* | 0 | 1 | 1.8 | 2.2 | 7 | -2 | 1900 |
| *y1* | 1 | 0 | -0.6 | -0.4 | -4 | 4 | 1200 |
|  | 0 | 0 | -42 | -28 | -580 | -120 | ***Z*=386000** |

Since all *Cj*<0, then the optimal solution is arrived with value of variables: *y1*=1200, *y2*=1900, *y3*=0, *y4*=0 with Max *Z*=386000. (0.25p)