**UNIVERSITE Mohamed Khider de Biskra**

**Département D’Informatique**

**Module : Programmation linéaire**

**Solution TD 05**

**Exemple (01) :**

Max Z(X)= x1 + 5x2 -M A1-M A2  x1 + 3x2 + e1 = 10 2x1 -3x2 - e2 +A1=5 x1 +x2 +A2 =4 où xi,ei, Ai≥ 0 pour i =1; 2

1. Max Z(X)= x1 +5x2 x1 + 3x2 ≤ 10 **P’** 2x1- 3x2 ≥ 5 x1 + x2  = 4 où xi ≥ 0 pour i =1; 2

**P**

**Cj’=Cj+**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **x1** | **x2** | **e1** | **e2** | **A1** | **A2** | **B** |
| **e1** | 1 | **3** | **1** | **0** | **0** | **0** | **10** |
| **A1** | **2** | **-3** | **0** | **-1** | **1** | **0** | **5** |
| **A2** | **1** | **1** | **0** | **0** | **0** | **1** | **4** |
| **Cj** | **1** | **5** | **0** | **0** | **0** | **0** | **0** |
| **Cj’**  Max | **1+3M** | **5-2M** | **0** | **-M** | **M** | **M** | **9M** |

min

10

5/2

4

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **x1** | **x2** | **e1** | **e2** | **.** | **A2** | **B** |
| **e1** | 0 | **9/2** | **1** | **1/2** |  | **0** | **15/2** |
| **x1** | **1** | **-3/2** | **0** | **-1/2** |  | **0** | **5/2** |
| **A2** | **0** | **5/2** | **0** | **1/2** |  | **1** | **3/2** |
| **Cj** | **0** | **13/2** | **0** | **1/2** |  | **0** | **-5/2** |
| **Cj’** | **0** | **13/2+5/2M** | **0** | **½+½M** |  | **M** | **-5/2+3/2M** |

5/3

--

3/5

Max

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **x1** | **x2** | **e1** | **e2** | **.** | **.** | **B** |
| **e1** | 0 | **0** | **1** | **-2/5** |  |  | **24/5** |
| **x1** | **1** | **0** | **0** | **-1/5** |  |  | **17/5** |
| **x2** | **0** | **1** | **0** | **1/5** |  |  | **3/5** |
| **Cj** | **0** | **0** | **0** | **-4/5** |  |  | **-32/5** |
| **Cj’** | **0** | **0** | **0** | **-4/5** |  |  | **32/5** |

Les **Cj’** sont indépendante de **M**, on dit que une solution réalisable est obtenue ( condition nécessaire)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **x1** | **x2** | **e1** | **e2** | **B** |
| **e1** | 0 | **0** | **1** | **-2/5** | **24/5** |
| **x1** | **1** | **0** | **0** | **-1/5** | **17/5** |
| **x2** | **0** | **1** | **0** | **1/5** | **3/5** |
| **Cj** | **0** | **0** | **0** | **-4/5** | **-32/5** |

Toute les **Cj ≤0**, donc le critère d’arrêt est satisfait

La solution est : X1 = 17/5, X2= 3/5, Z = 32/5

Exemple (02) :

Min Z(Y) = 14 y1 + 10y2 + 3y3+MA1 +MA2 y1 + 2y2 + y3-e1+A1 = 2 2 y1 - y2 - y3 -e2+A2 = 1 Où yi ≥ 0 pour i =1;3. A1, A2≥0 ; e1,e2 ≥0

Min Z(Y) = 14 y1 + 10y2 + 3y3 y1 + 2y2 + y3 ≥ 2 2 y1 - y2 - y3 ≥ 1 Où yi ≥ 0 pour i =1;3.

**Cj’=Cj-**

Min

2

1/2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **y1** | **y2** | **y3** | **e1** | **e2** | **A1** | **A2** | **B** |
| **A1** | **1** | **2** | **1** | **-1** | **0** | **1** | **0** | **2** |
| **A2** | **2** | **-1** | **-1** | **0** | **-1** | **0** | **1** | **1** |
| **Cj** | **14** | **10** | **3** | **0** | **0** | **0** | **0** | **0** |
| **Cj’** | **14-3M** | **10-M** | **3** | **M** | **M** | **-M** | **-M** | **-3M** |

Min

3/5

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Min

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **y1** | **y2** | **y3** | **e1** | **e2** | **A1** |  | **B** |
| **A1** | **0** | **5/2** | **3/2** | **-1** | **1/2** | **1** |  | **3/2** |
| **y1** | **1** | **-1/2** | **-1/2** | **0** | **-1/2** | **0** |  | **1/2** |
| **Cj** | **0** | **17** | **10** | **0** | **7** | **0** |  | **-7** |
| **Cj’** | **0** | **17-5/2M** | **10-3/2M** | **M** | **7-1/2M** | **-M** |  | **-7-3/2M** |

Min

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **y1** | **y2** | **y3** | **e1** | **e2** |  |  | **B** |
| **y2** | **0** | **1** | **3/5** | **-2/5** | **1/5** |  |  | **3/5** |
| **y1** | **1** | **0** | **-1/5** | **-1/5** | **-2/5** |  |  | **4/5** |
| **Cj** | **0** | **0** | **-1/5** | **34/5** | **18/5** |  |  | **-86/5** |
| **Cj’** | **0** | **0** | **-1/5** | **34/5** | **18/5** |  |  | **-86/5** |

Les **Cj’** sont indépendante de **M**, on dit que une solution réalisable est obtenue ( condition nécessaire)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **y1** | **y2** | **y3** | **e1** | **e2** | **B** |
| **y2** | **0** | **1** | **3/5** | **-2/5** | **1/5** | **3/5** |
| **y1** | **1** | **0** | **-1/5** | **-1/5** | **-2/5** | **4/5** |
| **Cj** | **0** | **0** | **-1/5** | **34/5** | **18/5** | **-86/5** |

Min

Min

1

--

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **y1** | **y2** | **y3** | **e1** | **e2** | **B** |
| **y3** | **0** | **5/3** | **1** | **-2/3** | **1/3** | **1** |
| **y1** | **1** | **1/3** | **0** | **-1/3** | **-1/3** | **1** |
| **Cj** | **0** | **1/3** | **0** | **20/3** | **11/3** | **-17** |

Toute **les Cj ≥ 0 (problème de minimisation),** donc le critère d’arrêt est satisfait.

La solution est : y1 = 1, y2= 0, y3=1 Z(Y) = 17.