

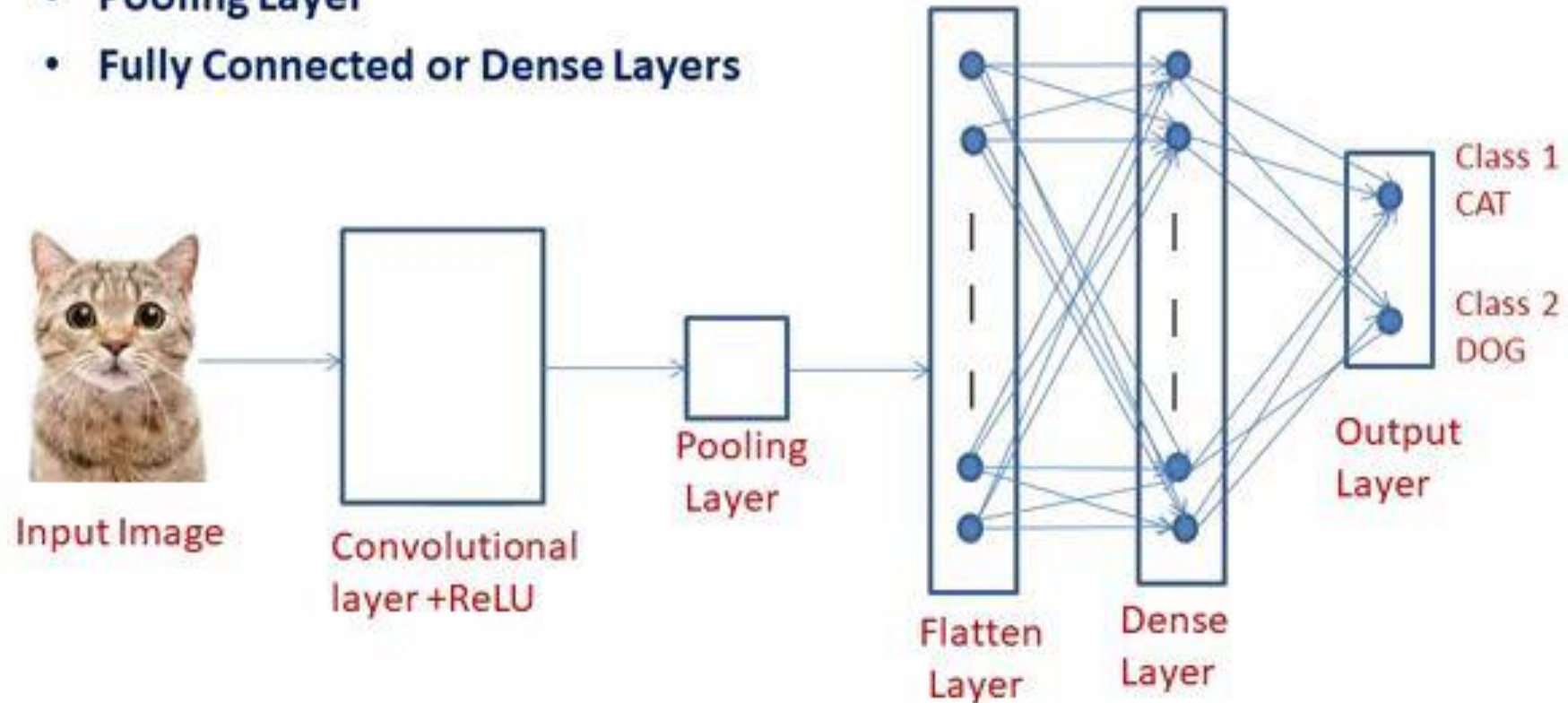
CONVOLUTIONAL NEURAL NETWORK

CONVOLUTIONAL NEURAL NETWORK

- Convolutional Neural Networks (CNNs) are a category of Neural Network that have proven very effective in areas such as image recognition and classification.
- CNNs have been successful in identifying faces, objects and traffic signs apart from powering vision in robots and self driving cars.
- CNNs are also used in Smart Grid applications as well.

CNN ARCHITECTURE

- CNN has three types of layers to build architectures apart from input layer:
 - Convolutional Layer (with ReLU activation)
 - Pooling Layer
 - Fully Connected or Dense Layers



CONVOLUTIONAL LAYER

- Convolutional Layer:** The primary purpose of Convolution is to extract features from the input image. Convolution preserves the spatial relationship between input by learning input features.

$$\text{Convolution layer output } y = \sum_i^n x * f$$

1	1	1	0	0
0	1	1	1	0
0	0	-1	1	1
0	0	-1	1	0
0	1	-1	0	0

Input Image(x)
(nxn)

*

1	0	1
0	1	0
1	0	1

Filter
(fxf)

=

2	3	2
0	2	1
-2	1	0

Output Image (y)
(n-f+1) x (n-f+1)

CONVOLUTIONAL LAYER OPERATION

1	1	1	0	0
0	1	1	1	0
0	0	-1	1	1
0	0	-1	1	0
0	1	-1	0	0

Input Image(x)
(5x5)

*

1	0	1
0	1	0
1	0	1

Filter
(3x3)

2	3	2
0	2	1
-2	1	0

Output Image (y)
(3x3)

1	1	1	0	0
x1	x0	x1		
0	1	1	1	0
x0	x1	x0		
0	0	-1	1	1
x1	x0	x1		
0	0	-1	1	0
0	1	-1	0	0

$$\begin{aligned}
 y_{11} &= (1x1) + (1x0) \\
 &+ (1x1) + (0x0) \\
 &+ (1x1) + (1x0) \\
 &+ (0x1) + (0x0) \\
 &+ (-1x1) = 2
 \end{aligned}$$

1	1	1	0	0
	x1	x0	x1	
0	1	1	1	0
	x0	x1	x0	
0	0	-1	1	1
	x1	x0	x1	
0	0	-1	1	0
0	1	-1	0	0

$$\begin{aligned}
 y_{12} &= (1x0) + (1x1) \\
 &+ (0x1) + (1x0) \\
 &+ (1x1) + (1x0) \\
 &+ (0x1) + (-1x0) \\
 &+ (1x1) = 3
 \end{aligned}$$

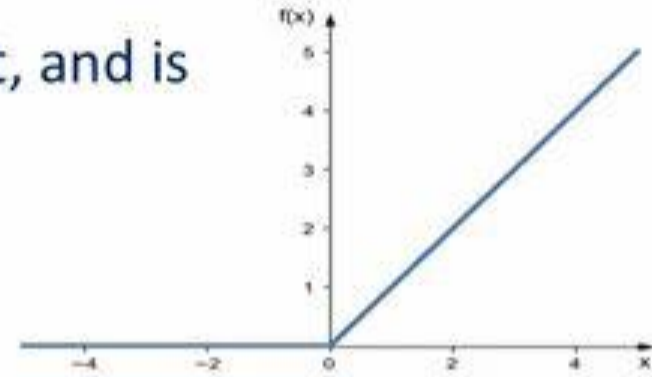
1	1	1	0	0
0	1	1	1	0
x1	x0	x1		
0	0	-1	1	1
x0	x1	x0		
0	0	-1	1	0
x1	x0	x1		
0	1	-1	0	0

$$\begin{aligned}
 y_{21} &= (0x1) + (1x0) \\
 &+ (1x1) + (0x0) \\
 &+ (0x1) + (-1x0) \\
 &+ (0x0) + (0x0) \\
 &+ (-1x1) = 0
 \end{aligned}$$

ReLU Layer

ReLU stands for rectified linear unit, and is a type of **activation function**.

$$f(x) = \max(0, x)$$



2	3	2
0	2	1
-2	1	0

Input of ReLU



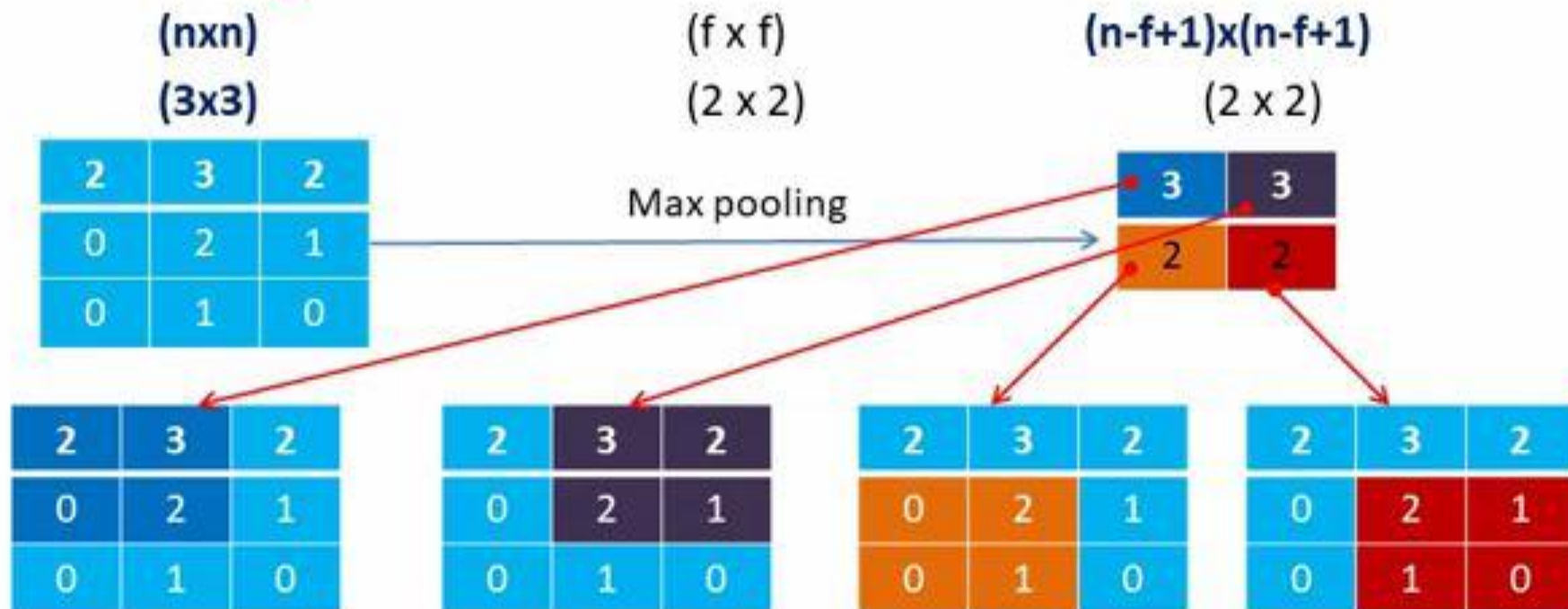
2	3	2
0	2	1
0	1	0

Output of ReLU

Pooling layers

- Pooling layers would reduce the number of parameters when the inputs are too large.
- Pooling also called down sampling which reduces the dimensionality of each map but retains important information.
- There are three types of pooling namely, Max Pooling, Average Pooling, Sum Pooling.

Max Pooling:

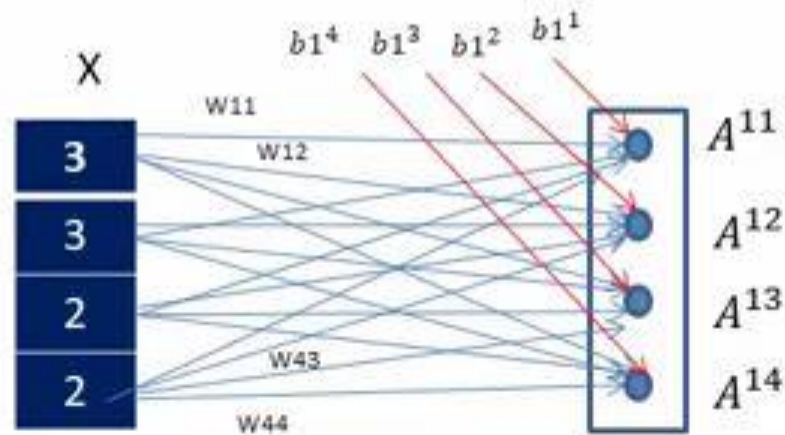


Flatten layer

- The feature matrix will be converted as vector (x_1, x_2, x_3, \dots) .

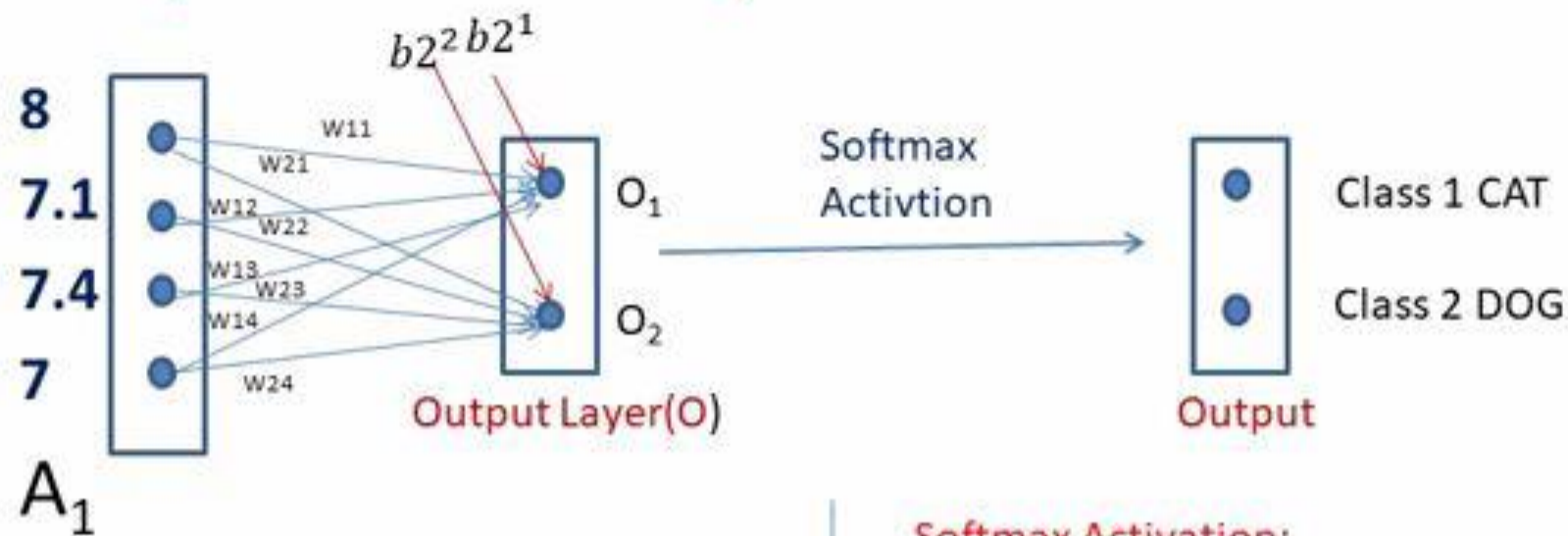


Fully Connected layer with ReLU



$$A_1 = W_1 X + b_1$$
$$\begin{bmatrix} A_{11} \\ A_{12} \\ A_{13} \\ A_{14} \end{bmatrix} = \begin{bmatrix} 1.0 & 1.0 & 0.2 & 0.8 \\ 1.0 & 0.5 & 0.5 & 0.8 \\ 0.8 & 1.0 & 0.2 & 0.8 \\ 0.5 & 0.5 & 1.0 & 1.0 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 8 \\ 7.1 \\ 7.4 \\ 7 \end{bmatrix}$$
$$\text{ReLU} \begin{bmatrix} 8 \\ 7.1 \\ 7.4 \\ 7 \end{bmatrix} = \begin{bmatrix} 8 \\ 7.1 \\ 7.4 \\ 7 \end{bmatrix}$$

Fully Connected layer & Softmax Activation



$$O = W_2 A_1 + b_2$$

$$\begin{bmatrix} O_1 \\ O_2 \end{bmatrix} = \begin{bmatrix} 0.2 & 0.1 & 0.5 & 0.5 \\ 0.1 & 0.5 & 0.1 & 0.2 \end{bmatrix} \begin{bmatrix} 8 \\ 7.1 \\ 7.4 \\ 7 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 9.61 \\ 6.49 \end{bmatrix}$$

Softmax Activation:

$$\sigma(O_i) = \frac{e^{O_i}}{\sum e^{O_i}}$$

$$\sigma(O_1) = \frac{e^{O_1}}{e^{O_1} + e^{O_2}} = \frac{14913.17}{14913.17 + 658.52} = 0.95$$

$$\sigma(O_2) = \frac{e^{O_2}}{e^{O_1} + e^{O_2}} = \frac{658.52}{14913.17 + 658.52} = 0.04$$

STACKED CNN ARCHITECTURE

