University of Biskra
Mathematics Department
Module: Analysis 1

First year license
2023/2024

Catch-up Exam

Exercise 1 *(/7pts)*

Lets consider the following polynomial $P(z) = z^3 - 2iz^2 + (i-2)z + i + 1$, with $Z \in \mathbb{C}$.

1. Demonstrate that the equation $P(z) = 0$ admits an imaginary solution z_1.
2. Determine a polynomial $Q(z)$ such that $P(z) = (z - z_1)Q(z)$.
3. Solve the equation $P(z) = 0$ in $\mathbb{C}$.
4. Demonstrate that the points of the complex plane corresponding to the solutions of the equation $P(z) = 0$ are not aligned.

Exercise 2 *(/6pts)*
Important note: in this exercise each question is independent of the others.

Let's consider a recursive sequence defined on $\mathbb{N}^*$ by its first term u_1 and the expression

$$u_{n+1} = au_n + \frac{\alpha n + \beta}{n}, \text{ with } \alpha > 0 \text{ and } \beta > 0.$$

1. Suppose that u_n converge then gives the limit of u_n against α and β.
2. Suppose that the limit of u_n exist and equal to l then give the value of α against l.
3. If $u_1 = 1$, $u_2 = 6$ and $u_3 = 31/2$ then find the exact expression of u_n (find α and β).
4. If $u_3 = a$ then find the expression of the first term of the sequence u_n.
5. Check that u_n converge only if $0 < \alpha < 1$.

Exercise 3 *(/7pts)*

Consider the function f defined by:

$$f(x) = \left(\frac{2\sqrt{1 - \cos(x)}}{\sin(2x)} \right)^m, \quad m \in \mathbb{N}^*.$$

1. Determine the domain of the function f.
2. Discuss the parity (even or odd) of f according to the values of the parameter m.
3. Verify that f is a 2π-periodic function, then discuss the limit of f at all bounds of its domain, according to the values of the parameter m.
4. Check, according to m if f have a removable discontinuity.