

Exercise series-course

Exercise 1 The plan is related to an orthonormal reference frame. Let f be the real variable function defined by

$$f(x) = \frac{x^2 - x - 2}{x^2 - x + 1}.$$

Let $\mathcal{C}_f$ its graphic representation in an orthonormal reference frame.

1. What is the domain D_f of f ?
2. What is the differentiability set of f ?
3. Calculate the derivative f' of the function f .
4. Determine the sign of f' , then deduce the variations direction of f .
5. Determine the limits of f at the bounds of D_f .
6. Determine possible asymptotes and their relative position to $\mathcal{C}_f$.
7. Determine the equation of the tangent T to $\mathcal{C}_f$ at its point of abscissa 2.
8. Study the relative position of $\mathcal{C}_f$ and T .
9. Construct, on the same drawing, the curve $\mathcal{C}_f$, the tangent T and the asymptotes.

Exercise 2 The plan is related to an orthonormal reference frame. Let f be the real function defined by

$$f(x) = \frac{x^3 - 4x^2 + 8x - 4}{(x - 1)^2}$$

. Let $\mathcal{C}_f$ its graphic representation in an orthonormal reference frame.

1. What is the domain D_f of f ?
2. Determine three real numbers a , b and c such that for all $x \in D_f$:

$$f(x) = x + a + \frac{b}{x - 1} + \frac{c}{(x - 1)^2}.$$

3. Specify the position of $\mathcal{C}_f$ to the line D with the equation $y = x - 2$.
4. Determine the differentiability set of f . Calculate the derivative f' of the function f .
5. Determine the sign of f' , then deduce the variations direction of f .
6. Determine the limits of f across D_f .
7. Determine possible asymptotes and their relative position to $\mathcal{C}_f$.
8. Determine the equation of the tangent T to $\mathcal{C}_f$ at its point of abscis $x = 2$.
9. Construct, on the same frame, the curve $\mathcal{C}_f$, the line D as well as the line with equation $x = 1$ and the tangent T .

Exercise 3

1. Find the first 3 terms of the Taylor series for the function $\sin(\pi x)$ centered at $a = 0.5$. Then, use your answer to find an approximate value to $\sin(\frac{\pi}{2} + \frac{\pi}{10})$
2. Find the first 4 terms in the Taylor series for $(x - 1)e^x$ near $x = 1$.
3. Find the Maclaurin series for $x \sin x$.
4. Find the Maclaurin series for $\ln\left(\frac{1+x}{1-x}\right)$.
5. (a) Find the Maclaurin series for $(1 + x)^m$ where m is not necessarily an integer and hence show that the formula for the binomial series works for non-integral exponents as well. (b) Use your answer to find the expansion of $\frac{1}{\sqrt{1-x^2}}$ up to the term in x^6 .
6. Find the first 3 terms in the Maclaurin series for $\cos(\sin x)$. Hence or otherwise find

$$\lim_{x \rightarrow 0} \frac{1 - \cos(\sin(x))}{x^2}.$$

7. Using the Taylor series, determine the following limits:

$$1) \lim_{x \rightarrow 0} \frac{e^{3x-2} - e^2}{x} \quad 2) \lim_{x \rightarrow 1} \frac{\ln(2-x)}{x-1} \quad 3) \lim_{x \rightarrow \pi} \frac{\sin(x)}{x^2 - \pi^2} \quad 4) \lim_{x \rightarrow \frac{\pi}{2}} \frac{e^{\cos(x)}}{x - \frac{\pi}{2}} \quad 5) \lim_{x \rightarrow 0} \frac{\ln(1 - \sin(x))}{x}$$