Tutorial 3: Solution

Exercise 1

$$\begin{cases} x = 2\cos(3t+2) \\ y = 2\sin(3t+2) \end{cases}$$

Equation of trajectory:

$X^2+y^2 = 2^2\,[\cos^2(3t+2)+/\sin^2(3t+2)]$

$X^2 + y^2 = 2^2$ from The form $(x-x_0)^2\ (y-y_0)^2 = R^2$

$X_0=y_0=0$

It is the equation of a circle Centered at 0(0,0)and have a radius R = 2.

Velocity : $\vec{V}=\frac{d\overrightarrow{OM}}{dt}$

$$\vec{V}=\begin{cases} v_x = \dot{x} = 2[3(-\sin 3t+2)] = -6\sin(3t+2) \\ v_y = \dot{y} = 2[3\cos(3t+2)] = 6\cos(3t+2) \end{cases}$$

$$V=\sqrt{{v_x}^2+{v_y}^2} = \sqrt{[-6\sin(3t+2)]^2+[6\cos(3t+2)]^2}$$

$$=\sqrt{6^2[\sin^2(3t+2)+\cos^2(3t+2)}$$

$$=\sqrt{6^2} = 6\ \text{m/s}$$

Acceleration: $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\overrightarrow{OM}}{dt^2}$

$$\vec{a}=\begin{cases} a_x = \dot{v}_x = \ddot{x} = -6[\cos(3t+2)] = -18\cos(3t+2) \\ a_y = \dot{v}_y = \ddot{y} = 6[-3\sin(3t+2)] = -18\sin(3t+2) \end{cases}$$

$$a=\sqrt{{a_x}^2+{a_y}^2} = 18\ \text{m/s}^2$$

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Exercise 2

$\rho(t) = \frac{t^2}{4}$;$\varphi(t) = \frac{\pi}{4}t$

1- Position:$\overrightarrow{OM}$=$\rho\vec{U}_\rho$=$\frac{t^2}{4}\vec{U}_\rho$

-velocity: $\vec{V}$=$\dot{\rho}\vec{U}_\rho + \rho\dot{\vec{U}}_\rho$=$\dot{\rho}\vec{U}_\rho + \rho\dot{\varphi}\vec{U}_\varphi$

$=2\frac{t}{4}\vec{U}_\rho+\frac{t^2}{4}\frac{\pi}{4}\vec{U}_\varphi$

$=\frac{t}{2}\vec{U}_\rho+\frac{t^2\pi}{16}\vec{U}_\varphi$

Acceleration: $\vec{a}$= $(\ddot{\rho} - \rho\dot{\varphi}^2)\,\vec{U}_\rho + (2\dot{\rho}\dot{\varphi}+\rho\ddot{\varphi})\,\vec{U}_\varphi$

$\dot{\rho} = \frac{t}{2}$; $\ddot{\rho} = \frac{1}{2}$; $\dot{\varphi} = \frac{\pi}{4}$; $\ddot{\varphi} = 0$

$\vec{a}$= $[\frac{1}{2} - \frac{t^2}{4}(\frac{\pi^2}{4})]\,\vec{U}_\rho$ + $[2(\frac{1}{2})(\frac{\pi}{4})]\,\vec{U}_\varphi$

$\vec{a} = \left(\frac{-\pi^2t^2}{16} + \frac{1}{2}\right)\vec{U}_\rho + \frac{\pi}{4}\vec{U}_\varphi$

2) Magnitude: at t = 6s

Velocity:

$\vec{V}$= $\frac{6}{2}\vec{U}_\rho$+$\frac{6^2\pi}{16}\vec{U}_\varphi = 3\vec{U}_\rho + \frac{9\pi}{4}\vec{U}_\varphi$

V = $\sqrt{3^2 + (\frac{9\pi}{4})^2}$ = $\sqrt{9 + \frac{9^2\pi^2}{4^2}} = 3\sqrt{1 + \frac{9\pi^2}{16}}$

$=3\sqrt{\frac{16+9\pi^2}{16}} = \frac{3}{4}\sqrt{16 + 9\pi^2}$

$\vec{a} = \left[\frac{1}{2} - \frac{\pi^2(6)^2}{16}\right]\vec{U}_\rho + \frac{\pi}{4}\vec{U}_\varphi$

$=(\frac{1}{2} - \frac{36\pi^2}{16})\,\vec{U}_\rho + \frac{\pi}{4}\vec{U}_\varphi$ $=(\frac{8-36\pi^2}{16})\vec{U}_\rho + \frac{\pi}{4}\vec{U}_\varphi$

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$$a = \sqrt{\left(\frac{8-36\pi^2}{16}\right)^2 + (\frac{\pi}{4})^2} \quad = \sqrt{\frac{8^2-8*2*36\pi^2+36^2\pi^4}{16^2} + \frac{\pi^2}{16}}$$

$$= \frac{1}{4}\sqrt{\frac{8^2-8*2*36\pi^2+36^2\pi^4+16\pi^2}{16}}$$

$$=\frac{1}{16}\sqrt{64 - 416\pi^2 + 1296\pi^4}$$

3) Cartesian coordinates of point M:

X= $\rho \cos \varphi = \frac{t^2}{4} \cos \ (\frac{\pi}{4} t)$

Y= $\rho \sin \varphi = \frac{t^2}{4} \sin \ (\frac{\pi}{4})t$

4) The expression of the velocity vector in Cartesian coordinates:

$$\vec{V} = \begin{cases} \dot{x} = v_x = -\frac{t^2\pi}{16} \sin(\frac{\pi}{4} t) \\ \dot{y} = v_y = -\frac{t^2\pi}{16} \cos(\frac{\pi}{4} t) \end{cases} - \frac{t^2\pi}{16}$$