Geometric properties

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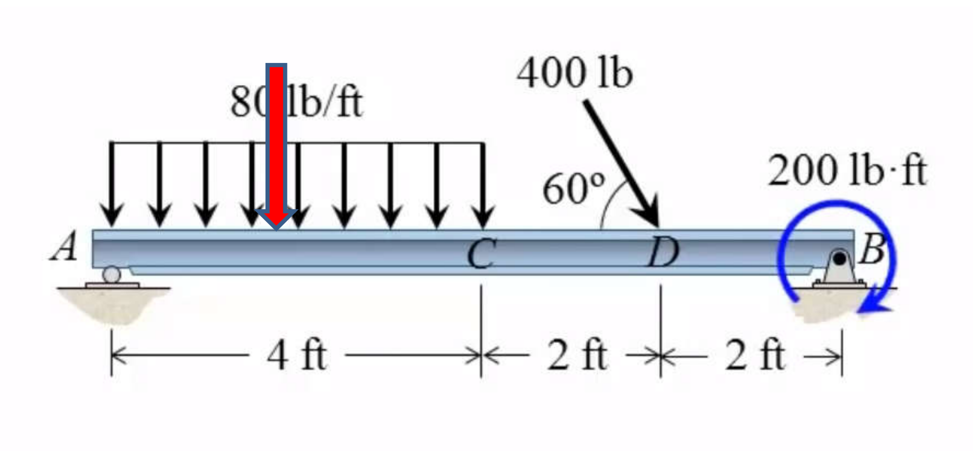
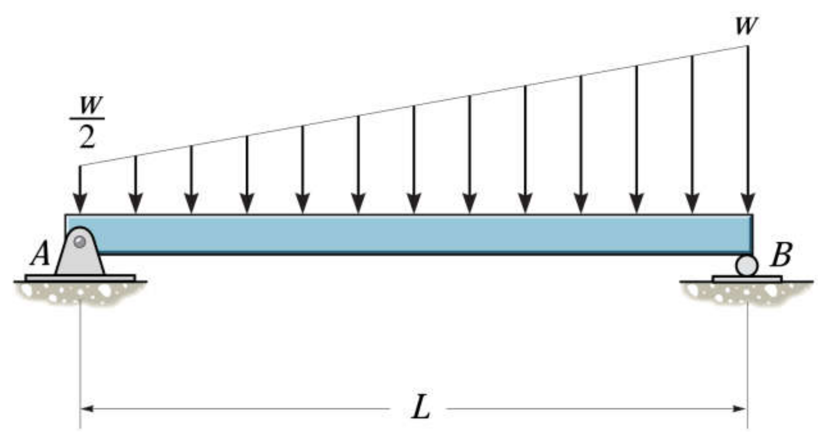
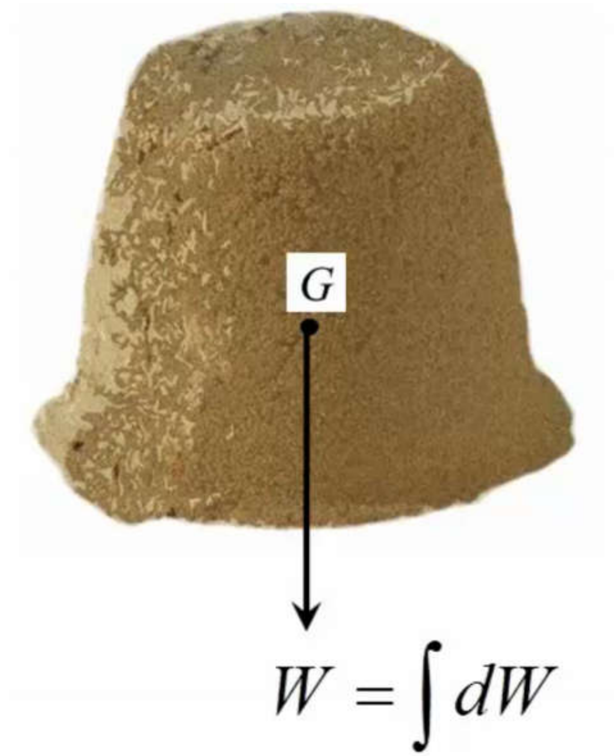
Centre of Gravity

Objectives:

- You will be able how to find the center of gravity of a rigid body



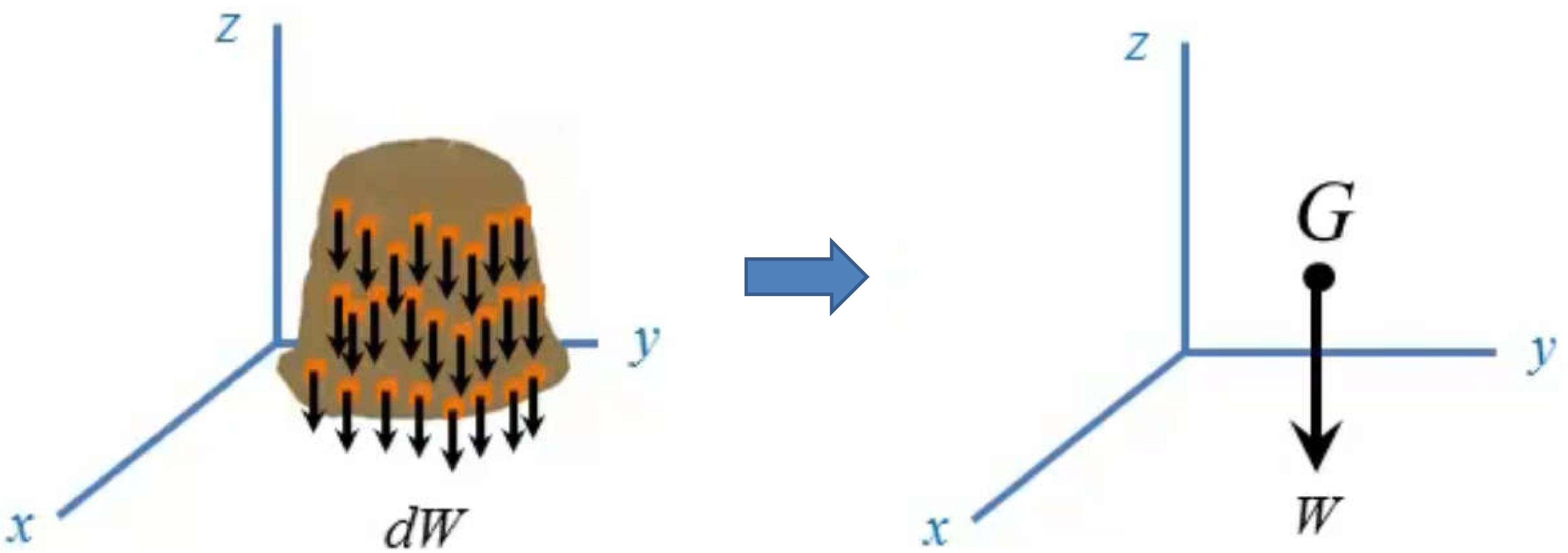




This is **undeformable rigid** material

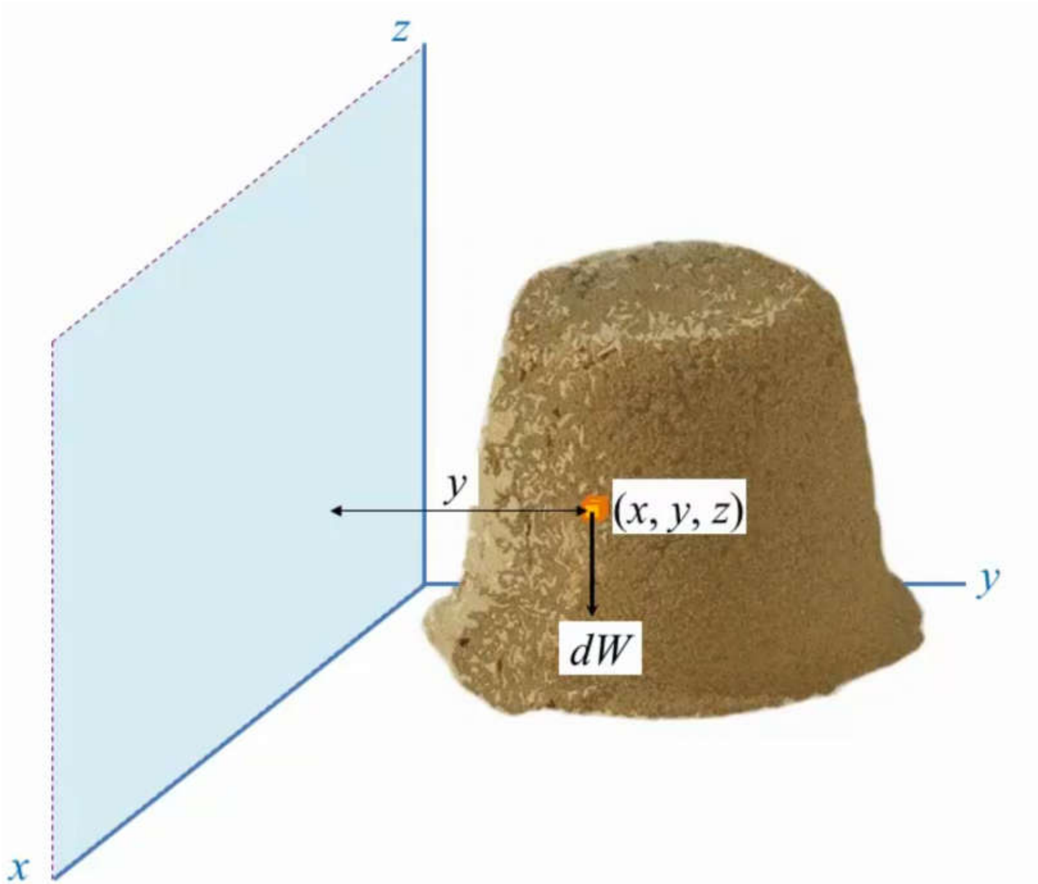
W is the total weight of this solid

1- We can simplify the analysis by replacing the rigid body by a particle of the same total weight located at this point

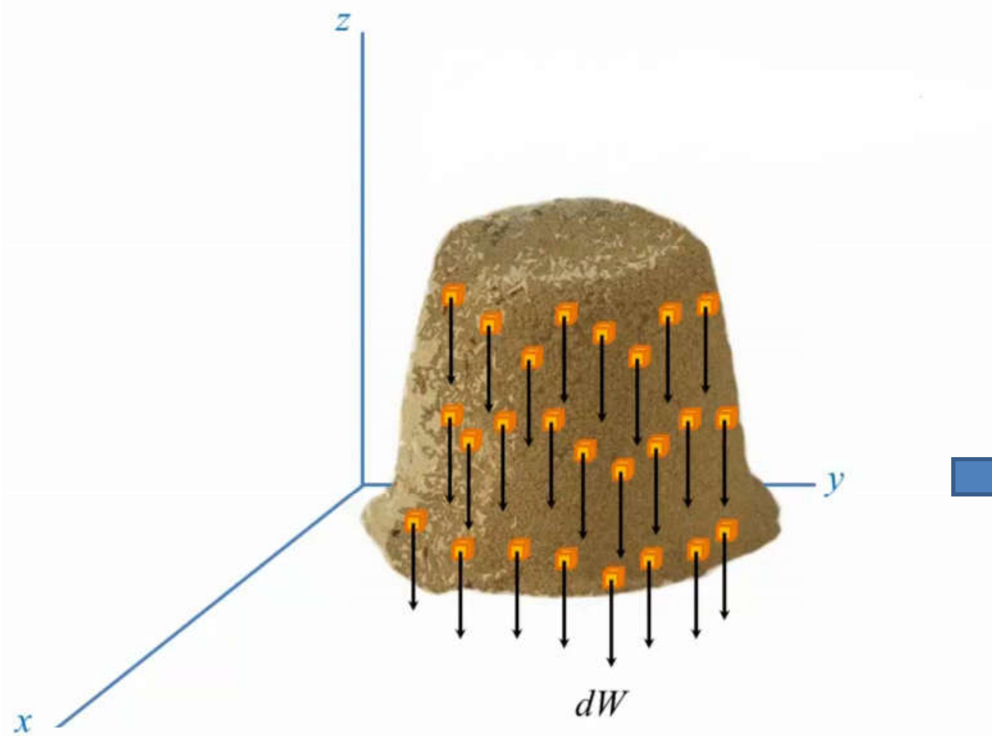


2- For a rigid body, in near-earth gravity, its center of gravity superimposes with its center of mass, and is a unique point fixed in relation to the body.

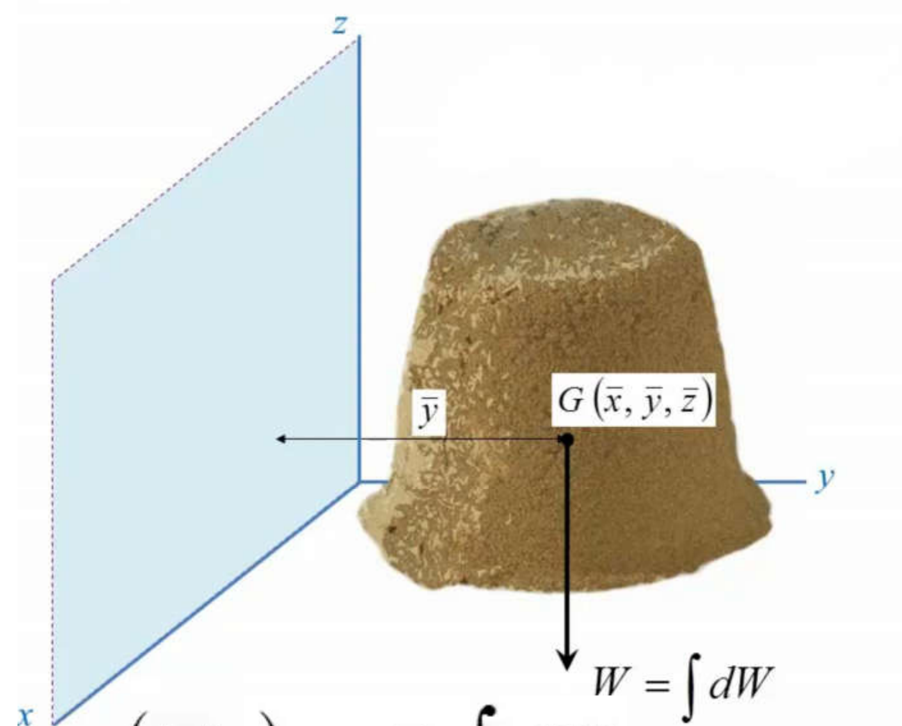
3- When the body has uniform density, center of gravity and center of mass is also centroid of the volume



$$\circlearrowleft + \ dM_x = y \cdot dW$$



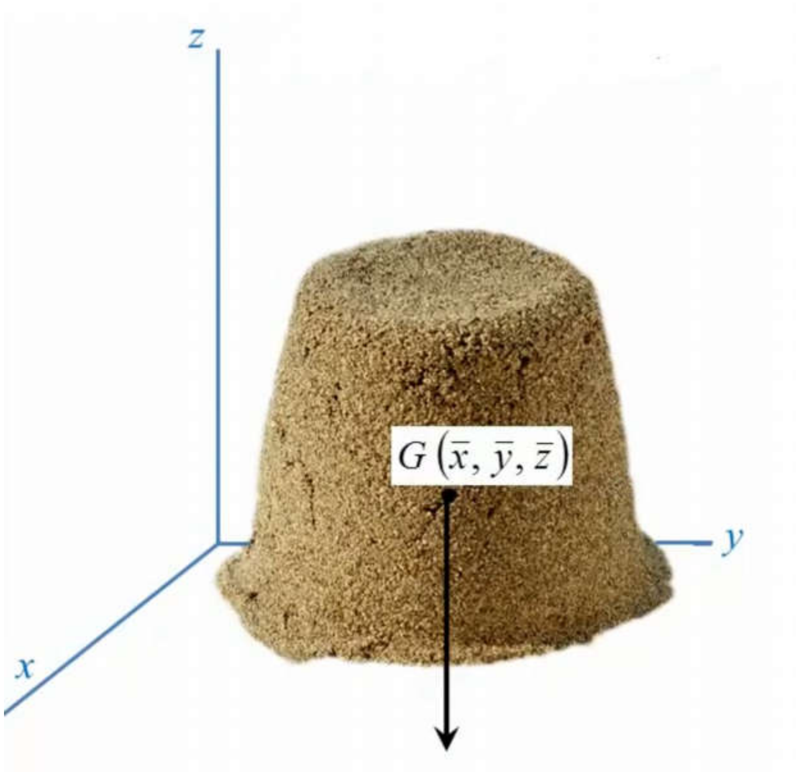
$$\circlearrowleft + (M_R)_x = \int dM_x = \int y \cdot dW$$



$$(M_R)_x = \bar{y} \cdot \int dW$$

$$\bar{y} = \frac{\int y \cdot dW}{\int dW}$$

Center of Gravity:



$$\bar{x} = \frac{\int x \cdot dW}{\int dW}$$

$$\bar{y} = \frac{\int y \cdot dW}{\int dW}$$

$$\bar{z} = \frac{\int z \cdot dW}{\int dW}$$

$$W = mg$$

g **gravitational acceleration constant , 9.81 m/s^2**

$$\bar{x} = \frac{\int x \cdot dW}{\int dW} = \frac{\int x \cdot dm \cdot g}{\int dm \cdot g} = \frac{\int x \cdot dm}{\int dm}$$

$$\bar{y} = \frac{\int y \cdot dm}{\int dm}$$

$$\bar{z} = \frac{\int z \cdot dm}{\int dm}$$

Centroid of Mass

$$\bar{x} = \frac{\int x \cdot dW}{\int dW}$$

$$\bar{y} = \frac{\int y \cdot dW}{\int dW}$$

$$\bar{z} = \frac{\int z \cdot dW}{\int dW}$$

$$m = \rho \cdot V$$

$$\bar{z} = \frac{\int z \cdot dm}{\int dm}$$

p **constant density**

$$\bar{x} = \frac{\int x \cdot dm}{\int dm} = \frac{\int x \cdot \rho \cdot dV}{\int \rho \cdot dV} = \frac{\int x \cdot dV}{\int dV}$$

$$\bar{x} = \frac{\int x \cdot dV}{\int dV} \qquad \bar{y} = \frac{\int y \cdot dV}{\int dV} \qquad \bar{z} = \frac{\int z \cdot dV}{\int dV}$$

Centroid of Volume

$$V = t \cdot A$$

t **constant thickness**

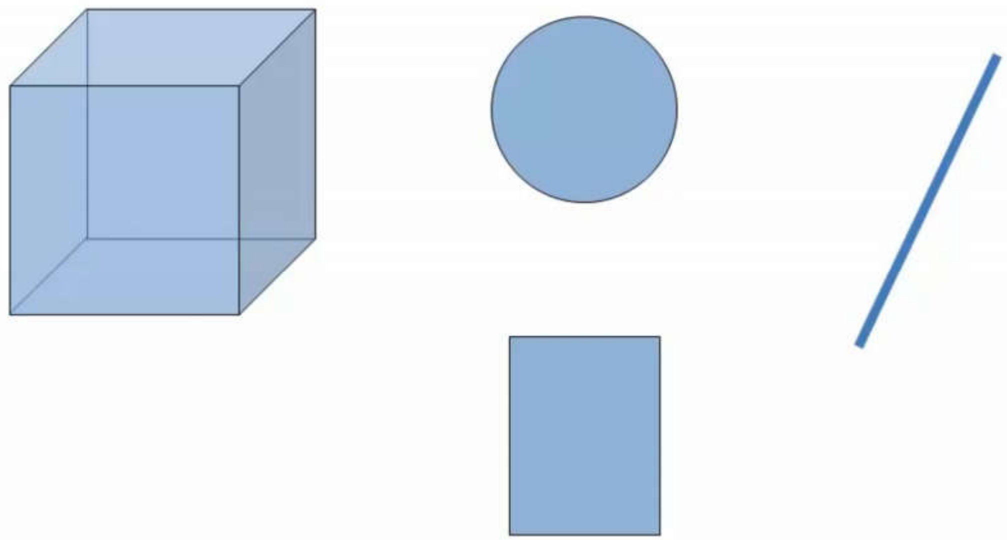
$$\bar{x} = \frac{\int x \cdot dV}{\int dV}$$

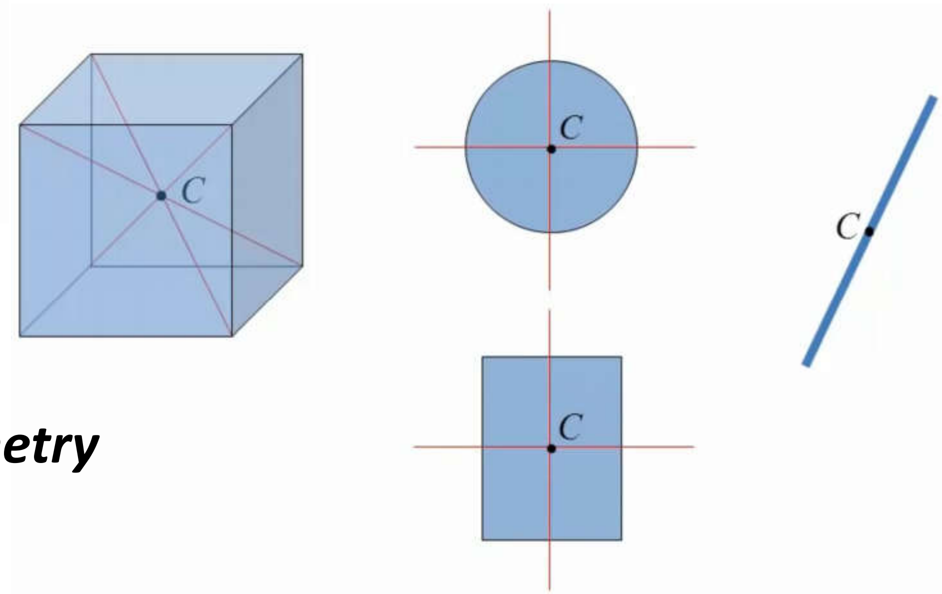
$$\bar{x} = \frac{\int x \cdot dV}{\int dV} = \frac{\int x \cdot t \cdot dA}{\int t \cdot dA} = \frac{\int x \cdot dA}{\int dA}$$

$$\bar{x} = \frac{\int x \cdot dA}{\int dA}$$

$$\bar{y} = \frac{\int y \cdot dA}{\int dA}$$

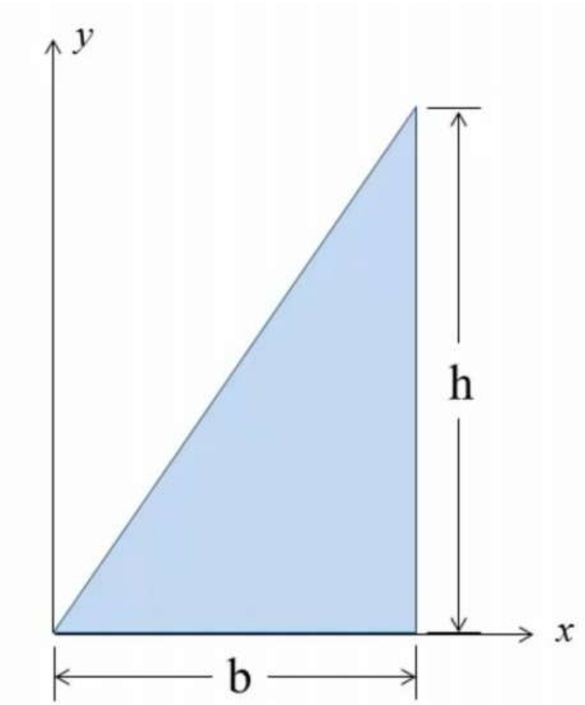
Centroid of Area





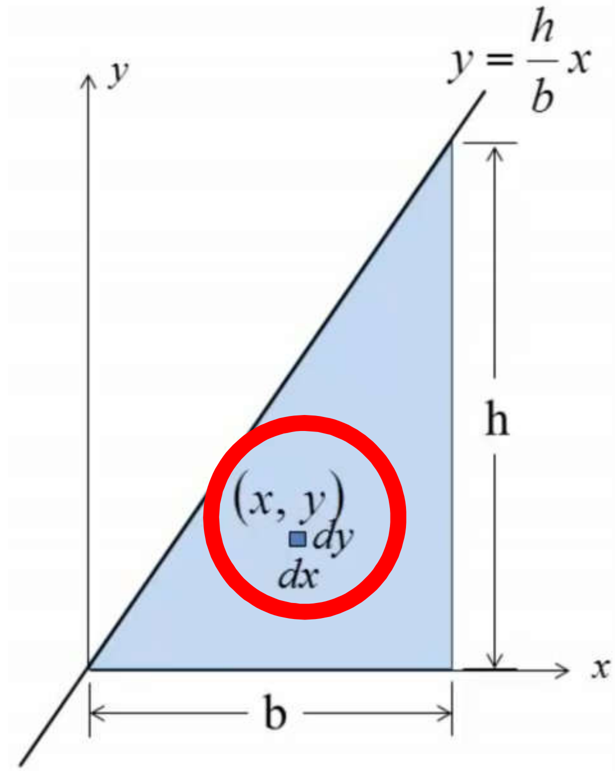
Centroid is always located in axis of symmetry

Example: ***find the centroid of this rectangle:***



$$\bar{x} = \frac{\int x \cdot dA}{\int dA}$$

$$\bar{y} = \frac{\int y \cdot dA}{\int dA}$$



$$dA = dx \cdot dy$$

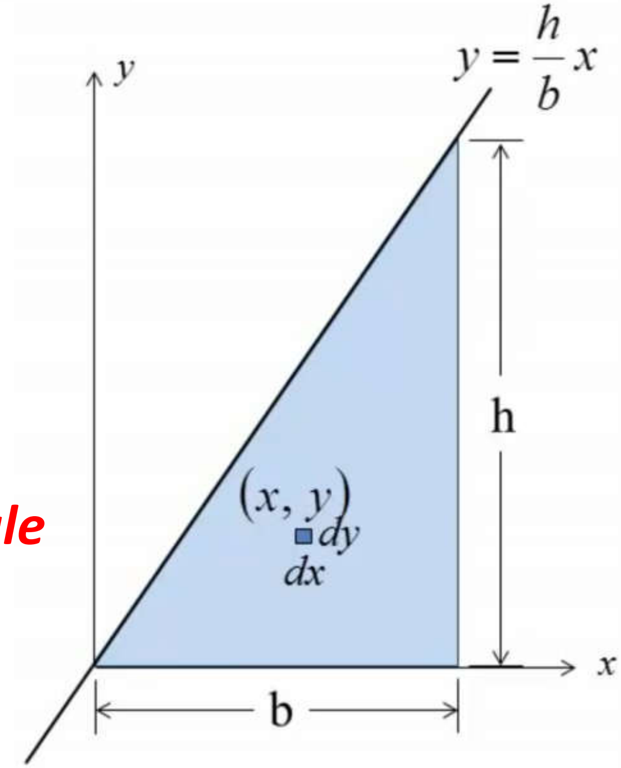
$$\int_A x\,dA = \int_0^b \int_0^{\frac{h}{b}x} x\,dy\,dx = \int_0^b \frac{h}{b}x^2\,dx = \frac{h}{b}\cdot\frac{1}{3}x^3\Big|_0^b = \frac{1}{3}hb^2$$

$$dA = dx\cdot dy$$

$$\int_A dA = \int_0^b \int_0^{\frac{h}{b}x} dy\,dx = \int_0^b \frac{h}{b}x\,dx = \frac{h}{b}\cdot\frac{1}{2}x^2\Big|_0^b = \frac{1}{2}hb$$

Area of triangle

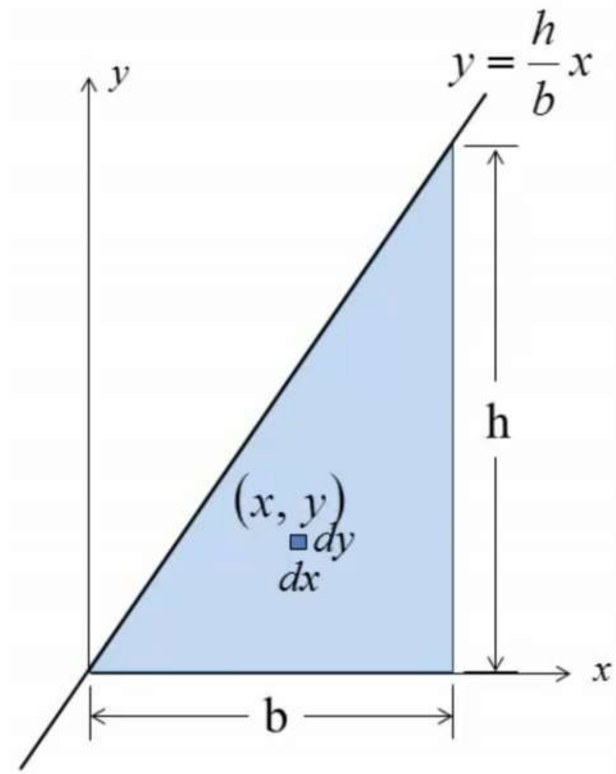
$$\bar{x} = \frac{\int x\cdot dA}{\int dA} = \frac{2}{3}b$$



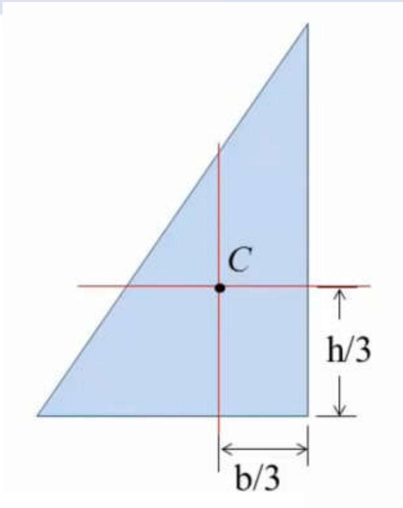
$$\int_A y\,dA = \int_0^b \int_0^{\frac{h}{b}x} y\,dy\,dx = \int_0^b \frac{h^2}{2b^2} x^2\,dx$$

$$= \frac{h^2}{b^2} \cdot \frac{1}{6} x^3 \Big|_0^b = \frac{1}{6} h^2 b$$

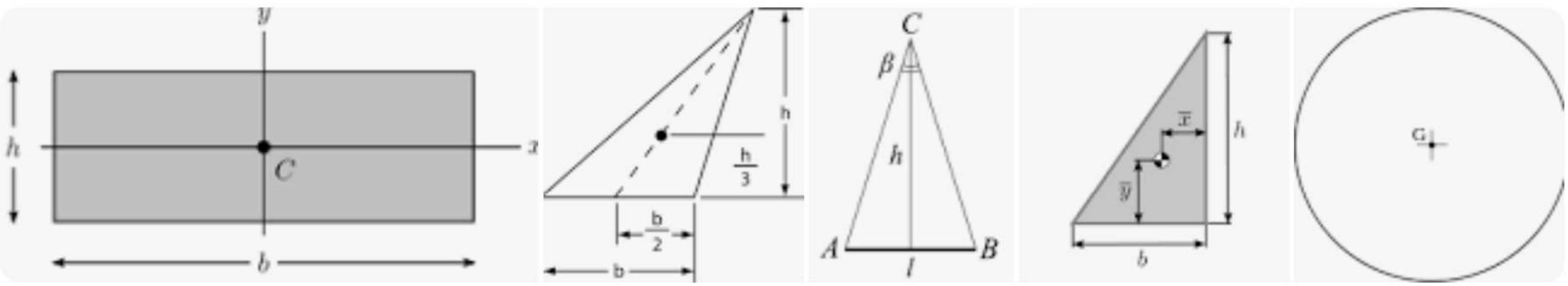
$$\int_A dA = \frac{1}{2} hb \qquad \bar{y} = \frac{\int y \cdot dA}{\int dA} = \frac{1}{3} h$$



The centroid of this rectangle:



List of centroids - Wikipedia



The following is a **list of centroids** of various two-dimensional and three-dimensional ... List of centroids. Article Talk · Language · Watch · Edit. The following is a list of centroids of various two-dimensional an…

2-D Centroids · 3-D Centroids

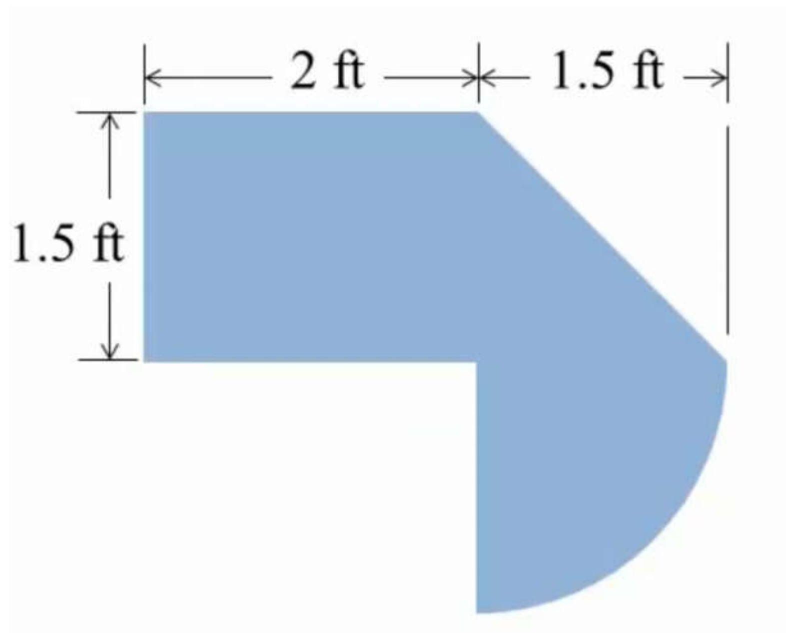
Centroid of composite areas:

$$\bar{x} = \frac{\int \tilde{x} \cdot dA}{\int dA} \quad \Rightarrow \quad \bar{x} = \frac{\sum \tilde{x} \cdot A}{\sum A}$$

$$\bar{y} = \frac{\int \tilde{y} \cdot dA}{\int dA} \quad \Rightarrow \quad \bar{y} = \frac{\sum \tilde{y} \cdot A}{\sum A}$$

Centroid of composite areas:

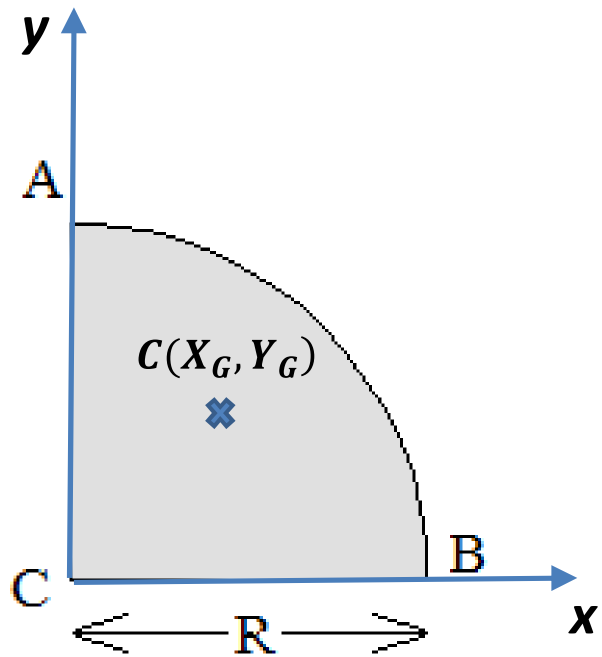
Example: find the centroid of the area of the composite shape:

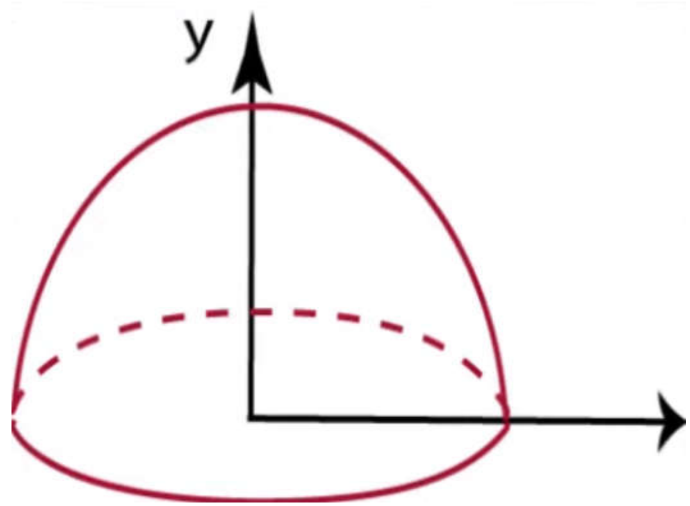


Centroid of an Area:

Guldin theorem:

$$\begin{cases} X_G = \dfrac{V_y}{2\pi S} \\ Y_G = \dfrac{V_x}{2\pi S} \end{cases}$$

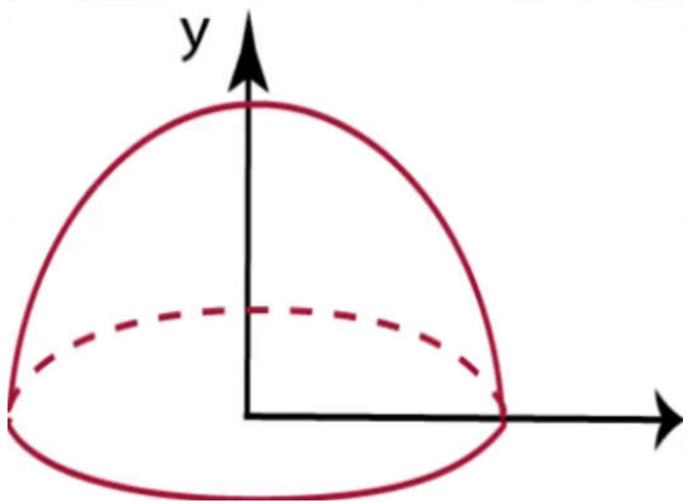




V_y ***Is the Volume created by the shape around the X axis***

Centroid of an Area:

Guldin theorem:



$$X_G = \frac{V_y}{2\pi S}$$

V_y= the volume of the half of the sphere

S= the surface of the quarter-circle

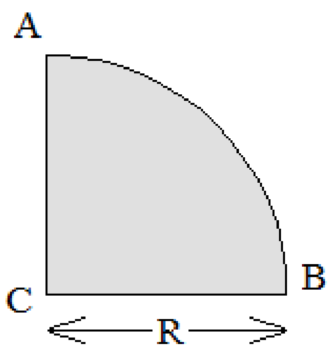
V_y ***Is the Volume created by the shape around the X axis***

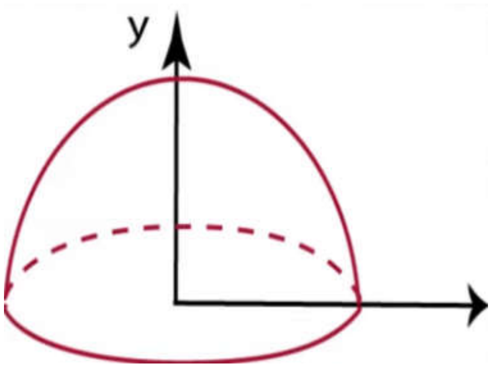
S ***Is the Initial surface of the shape***

$$X_G = \frac{V_y}{2\pi S}$$

V_y= the volume of the half of the sphere

S= the surface of the quart-circle





$$S = \frac{\pi R^2}{4}$$

$$V_y = \frac{2\pi R^3}{3}$$

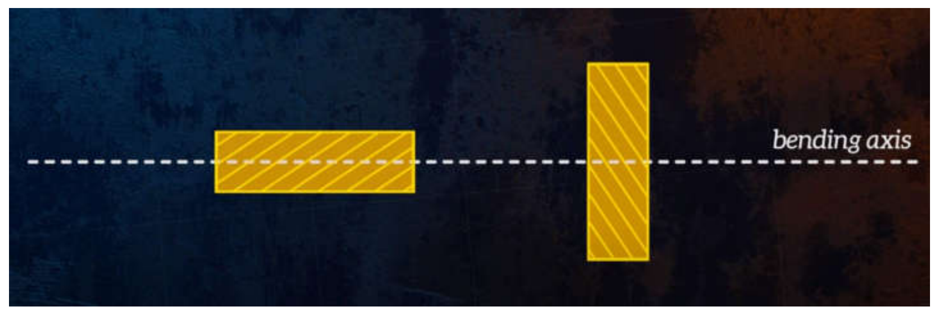
$$X_G = \frac{\frac{2\pi R^3}{3}}{2\pi \frac{\pi R^2}{4}}$$

$$X_G, Y_G = \frac{4R}{3\pi}$$

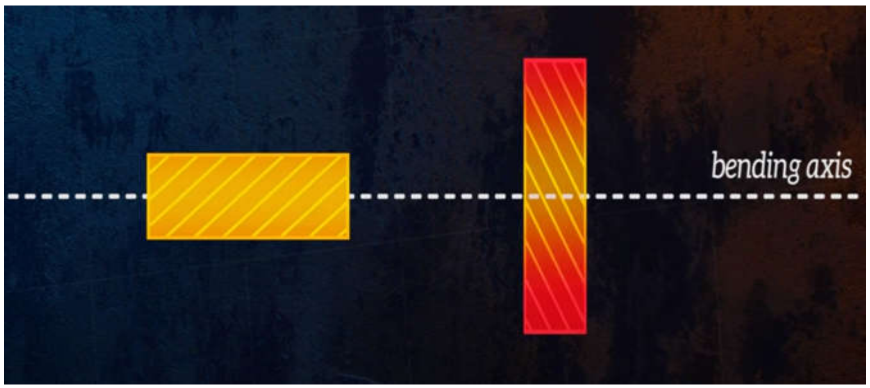
Moment of inertia

Objectives:

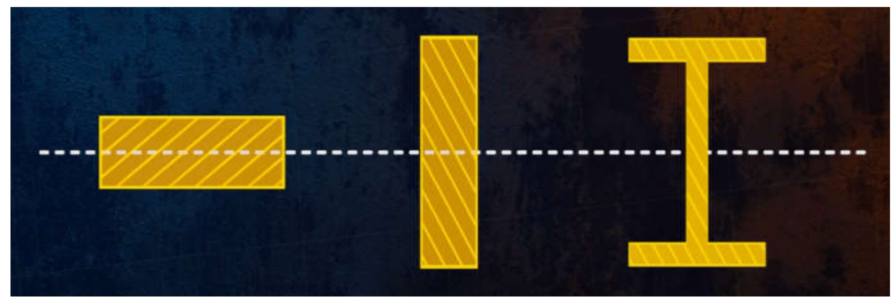
- You will be able how to find the Moment of inertia of any shape



Moment of inertia

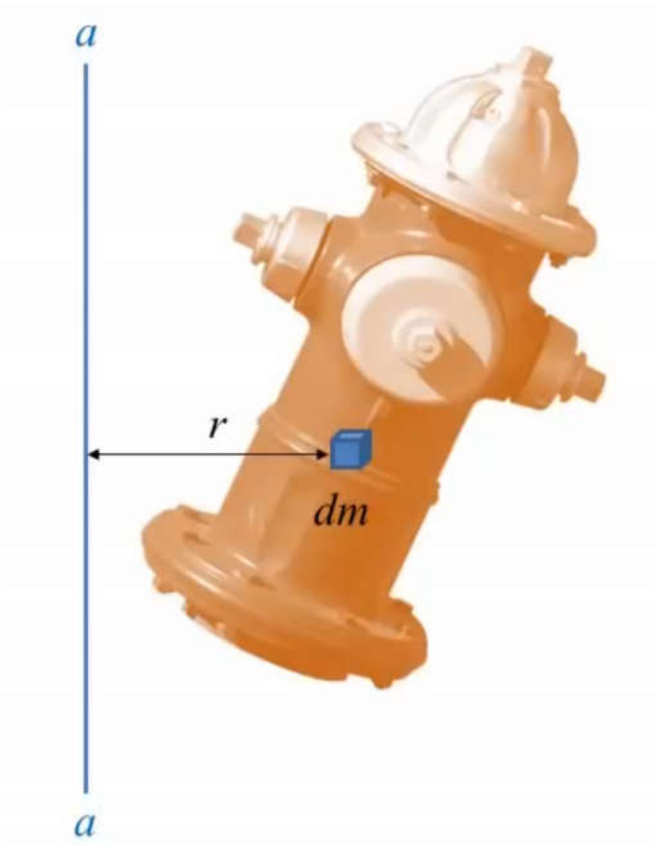


Resistance to bending (rotation) = Area moment of inertia (second moment of Area)



More material located fare from the bending axis, it better resists the bending.
Even thought , we have the same section of material

Moment of inertia



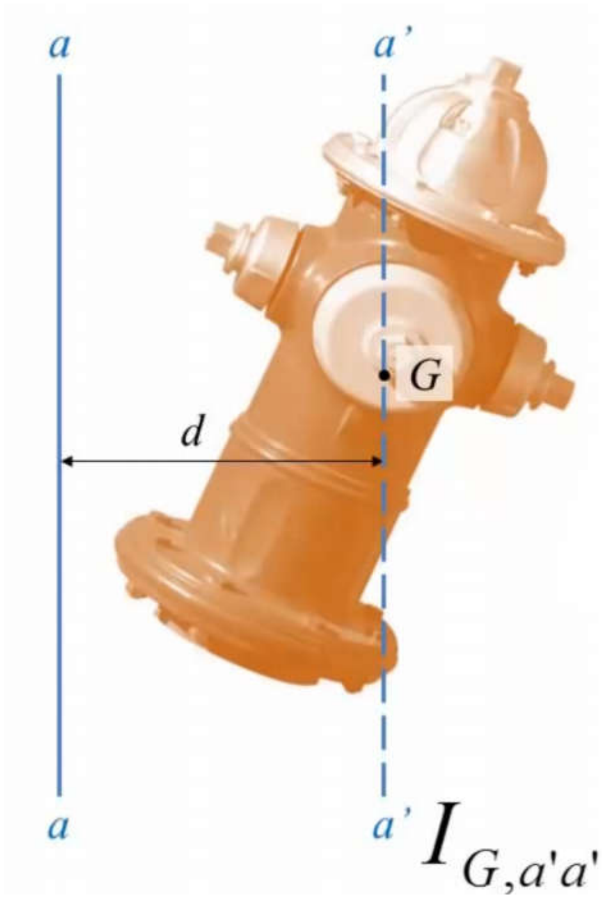
$$I_{aa} = \int_m r^2 dm$$

The moment of inertia is relative, different when calculated about different axis

Radius of gyration

$$k_{aa} = \sqrt{\frac{I_{aa}}{m}}$$

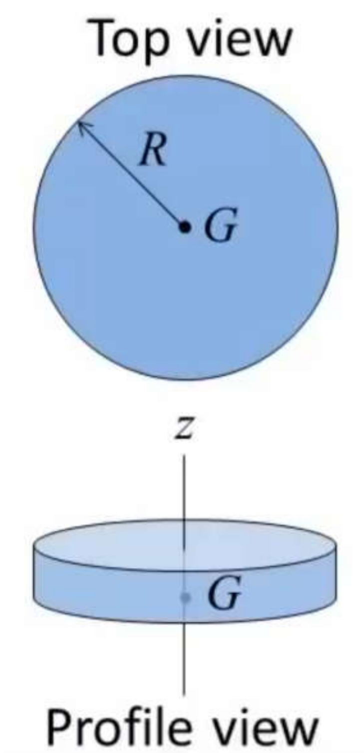
Parallel axis theorem:



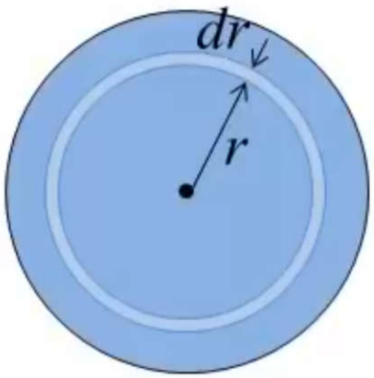
$$I_{aa} = I_{G,aa'} + d^2 m$$

Example:

For a uniform thin disk of mass m, determine its mass moment of inertia abut the z axis, which passes through its center of gravity G and is perpendicular to the disk



Solution:



$$dm = \rho dV = \rho.t.dA$$

$$dm = \rho.t.2\pi r dr$$

$$I_z = \int_m r^2 dm$$

$$I_z = \rho.t.2\pi \int_m r^3 dm$$

$$I_z = \rho.t.2\pi \int_m r^3 dm$$

$$I_z = \rho.t.2\pi t \frac{1}{4} R^4 = \frac{1}{2} \pi \rho t R^4$$

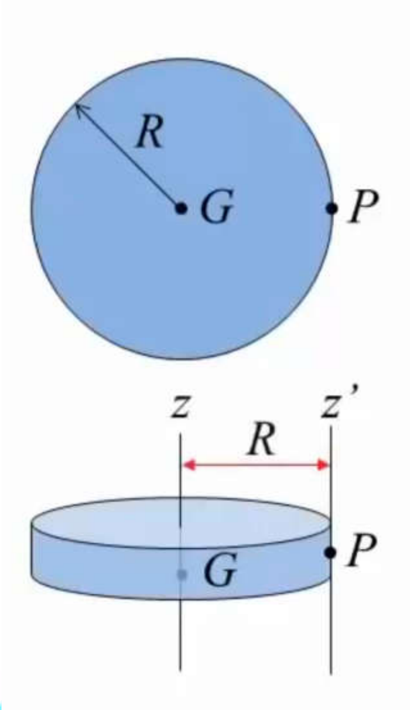
$$V = \pi R^2 t$$

$$m = \rho V = \rho \pi R^2 t$$

$$I_z = \frac{1}{2} m R^2$$

Example 2:

For a uniform thin disk of mass m, determine its mass moment of inertia abut the z' axis, which passes through point P and is perpendicular to the disk



We have $$I_z = \frac{1}{2} m R^2$$

Parallel axis theorem:

$$I_{z'} = \frac{1}{2} m R^2 + m R^2$$

$$I_{z'} = \frac{3}{2} m R^2$$