Simplification of Force and Moment System

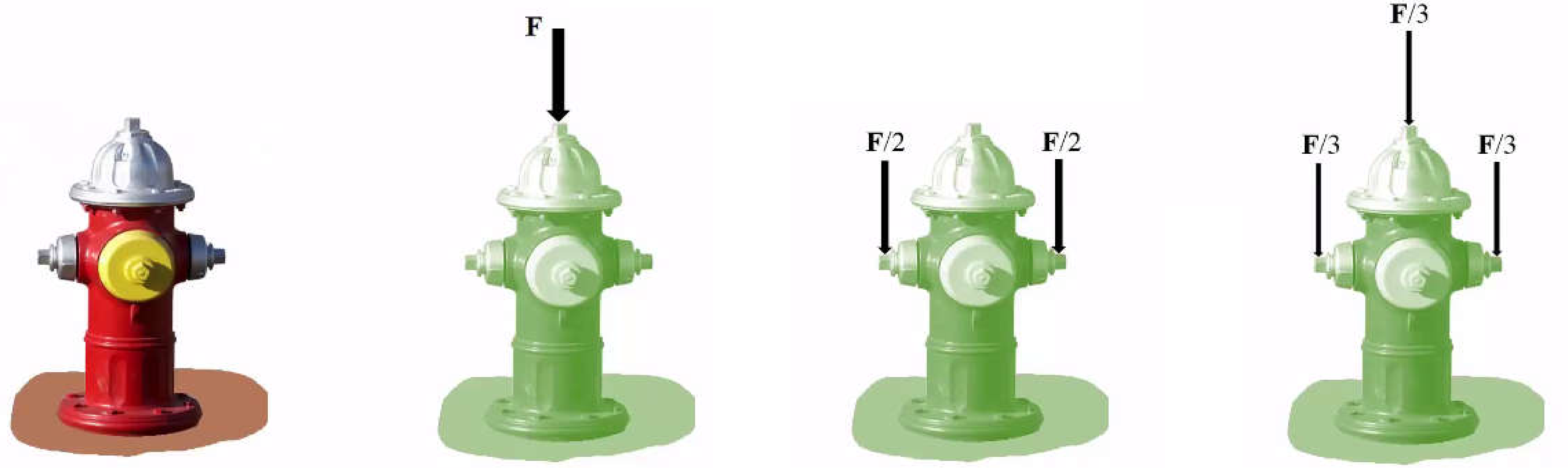
Dr. Djedoui . Dr. Khechai

Objectives :

- To calculate the resultant force and resultant moment of a given multiple-force-moment system.
- To replace the original multiple-force-moment system by its equivalent single-force-moment system, or a single-force-only system in some cases.

Question 1: In particle equilibrium you've learned how to find the resultant force of multiple forces (and subsequently apply Newton's 1^{st} law to solve problems). Similarly, for a rigid body subjected to forces AND moments, how can you find the resultant moment? For any given system, is there only one correct resultant moment?

Imagine this fire hydrant, fixed to the ground, and there is a force **F** acting on it (Fig.1).

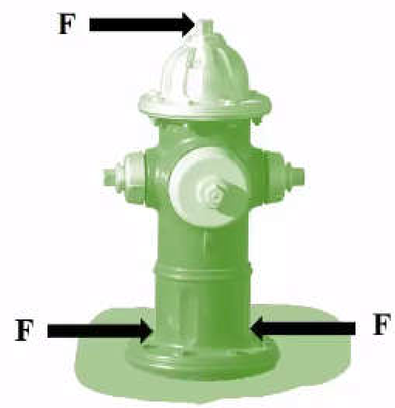


From experience, we can say that if we replace this force **F** by **two forces** (Fig.2), each **with half the magnitude**, placed **symmetrically** about the **central axis**, these two forces will create the same effect as the original **F** force. Even if we replace the forces by these three (Fig.3), again, they create the same effect.

By the **same effect,** I mean that the forces create the **same tendency to push** the fire hydrant down, and also, the **ground** will generate **the same** force **to support** the fire hydrant, preventing it from going down.

These several force systems are known to be equivalent systems.

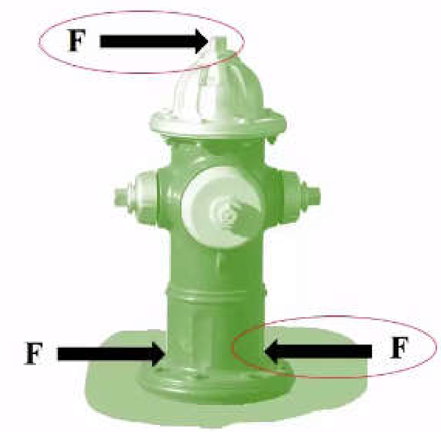
Now, let's imagine the force **F** acts on the fire hydrant this way. Now the force creates a **translational tendency to push** it to the right, and also it creates **a clockwise rotational tendency** for the fire hydrant **to fall** to the right.



For this fire hydrant to **stay still**, as a response, the ground must **create a force** supporting the fire hydrant, pointing to the left, and also a moment to cancel out **the rotational effect.**

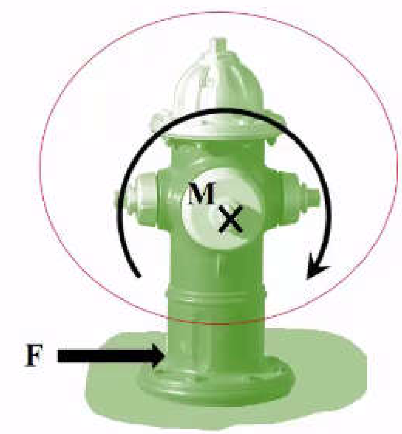
We can add **a pair of cancelling forces** to this fire hydrant without changing the load **status.**

These two forces now create a **Couple moment.**



Now this force still provides a **translational tendency** to push the fire hydrant to the right, while **the couple moment** creates the **clockwise rotational effect**.

So, in order to keep the fire hydrant **statics**, the ground must create **a force pointing to the left**, and **counterclockwise moment** to cancel out the rotational effect.



This **force-moment system** is the equivalent system as the previous **single force system**.

Equivalent system

- A system is equivalent if the external effects it produces on a body are the same as those caused by the original force and couple moment system.

- A load with multiple forces and couple moments acting on multiple locations can be replaced by a single force and a single couple moment acting on a single point.

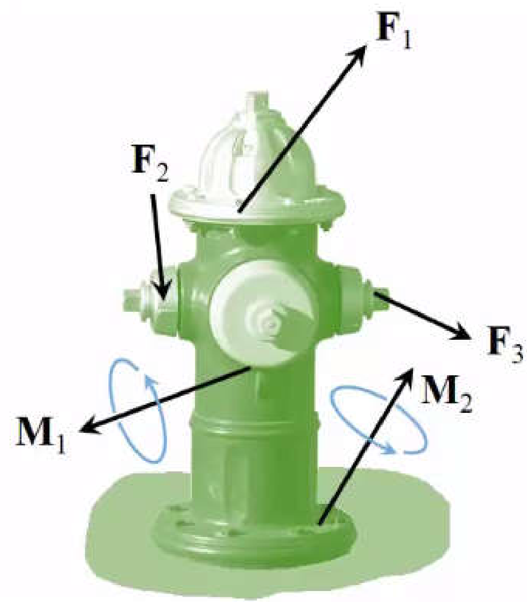
We want to do so to help calculate the support reactions.

Let's imagine the fire hydrant subjected to **multiple forces** and **multiple moments** acting on **multiple points**.

We want to replace all of these by **a single force** and **a single moment** placed at **a certain point** say point *O*.

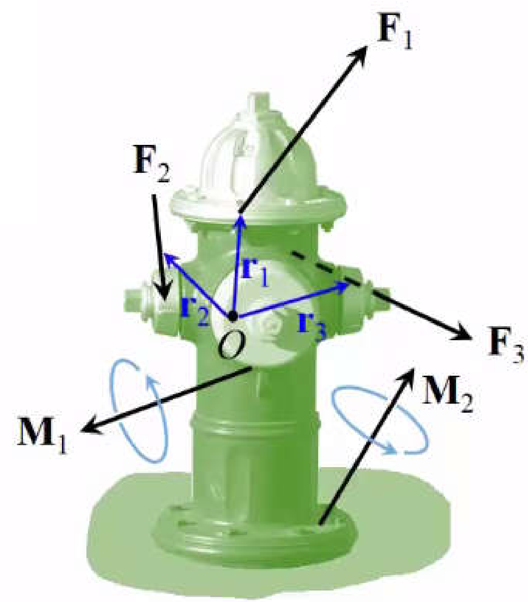
The single force is simply **the resultant force**:

$$\mathbf{F}_R = \sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$



For the resultant moment, we need to first calculate **the individual moment** caused by **each force** about point *O*, add them together:

$$\sum \mathbf{M}_{F,O} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \mathbf{r}_3 \times \mathbf{F}_3$$

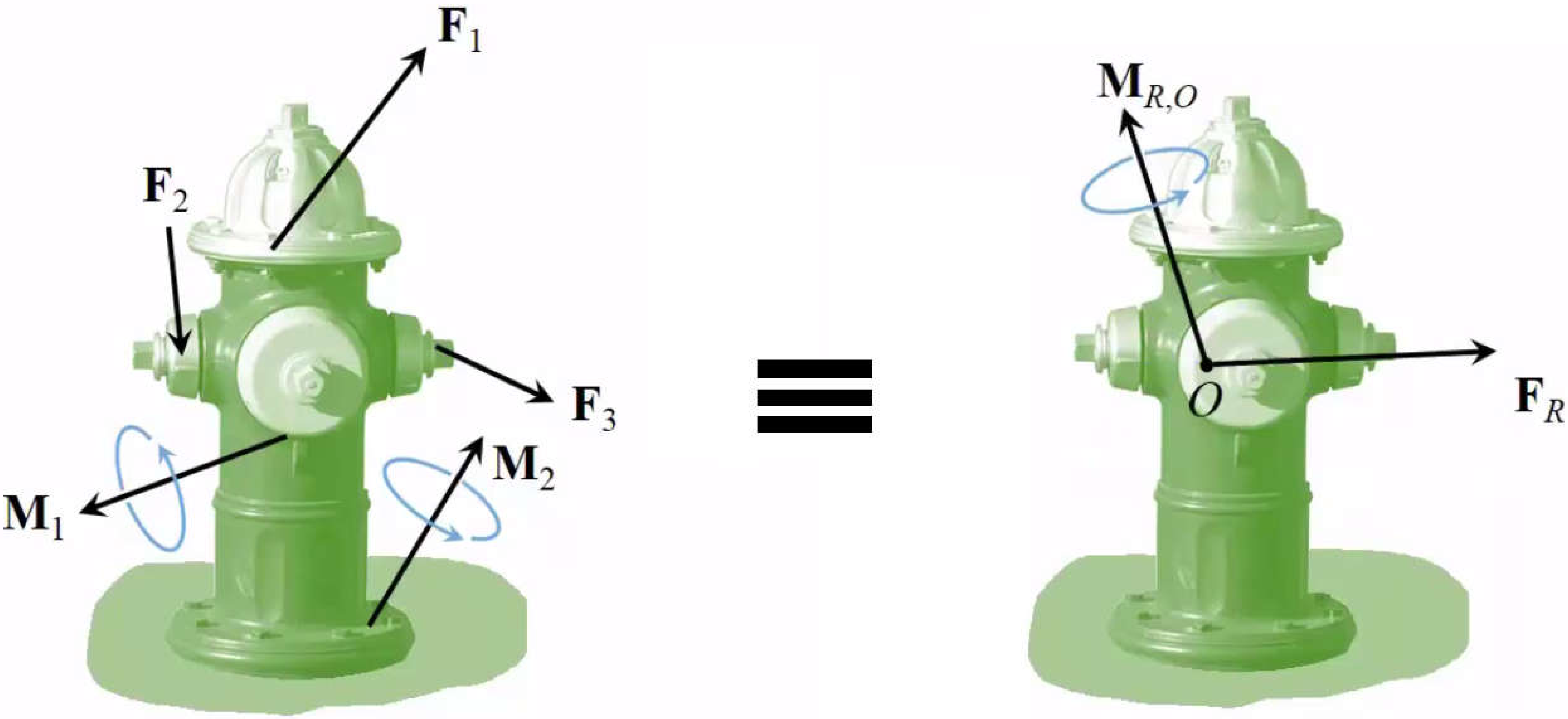


Then, add all of the free couple moments together **M1** and **M2**:

$$\sum \mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$$

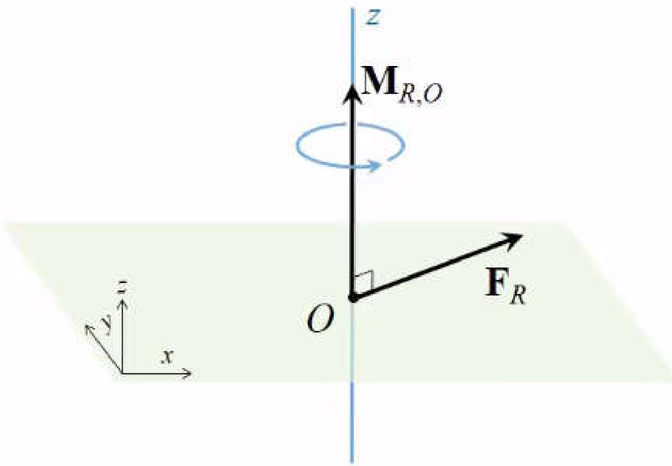
Then, we add **the total moment caused by the forces** and the **couple moments** together:

$$\mathbf{M}_{R,O} = \sum \mathbf{M}_{F,O} + \sum \mathbf{M}$$



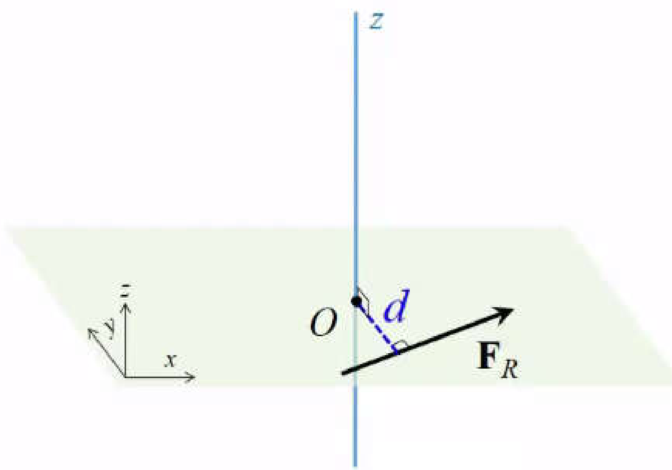
We replaced the **original multi-force multi-moment load system** by **a single force single moment system**.

In some special situation, the **resultant force vector** and **the resultant moment vector** are perpendicular to each other.



We can further reduce the moment by placing the force away from point *O*, say at distance *d*:

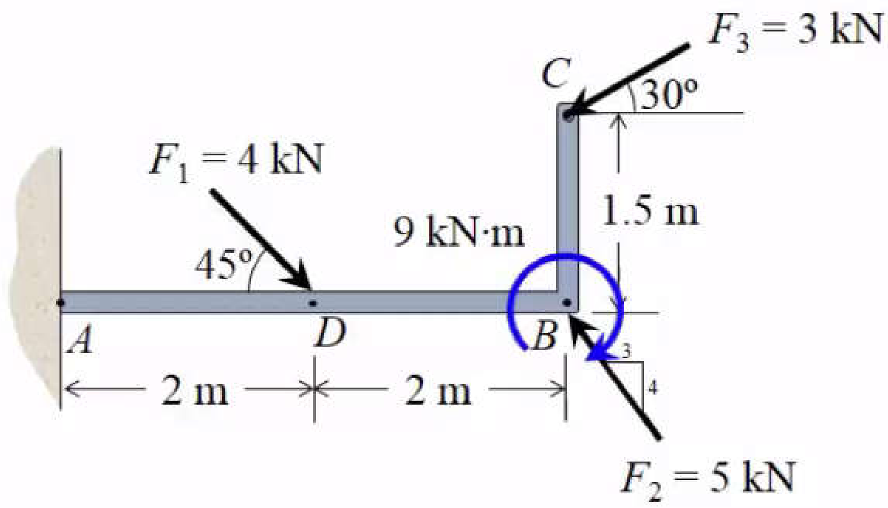
$$d = \frac{M_{R,O}}{F_R}$$



The reason is because, this way, the resultant force is creating a moment about point *O* that equals to :

$$M_{R,O} = F_R\, d$$

Example: Replace the shown force-moment system with an equivalent single force placed on the AB segment. Neglect the thickness of the member.

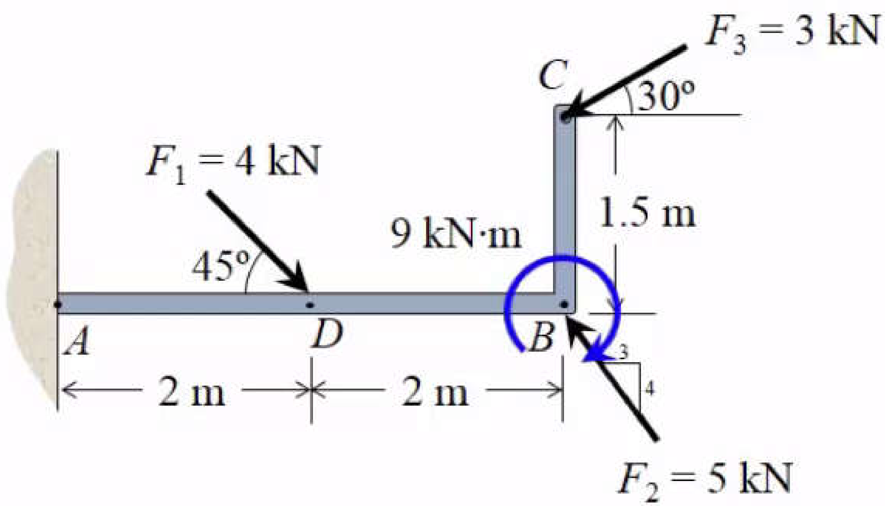


Example: Replace the shown force-moment system with an equivalent single force placed on the AB segment. Neglect the thickness of the member.

A force & a couple moment:

$$\mathbf{F}_R = \sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

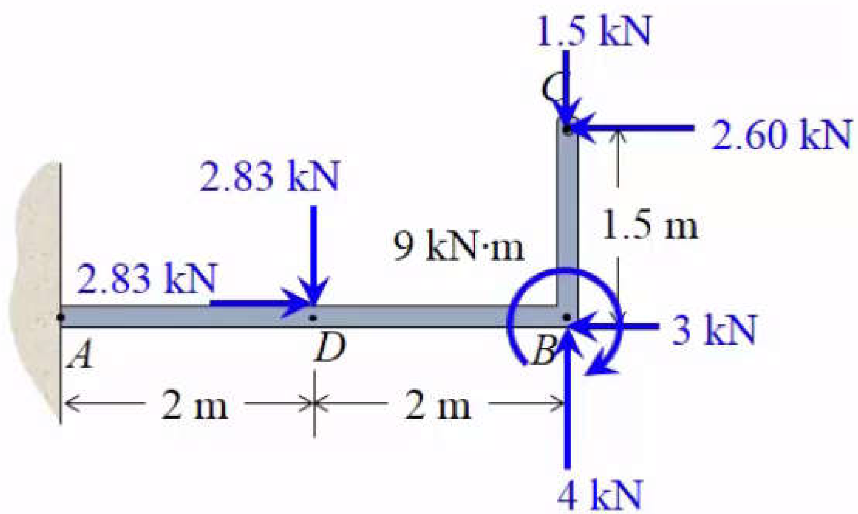
$$\mathbf{M}_{R,A} = \sum \mathbf{M}_{F,A} + \sum \mathbf{M}$$



$$\overset{+}{\rightarrow} F_x = 2.83 - 2.60 - 3 = -2.77\,(\text{kN})$$

$$+\uparrow F_y = -2.83 - 1.5 + 4 = -0.33\,(\text{kN})$$

$$\circlearrowleft + M_A = 8.24 - 9 = -0.76\,(\text{kN}\cdot\text{m})$$



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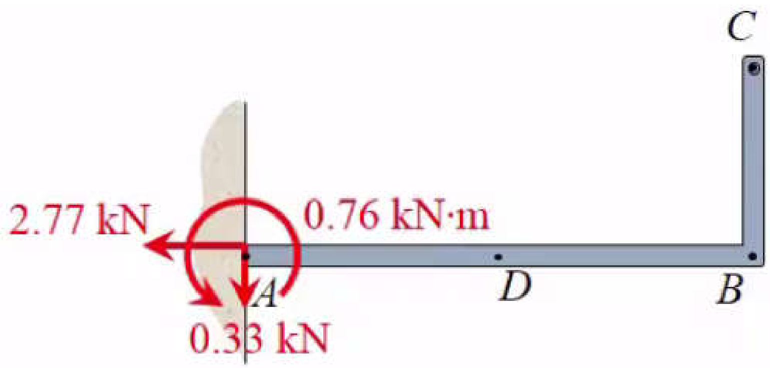


$$\overset{+}{\rightarrow} F_x = -2.77 \text{ kN}$$

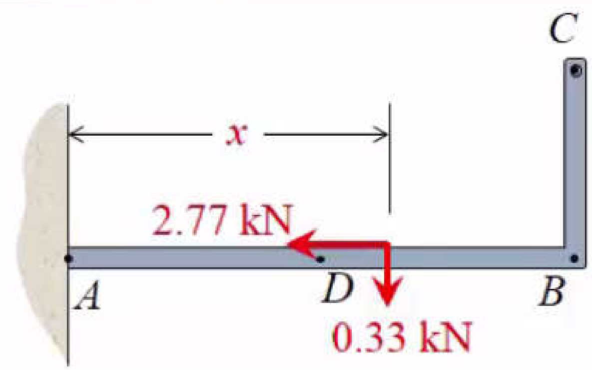
$$+\uparrow F_y = -0.33 \text{ kN}$$

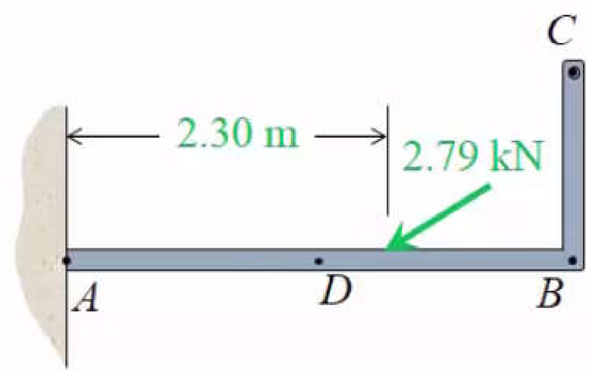
$$\circlearrowleft + M_A = -0.76 \text{ kN}\cdot\text{m}$$

$$x = \left|\frac{M_A}{F_y}\right| = \left|\frac{-0.76}{-0.33}\right| = 2.30\,(\text{m})$$

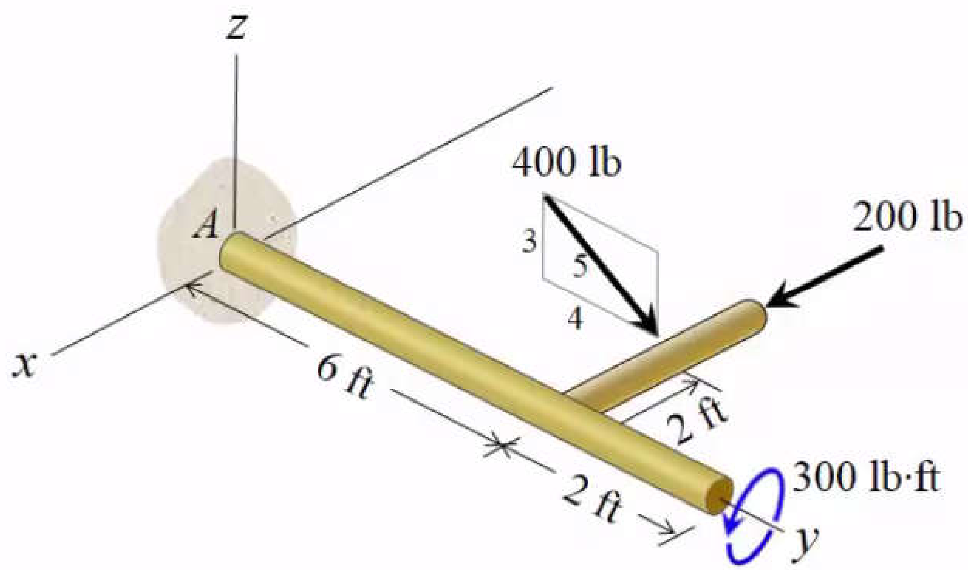
$$F = \sqrt{F_x^2 + F_y^2} = 2.79\,(\text{kN})$$

Ans.





Question 2: What is the resultant **applied** force (not including support) in Cartesian vector form?



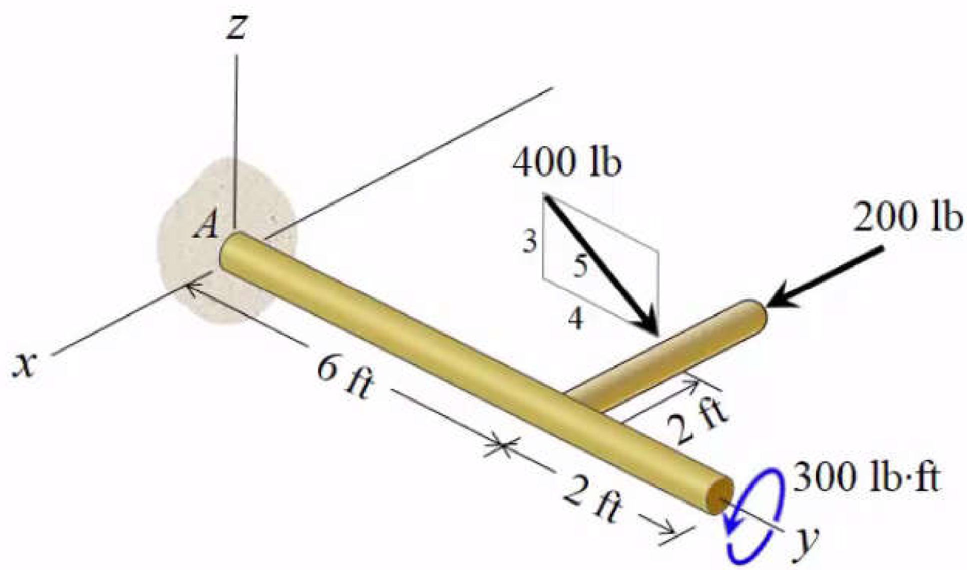
(a) $\{200\mathbf{i} - 20\mathbf{j} - 240\mathbf{k}\}$ lb

(b) $\{200\mathbf{i} - 320\mathbf{j} - 240\mathbf{k}\}$ lb

(c) $\{200\mathbf{i} + 620\mathbf{j} - 240\mathbf{k}\}$ lb

(d) $\{200\mathbf{i} + 320\mathbf{j} - 240\mathbf{k}\}$ lb

Question 3: What is the resultant **applied** moment (not including support) about point A in Cartesian vector form?



(a) $\{-1440\mathbf{i}-480\mathbf{j}-1840\mathbf{k}\}\text{lb}\cdot\text{ft}$ (b) $\{-1440\mathbf{i}-480\mathbf{j}-640\mathbf{k}\}\text{lb}\cdot\text{ft}$

(c) $\{-1440\mathbf{i}-180\mathbf{j}-1840\mathbf{k}\}\text{lb}\cdot\text{ft}$ (d) $\{-1440\mathbf{i}-180\mathbf{j}-640\mathbf{k}\}\text{lb}\cdot\text{ft}$