

Rigid body equilibrium: Conditions

Objective :

To introduce the general conditions for 2D and 3D rigid body equilibrium problems.

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Particle equilibrium

First let's recall the conditions for **particle equilibrium**.

According to **Newton's first law**, an object will have a **linear acceleration of zero** when there is **no unbalanced force** acting on it.

$$\mathbf{F}_R = \sum \mathbf{F} = \mathbf{0}$$

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Particle equilibrium $\mathbf{F}_R = \sum \mathbf{F} = \mathbf{0}$

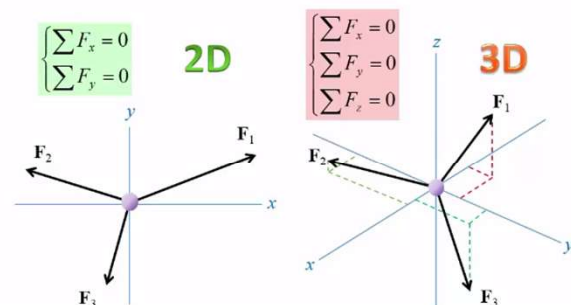
Since **Particle** is an idealized object with **no size** or **shape**, and is only represented by a **dot** in space, the forces acting on the particle will be **concurrent**.

For a 2D problem, the vector equation can be written as **2 scalar equations**.

For a 3D problem, the vector equation becomes **3 scalar equations**.

2D problem = Solve for 2 unknowns

3D problem = Solve for 3 unknowns



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Recall: moment of a couple

However, a **rigid body** has **shape** and **size** and it is **not necessarily static** even if **the resultant force** acting on it is indeed **zero**.

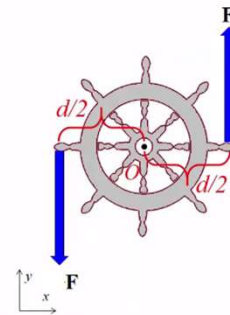
The two forces acting on this wheel are indeed in **equilibrium**.

$$\sum F_y = F + (-F) = 0$$

But this only means that they don't cause **translational motion**. We already learned that these two forces make a **couple moment**.

$$\begin{aligned}\sum M_O &= F \cdot \frac{d}{2} + F \cdot \frac{d}{2} \\ &= F \cdot d\end{aligned}$$

This moment causes **rotational effect** on this wheel.



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Therefore, for a **rigid body** to be **static**, it is not enough to only have **unbalanced force**, but the **resultant moment** summarized about **any arbitrary point** must be **Zero** as well.

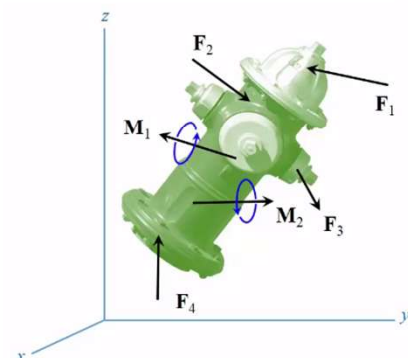
Otherwise, the object will rotate.

For a **rigid body** that is subjected to multiple forces and couple moments, **the first condition for equilibrium is:**

$$\mathbf{F}_R = \sum \mathbf{F} = \mathbf{0}$$

Then, **the resultant moment** summarized about any point, must also be **zero**, includes both the **total moment** caused **by the forces** and the **total couple moments**.

$$\mathbf{M}_{R,O} = \sum \mathbf{M}_{F,O} + \sum \mathbf{M} = \mathbf{0}$$



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Conditions for rigid body equilibrium

As a summary, for rigid body equilibrium, we can have **two vector equations**, one for force and one for moment.

$$\begin{cases} \mathbf{F}_R = \sum \mathbf{F} = \mathbf{0} \\ \mathbf{M}_{R,O} = \sum \mathbf{M}_{F,O} + \sum \mathbf{M} = \mathbf{0} \end{cases}$$

For a 2D problem, based on one free body diagram, we can write a maximum of **3 independent scalar equations** and then solve for **3 unknowns**.

2-D problems:

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_O = 0 \end{cases}$$

2-D problems:

$$\begin{cases} \sum F_x = 0 \\ \sum M_A = 0 \\ \sum M_B = 0 \end{cases}$$

2-D problems:

$$\begin{cases} \sum M_A = 0 \\ \sum M_B = 0 \\ \sum M_C = 0 \end{cases}$$

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Conditions for rigid body equilibrium

As a summary, for rigid body equilibrium, we can have **two vector equations**, one for force and one for moment.

$$\begin{cases} \mathbf{F}_R = \sum \mathbf{F} = \mathbf{0} \\ \mathbf{M}_{R,O} = \sum \mathbf{M}_{F,O} + \sum \mathbf{M} = \mathbf{0} \end{cases}$$

For a 3D problem, based on one free body diagram, we can write a maximum of **6 independent scalar equations** and solve for a maximum of **6 unknowns**.

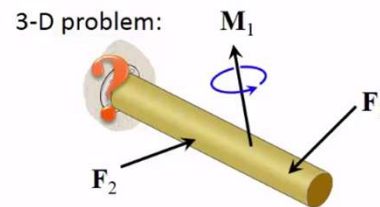
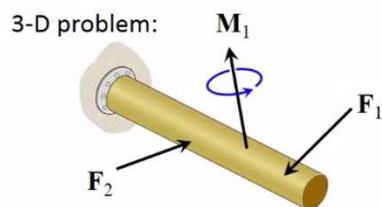
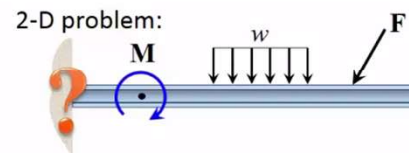
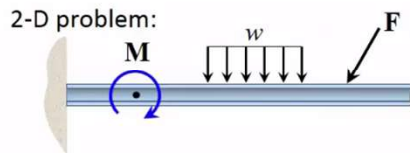
3-D problems:

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases} \quad \begin{cases} \sum M_x = 0 \\ \sum M_y = 0 \\ \sum M_z = 0 \end{cases}$$

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Here are **two examples** of rigid body equilibrium problems. Normally the **applied loadings** are **known**, and we will need to use **the equilibrium equations** to find the **unknown support reactions**.

The support reactions are also external force or moment acting on the body.



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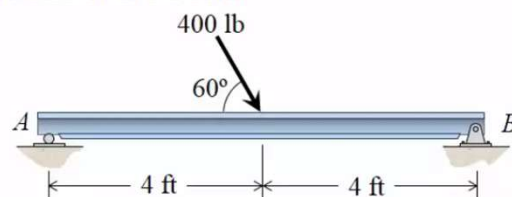
Rigid body equilibrium: 2D Supports

Engineering Mechanics: Statics**Objectives :**

- To explain how to identify 2D support reactions.
- To demonstrate how to solve 2D rigid body equilibrium problems.

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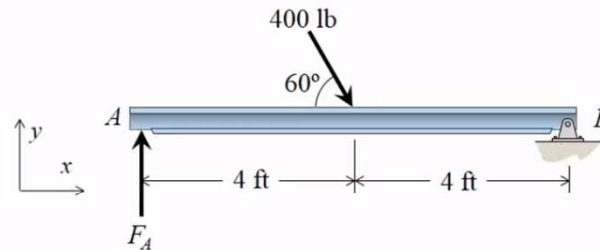
Example 3: Determine the support reactions at the roller at A and the pin at B . Neglect the weight and size of the beam.



We need to determine the support reactions at the **roller** and the **pin**.

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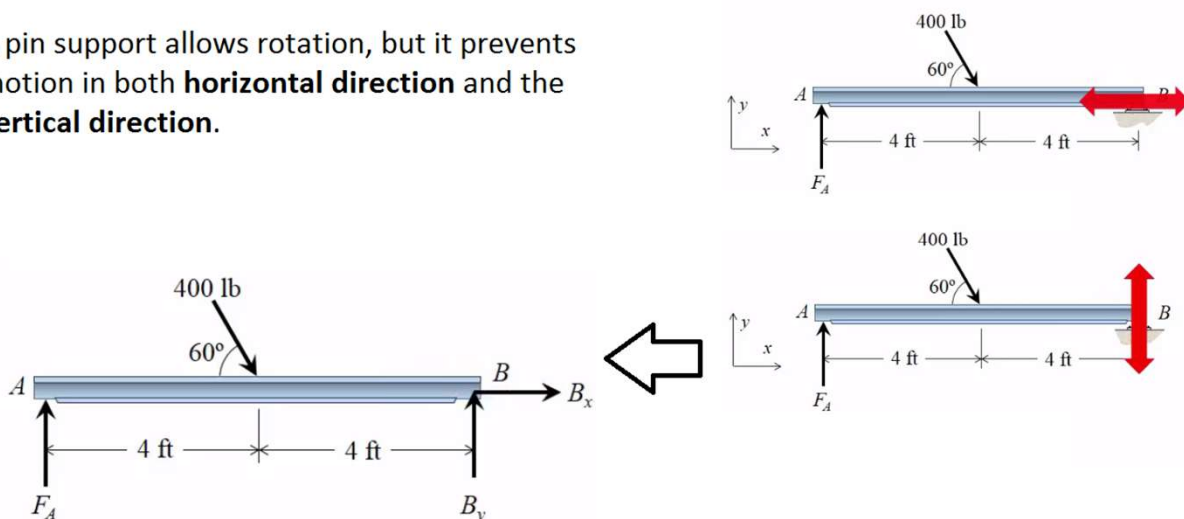
Free body diagram (FBD)



A **roller is similar to a rocker**, or a **simple contact** support. It exerts a force that is perpendicular to the contacting surface.

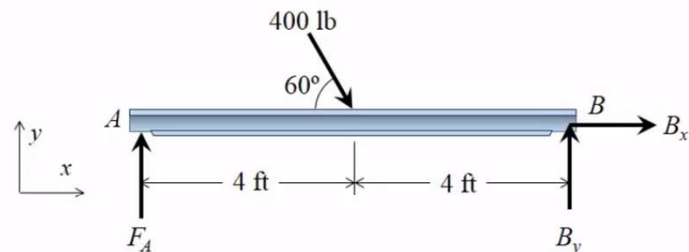
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A pin support allows rotation, but it prevents motion in both **horizontal direction** and the **vertical direction**.



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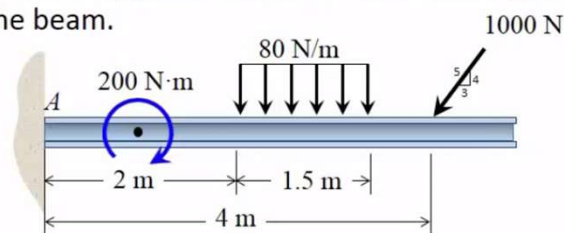
Free body diagram (FBD)



$$\begin{aligned} \sum F_x &= 400 \text{ lb} \cdot \cos 60^\circ + B_x = 0 \\ \sum F_y &= F_A - 400 \text{ lb} \cdot \sin 60^\circ + B_y = 0 \\ \sum M_B &= -F_A \cdot 8 \text{ ft} + 400 \text{ lb} \cdot \sin 60^\circ \cdot 4 \text{ ft} = 0 \end{aligned} \quad \therefore \begin{cases} F_A = 173 \text{ lb} \\ B_x = -200 \text{ lb} \\ B_y = 173 \text{ lb} \end{cases}$$

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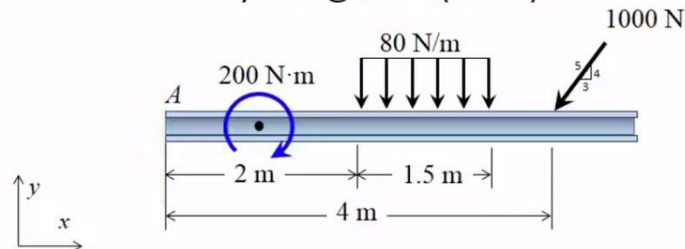
Example 1: Determine the support reactions at the fixed support, A. Neglect the weight and size of the beam.



The applied loadings acting on this beam is known and we need to determine **the support reactions** at the fixed support A.

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Free body diagram (FBD)

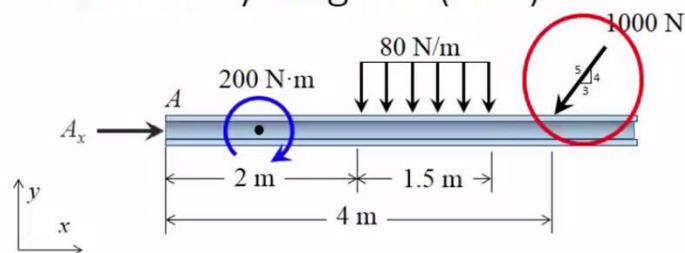


- 1) We need to first isolate our object. Therefore, the wall needs to disappear.
- 2) We need to note **all external forces** and moments on the diagram.

First, we need to be clear that the body is in equilibrium, in other words, **it is not moving**.

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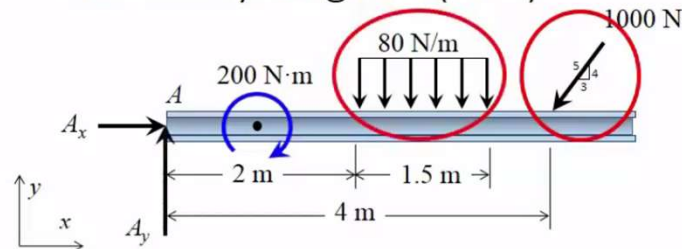
Free body diagram (FBD)



We see that there is a **horizontal component** of this force that will **push** the member to the **left**. Therefore, the wall must exert a horizontal force to the **right** to prevent this motion to the **left**.

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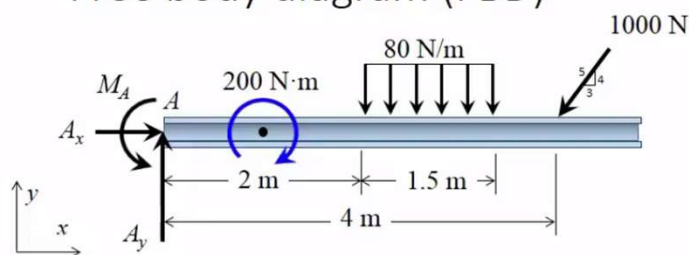
Free body diagram (FBD)



Because of the vertical component of the **concentrated force**, as well as **the distributed force**, causing the member **to go down**. Therefore, the wall must exert **a vertical force up** in order to prevent this downward motion.

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Free body diagram (FBD)

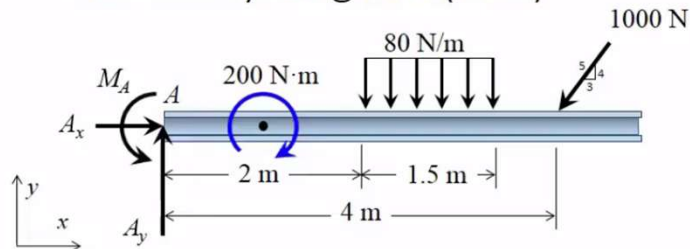


Finally, the forces and **the moment** all cause the member to **rotate clockwise**. Therefore, the wall also must exert **a moment support** that is **counterclockwise**, in order to prevent this **rotational motion**.

Since this is a **2D problem**, all motions can only occur within the **xy plane**, therefore there can not be any other type of motion, and the wall **will not provide** any other type of **support reactions**.

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Free body diagram (FBD)

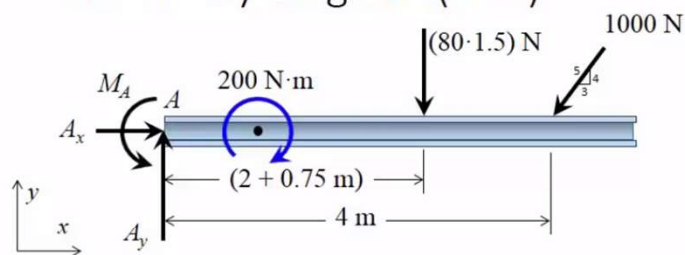


We have 3 unknowns: A_x , A_y and M_A .

We can write the three equilibrium equations to solve for all of them.

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Free body diagram (FBD)



$$\begin{aligned} \sum F_x &= A_x - \frac{3}{5} \cdot 1000\text{N} = 0 \\ \sum F_y &= A_y - 80 \cdot 1.5\text{N} - \frac{4}{5} \cdot 1000\text{N} = 0 \\ \sum M_A &= M_A - 200\text{N} \cdot \text{m} - 80 \cdot 1.5\text{N} \cdot 2.75\text{m} \\ &\quad - \frac{4}{5} \cdot 1000\text{N} \cdot 4\text{m} = 0 \end{aligned} \quad \therefore \begin{cases} A_x = 600\text{ N} \\ A_y = 920\text{ N} \\ M_A = 3730\text{ N} \cdot \text{m} \end{cases}$$

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Identify all the support reactions

- If a support prevents **translational** effect in a direction – exerts a **force**.
- If a support prevents **rotational** effect about an axis – exerts a **couple moment**.