

التمرين 05:

$$01) \int_0^1 \left(\frac{1}{x^2+1} - \frac{1}{\sqrt{1-x^2}} \right) dx = (\arctan x - \arcsin x)_0^1 \\ = \frac{\pi}{4} - \frac{\pi}{2} = -\frac{\pi}{4}.$$

$$02) \int_0^{\frac{\pi}{3}} (\tan x) dx = (-\ln|\cos x|)_0^{\frac{\pi}{3}} = \ln 2.$$

$$03) \int_0^{\pi} \left(\frac{\sin x}{1+\cos^2 x} \right) dx, \quad \left(\begin{array}{l} \cos x = t \Rightarrow -(\sin x) dx = dt \\ x = 0 \rightarrow t = 1, x = \pi \rightarrow t = -1 \end{array} \right)$$

↓

$$\int_0^{\pi} \left(\frac{\sin x}{1+\cos^2 x} \right) dx = \int_1^{-1} \left(\frac{-1}{1+t^2} \right) dt \\ = \int_{-1}^1 \left(\frac{1}{1+t^2} \right) dt = (\arctan x)_{-1}^1 = \frac{\pi}{4} - \left(-\frac{\pi}{4} \right) = \frac{\pi}{2}.$$

$$04) \int_1^e \left(\frac{\ln^3 x}{x} \right) dx = \left(\frac{\ln^4 x}{4} \right)_1^e = \frac{1}{4}.$$