

التمرين 05:

$$\begin{aligned} 01) \int_0^1 \left(\frac{1}{x^2 + 1} - \frac{1}{\sqrt{1-x^2}} \right) dx &= (\arctan x - \arcsin x)_0^1 \\ &= \frac{\pi}{4} - \frac{\pi}{2} = -\frac{\pi}{4}. \end{aligned}$$

$$02) \int_0^{\frac{\pi}{3}} (\tan x) dx = (-\ln |\cos x|)_0^{\frac{\pi}{3}} = \ln 2.$$

$$\begin{aligned} 03) \int_0^{\frac{\pi}{2}} \left(\frac{\sin x}{1 + \cos^2 x} \right) dx &\quad , \left(\begin{array}{l} \cos x = t \Rightarrow -(\sin x) dx = dt \\ x = 0 \rightarrow t = 1 , x = \pi \rightarrow t = -1 \end{array} \right) \\ &\quad \Downarrow \\ \int_0^{\frac{\pi}{2}} \left(\frac{\sin x}{1 + \cos^2 x} \right) dx &= \int_1^{-1} \left(\frac{-1}{1 + t^2} \right) dt \\ &= \int_{-1}^1 \left(\frac{1}{1 + t^2} \right) dt = (\arctan x)_{-1}^1 = \frac{\pi}{4} - \left(-\frac{\pi}{4} \right) = \frac{\pi}{2}. \end{aligned}$$

$$04) \int_1^e \left(\frac{\ln^3 x}{x} \right) dx = \left(\frac{\ln^4 x}{4} \right)_1^e = \frac{1}{4}.$$