

$$\begin{aligned} \int \cos^3 x \sin^2 x dx &= \int \sin^2 x (1 - \sin^2 x) \cos x dx \\ &= \int t^2 (1 - t^2) dt = \frac{t^3}{3} - \frac{t^5}{5} + c \\ &= -\frac{1}{5} \sin^5 x + \frac{1}{3} \sin^3 x + c \end{aligned}$$

$$4) \int \frac{1}{x^2 + 4} dx = \frac{1}{4} \int \frac{1}{\left(\frac{x}{2}\right)^2 + 1} dx = \frac{1}{2} \arctan\left(\frac{x}{2}\right) + c$$

$$5) \int \frac{(\arctan^3 x)}{1+x^2} dx = \int t^3 dt, \quad \begin{cases} t = \arctan x \\ dt = \frac{dx}{1+x^2} \end{cases}$$

$$= \frac{t^4}{4} + c = \frac{\arctan^4 x}{4} + c.$$

$$6) \int \frac{1}{1+e^x} dx = \int \left(1 - \frac{e^x}{1+e^x}\right) dx = x - \ln(1+e^x) + c.$$

مكاملة بالتجزئة: $[\int uv' = uv - \int u'v]$

التمرين الثالث

$$1) \int (\arctan x) dx = x \arctan x - \int \frac{x}{1+x^2} dx$$

$$= x \arctan x - \frac{1}{2} \ln(1+x^2) + c, \quad \begin{cases} u = \arctan x, v' = 1 \\ u' = \frac{1}{1+x^2}, v = x \end{cases}$$

$$2) \int x \ln x dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + c, \quad \begin{cases} u = \ln x, v' = x \\ u' = \frac{1}{x}, v = \frac{x^2}{2} \end{cases}$$