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Chapter 1

Real functions (Fonctions réelles)

Lesson objective:

1. How do you determine the domain of definition of a function; (Comment déterminer le domaine de définition d'une fonction);
2. How do you find Limits (Comment Calculer les limites);
3. How do you study the continuity and differentiability of a function (Comment étudier la continuité et la dérivabilité d'une fonction).

1.1 Definitions and properties

Definition 1 *real function (fonction réelle): Let $I \subset \mathbb{R}$ et $J \subset \mathbb{R}$. We call real function, denoted f, any application*

$$f : I \to J$$

$$x \to f(x),$$

where I is the starting set (the antecedents) and J the arriving set (the images).

Remark 2 *1. The function f denotes the function;*

2. The real variable $x \in D_f$ is the antecedent;

3. The real number $f(x)$ is the image of x by f.

Definition 3 *Domaine de définition (domain of function): Let $I \subset \mathbb{R}$ and $J \subset \mathbb{R}$. The set of elements of I which have exactly one image in J by the function f is called the domain of definition of I. We denote it D_f :*

$$D_f = \{x \in \mathbb{R} \diagup f(x) \text{ is defined}\}$$

1.1.1 Examples (exemples)

Polynomial function (Fonction polynomiale): The general form of this function is:

$$f(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$$

The domain of definition of polynomial functions is $\mathbb{R} =]-\infty, +\infty[$, as examples:

$$f(x) = x^2,\ g(x) = 3 + 5x + 2x^2, ...$$

where $f(x)$ and $g(x)$ are second degree polynomials and $D_f = \mathbb{R}$, $D_g = \mathbb{R}$.

Nth root function(Fonction racinen-ième): Let h a real fonction. The general form of root function is :

$$f(x) = h^{\frac{p}{q}} = \sqrt[q]{h(x)^p},$$

where $p \in \mathbb{Z}$ and $q \in \mathbb{N}^*$.

for $p=1$ and $q=2$ we obtain *square root function (la fonction racine carrée)*, for examples:

$$f(x)=\sqrt{x-1},\ f(x)=\sqrt{x^2+2x}.$$

The condition for f to be defined is that the function $h(x)$ is positive or zero, that's meaning: $D_f=\{x\in\mathbb{R}/h(x)\geq 0\}$.

The domain of the function $f(x)=\sqrt{x-1}$ is $D_f=\{x\in\mathbb{R}/x-1\geq 0\}$, so

$$x-1\geq 0\Rightarrow x\geq 1$$

then: $D_f=[1,+\infty[$.

for $p=1$ and $q=3$ we obtain *cube root function (fonction racine cubique)*, for example:

$$f(x)=\sqrt[3]{x-1},$$

The domain of *cube root function is* $\mathbb{R}=]-\infty,+\infty[$.

Rational function (Fonction rationnelle): If h and g are two real functions, then The general form of rational functions is given by:

$$f(x)=\frac{h(x)}{g(x)},\ g(x)\neq 0.$$

The domain of quotient function $f=\frac{h(x)}{g(x)}$ is the intersection of the domains of $h(x)$ and $g(x)$.

Remark 4 *If h is a polynomial function, then for f to be defined it is necessary that* $g(x)\neq 0$.

Example 5 *1. Let f_1 a function defined by:*

$$f_1(x)=\frac{h_1(x)}{g_1(x)}=\frac{1}{x^2-1},$$

so: f_1 *is défined*$\Leftrightarrow g_1(x) = x^2 - 1 \neq 0$, *or* $D_{f_1} = \{x \in \mathbb{R}/x^2 - 1 \neq 0\}$, *the solution of the equation* $x^2 - 1 = 0$ *gives us the points which do not belong to* D_{f_1}. *So we have*

$$x^2 - 1 = (x-1)(x+1)$$

and

$$(x-1)(x+1) = 0 \Rightarrow \begin{cases} (x-1) = 0 \\ ou \ (x+1) = 0 \end{cases}$$

then, $x \neq 1$ *et* $x \neq -1$

so

$$\begin{aligned} D_{f_1} &= \{x \in \mathbb{R}/x \neq -1 \ and \ x \neq 1\} \\ &= \mathbb{R} - \{-1, 1\} \\ &=]-\infty, -1[\cup]-1, 1[\cup]1, +\infty[\end{aligned}$$

2. *Let* f_2 *a function defined by:*

$$f_2(x) = \frac{h_2(x)}{g_2(x)} = \frac{x^2+1}{x^2+2x-1},$$

The function f_2 *is defined*$\Leftrightarrow g_2(x) = x^2 + 2x - 1 \neq 0$. *We search the solution of the equation* $x^2 + 2x - 1$, *so we calculate the determinant* Δ :

$$\begin{aligned} \Delta &= (2)^2 - 4(1)(-1) \\ &= 8 \end{aligned}$$

and $\Delta > 0$, *then we have two different solutions are:*

$$x_1 = \sqrt{2} - 1 \ and \ x_2 = -\sqrt{2} - 1$$

Then:

$$\begin{aligned} D_{f_2} &= \mathbb{R} - \left\{-\sqrt{2}-1, \sqrt{2}-1\right\} \\ &= \left]-\infty, -\sqrt{2}-1\right[\cup \left]-\sqrt{2}-1, \sqrt{2}-1\right[\cup \left]\sqrt{2}-1, +\infty\right[\end{aligned}$$

3. *Let f_3 a function defined by:*

$$f_3(x) = \frac{h_3(x)}{g_3(x)} = \frac{\sqrt{x-1}}{x^2+2x-1}.$$

We observe that the function $h_3(x)$ is a square root function then to find the domain of f_3 it is necessary to add the condition that $h_3(x) \geq 0$ and the intersection with $g_3(x) \neq 0$ gives us D_{f_3}.

$D_{f_3} = \left\{x \in \mathbb{R}/x-1 \geq 0 \ et \ x^2+2x-1 \neq 0\right\}$, *and we have:*

$D_{h_3} = [1, +\infty[\ et \ D_{g_3} = \left]-\infty, -\sqrt{2}-1\right[\cup \left]-\sqrt{2}-1, \sqrt{2}-1\right[\cup \left]\sqrt{2}-1, +\infty\right[.$

then the intersection gives us: $D_{f_3} = [1, +\infty[$.

Logarithmic function(Fonction logarithme): The ln function is defined as follows:

$$\ln : \mathbb{R}_+^* \to \mathbb{R}$$

$$x \to \ln(x)$$

therefore the domain of the function ln is $\mathbb{R}_+^* =]0, +\infty[$

Exponential function(**Fonction exponentielle**): exponential function exp is defined as following :

$$\exp : \mathbb{R} \to \mathbb{R}$$

$$x \to \exp(x)$$

and the domain of the exponential function is $\mathbb{R}$.

Sinus and Cosinus functions(Fonctions sinus et cosinus): The sinus function is defined as follows :

$$\sin : \mathbb{R} \to \mathbb{R}$$

$$x \mapsto \sin(x)$$

and its domain is $\mathbb{R}$

The sinus function is defined as follows:

$$\cos : \mathbb{R} \to \mathbb{R}$$

$$x \mapsto \cos(x)$$

and its domain is $\mathbb{R}$.

The functions sin and cos are bounded functions, i.e.

$$\text{for } x \in \mathbb{R}, -1 \leq \sin(x) \leq 1 \text{ and } -1 \leq \cos(x) \leq 1.$$

1.1.2 Composition of two functions:

Let : $f : D_1 \to \mathbb{R}$ and $g : D_2 \to \mathbb{R}$, then the function $f \circ g$ is defined by:

$$D_2 \to \mathbb{R}$$

$$(f \circ g)(x) = f(g(x))$$

Example 6 *1. Let:* $f(x) = \sin(x)$ *and* $g(x) = x^2 + 2x - 1$ *then* $(f \circ g)(x) = f(g(x)) = \sin(x^2 + 2x - 1)$.

2. *Let:* $f(x) = \ln(x)$ *and* $g(x) = \frac{1}{x}$ *then* $(f \circ g)(x) = f(g(x)) = \ln(\frac{1}{x})$. *the domain of* f *is* $\mathbb{R}_+^* =]0, +\infty[$ *and the domain of* g *is* $\mathbb{R}^*$, *then the domaine of* $f(g(x))$ *is* $\mathbb{R}_+^*$.

Definition 7 *graphical representation (Représentation graphique): Let* f *a real fonction, the graph of* f, *noté* $G(f)$, *is defined by:*

$$G(f) = \{(x, f(x), x \in D_f\}.$$

1.1.3 Function operations (Opérations sur les fonctions)

If f and g are two functions defined on the same interval $I \subset \mathbb{R}$, we then have the following results:

1. **Sum (Somme)**:The sum function $f + g$ is defined for any real x of the interval I by:

$$(f + g)(x) = f(x) + g(x).$$

2. **Produc**t (**Produit):**the product function fg is defined for all real x of the interval I by:

$$(fg)(x) = f(x)g(x).$$

3. **Quotiont:** when the function g does not equal 0 on the interval I, the quotient function f/g is defined for any real x of I by:

$$(\frac{f}{g})(x) = \frac{f(x)}{g(x)},\ g(x) \neq 0.$$

1.2 limit of a function (Limite d'une fonction)

Let $f :]a, b[\to \mathbb{R}$ and let $x_0 \in]a, b[$ where $x_0 \notin]a, b[$.

1. We say that $f(x)$ tends to a real limit l as x tends to x_0 on the left if:

$$\lim_{\substack{x \to x_0 \\ <}} f(x) = l$$

2. We say that $f(x)$ tends to a real limit l as x approaches x_0 on the right if:

$$\lim_{\substack{x \to x_0 \\ >}} f(x) = l'$$

3. If we have

$$\lim_{\substack{x \to x_0 \\ <}} f(x) = l = \lim_{\substack{x \to x_0 \\ >}} f(x)$$

Then we say that l is the limit of f at the point x_0.

We say that f tends to an infinite limit $+\infty(-\infty)$ when x approaches x_0 on the left and on the right if:

$$\lim_{\substack{x \to x_0 \\ <}} f(x) = \lim_{\substack{x \to x_0 \\ >}} f(x) = \begin{cases} +\infty \\ \\ or -\infty \end{cases}.$$

Sometimes, in the calculations of the limits we find forms called indeterminate forms (IF) when calculating the limits, these forms are:

$$+\infty - \infty,\ \frac{\infty}{\infty},\ \frac{0}{0},\ \frac{\infty}{0}, 0\infty,\ 0^{\infty}.$$

To remove the indeterminate form (IF) in the calculations of the limits, we use the following methods:

1.2.1 Method 01: Factoring higher degree polynomials (Factoriser le terme de plus haut degré)

We use this method when we have an indeterminate form of the type $(+\infty - \infty)$ for a polynomial function or $\left(\frac{\infty}{\infty}\right)$ for a rational function. This method consists of putting the highest degree term into a factor and if we obtain a fraction we simplify as much as possible.

Example 8 *1. We want to calculate* $\lim_{x\to\infty}(x^2-2x+1)$*, we have:* $\lim_{x\to\infty}(x^2-2x+1) = +\infty-\infty$*, so it's an FI of the type* $+\infty-\infty$*. To remove the FI we put* x^2 *as a factor, then:*

$$x^2-2x+1 = x^2(1-\frac{2}{x}+\frac{1}{x^2})$$

and

$$\lim_{x\to\infty} x^2 = +\infty$$
$$\lim_{x\to\infty}(\frac{2}{x}+\frac{1}{x^2}) = 0$$

by product:

$$\lim_{x\to\infty} x^2(1-\frac{2}{x}+\frac{1}{x^2}) = \infty.1 = \infty$$

2. We want to calculate $\lim_{x\to\infty}(\frac{x^2-2x+1}{x+1})$*, we have:* $\lim_{x\to\infty}(\frac{x^2-2x+1}{x+1}) = \frac{\infty}{\infty}$*, so it's an FI of the type* $\frac{\infty}{\infty}$*. To remove the FI we put the highest degree in the numerator and divide it by the highest degree in the denominator, then:*

$$\lim_{x\to\infty}(\frac{x^2-2x+1}{x+1}) = \lim_{x\to\infty}(\frac{x^2}{x}) = \lim_{x\to\infty}(x) = +\infty$$

1.2.2 Method 02: conjugate multiplication technique (Multiplier par l'expression conjuguée)

This method used when we have an indeterminate form of the type $(+\infty-\infty)$ in an expression with square roots $(\sqrt{A(x)}-\sqrt{B(x)})$. To remove the FI in this type we multiply and divide by the conjugate expression of $\left(\sqrt{A(x)}-\sqrt{B(x)}\right)$, it is $\left(\sqrt{A(x)}+\sqrt{B(x)}\right)$.

Example 9 *We want to calculate:* $\lim\limits_{x\to\infty}\left(\sqrt{x}-\sqrt{x+3}\right)$, *wz have:* $\lim\limits_{x\to\infty}\left(\sqrt{x}\right)=+\infty$ *and* $\lim\limits_{x\to\infty}\left(\sqrt{x+3}\right)=+\infty$ *but the sum of these two limits equals* $+\infty-\infty$.

so we multiply and divide by: $\sqrt{x}+\sqrt{x+3}$, *we obtain:*

$$\begin{aligned}\frac{\left(\sqrt{x}-\sqrt{x+3}\right)\left(\sqrt{x}+\sqrt{x+3}\right)}{\left(\sqrt{x}+\sqrt{x+3}\right)} &= \frac{x-x+3}{\sqrt{x}+\sqrt{x+3}}\\ &= \frac{3}{\sqrt{x}+\sqrt{x+3}}\end{aligned}$$

so

$$\begin{aligned}\lim_{x\to\infty}\left(\sqrt{x}-\sqrt{x+3}\right) &= \lim_{x\to\infty}\left(\frac{3}{\sqrt{x}+\sqrt{x+3}}\right)\\ &= \frac{3}{+\infty}\\ &= 0.\end{aligned}$$

1.2.3 Method 03: Comparaison (La comparaison)

This method consists of comparing between two functions. We summarize this method as follows:

Let considred two functions f and g defined on the interval I in $\mathbb{R}$,

1. If $f(x)\leq g(x)$ and $\lim\limits_{x\to x_0} g(x)=-\infty$,then

$$\lim_{x\to x_0} f(x)=-\infty,$$

2. If $g(x)\leq f(x)$ and $\lim\limits_{x\to x_0} g(x)=+\infty$,then

3. Let f, g and h three fonctions defined on the interval I in $\mathbb{R}$, if $h(x)\leq f(x)\leq g(x)$ and $\lim\limits_{x\to x_0} g(x)=\lim\limits_{x\to x_0} h(x)=l$, then

$$\lim_{x\to x_0} f(x)=l.$$

Example 10 *Let considred a function f: $f(x) = x^2\sin(x) - 3x^2$. We want to calculate* $\lim_{x\to+\infty}\left(x^2\sin(x) - 3x^2\right)$

We know that:

$$-1 \leq \sin(x) \leq 1$$

and $x^2 \geq 0$, so

$$x^2\sin(x) \leq x^2$$

$$x^2\sin(x) - 3x^2 \leq x^2 - 3x^2$$

$$x^2\sin(x) - 3x^2 \leq -2x^2$$

and $\lim_{x\to\infty}(-2x^2) = -\infty$, *We conclude that :* $\lim_{x\to 0}\left(x^2\sin(x)\right) = -\infty$.

1.2.4 Method 04: Derivation method (Méthode de dérivation)

This method used when we have an indeterminate form of the type($\frac{+\infty}{+\infty}, \frac{-\infty}{-\infty}$ et $\frac{0}{0}$). This method consists of using the derivative of the function (Hospital rule). We summarize this method as follows:

Let f and g two functions differentiable on an interval $I =]a, b[$ of $\mathbb{R}$ and $g'(x) \neq 0$, then

1. $$\text{if } \lim_{x\to x_0} f(x) = \lim_{x\to x_0} g(x) = 0 \text{ and } \lim_{x\to x_0}\frac{f'(x)}{g'(x)} = l \text{ then } \lim_{x\to x_0}\frac{f(x)}{g(x)} = l$$

2. $$\text{if } \lim_{x\to x_0} f(x) = \lim_{x\to x_0} g(x) = +\infty \text{ and } \lim_{x\to x_0}\frac{f'(x)}{g'(x)} = l \text{ then } \lim_{x\to x_0}\frac{f(x)}{g(x)} = l$$

Remark 11 *Sometimes you will have to use the hospital rule several times.*

Example 12 *1. Let h a function defined by:*

$$h(x) = \frac{\sqrt{x}}{\ln(x)},$$

we calculate $\lim_{x\to+\infty}\left(\frac{\sqrt{x}}{\ln(x)}\right) = \frac{+\infty}{+\infty}$, *so we use the hospital rule and we obtain*

$$\lim_{x\to+\infty}\left(\frac{\frac{1}{2\sqrt{x}}}{\frac{1}{x}}\right) = \lim_{x\to+\infty}\left(\frac{\sqrt{x}}{2}\right) = +\infty.$$

1.2.5 Properties of Limits

If L, M, c, and k are real numbers and

$$\lim_{x\to c} f(x) = L \text{ and } \lim_{x\to c} g(x) = M, \text{ then}$$

Sum Rule: The limit of the sum of two functions is the sum of their limits

$$\lim_{x\to c}(f(x) + g(x)) = L + M.$$

Difference Rule: The limit of the difference of two functions is the difference of their limits

$$\lim_{x\to c}(f(x) - g(x)) = L - M.$$

Product Rule: The limit of a product of two functions is the product of their limits.

$$\lim_{x\to c}(f(x).g(x)) = LM.$$

Constant Multiple Rule: The limit of a constant times a function is the constant times the limit of the function

$$\lim_{x\to c}(kf(x)) = kL.$$

Quotient Rule: The limit of a quotient of two functions is the quotient of their limits, provided the limit of the denominator is not zero

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M}, M \neq 0.$$

1.3 Continuity (Continuité)

Definition 13 *Continuity left and right (Continuité à gauche et à droite): Let f a function defined on un intervalle I $(I \subset \mathbb{R})$. Let x_0 a point of I.*

-The function f is continuous to the left of x_0 if and only if

$$\lim_{x \overset{<}{\to} x_0} f(x) = f(x_0).$$

-The function f is continuous to the right of x_0 if and only if

$$\lim_{x \overset{>}{\to} x_0} f(x) = f(x_0).$$

-The function f is continuous at a point x_0 if and only if

$$\lim_{x \overset{<}{\to} x_0} f(x) = \lim_{x \overset{>}{\to} x_0} f(x) = f(x_0).$$

Definition 14 *Continuity (Continuité): Let f be a function defined on an interval I $(I \subset \mathbb{R})$. We say that the function f is continuous in I if and only if f is continuous at each point of I.*

Remark 15 *-If $\lim_{x \overset{<}{\to} x_0} f(x) \neq \lim_{x \overset{>}{\to} x_0} f(x) \neq f(x_0)$ then the function f is discontinuous in x_0.*

-If $\lim_{x \overset{<}{\to} x_0} f(x) \neq \lim_{x \overset{>}{\to} x_0} f(x)$ and $\lim_{x \overset{<}{\to} x_0} f(x) = f(x_0)$ then f iscontinuous to the left of x_0.

-If $\lim_{x \overset{>}{\to} x_0} f(x) = f(x_0)$ then f is continuous to the right of x_0.

Example 16 *-Polynomial functions are continuous at every point in* $\mathbb{R}$*.*

-Rational fractions functions are continuous where they are defined.

-Let f *a function defined by:* $f(x) = \frac{x+2}{x}$ *and* $D_f = \mathbb{R} - \{0\}$*. The function* f *is continuous at* $x_0 = 1$ *because:*

$$\lim_{\substack{< \\ x \to 1}} \frac{x+2}{x} = \lim_{\substack{> \\ x \to 1}} \frac{x+2}{x} = f(1) = 3.$$

-Let g *a function defined by:* $g(x) = \begin{cases} \frac{|x|}{x} \text{ if } x \neq 0 \\ 1 \text{ if } x = 0 \end{cases}$, $D_f = \mathbb{R}$.

The fonction g *is dicontinuous at* $x_0 = 0$ *because:*

$$\lim_{\substack{< \\ x \to 0}} \frac{|x|}{x} = \lim_{\substack{< \\ x \to 0}} \frac{-x}{x} = -1$$
$$\lim_{\substack{> \\ x \to 0}} \frac{|x|}{x} = \lim_{\substack{> \\ x \to 0}} \frac{x}{x} = 1$$

but it is continuous to the right because: $\lim_{\substack{> \\ x \to 0}} \frac{|x|}{x} = \lim_{\substack{> \\ x \to 0}} \frac{x}{x} = 1 = g(0)$.

1.3.1 Extension by continuity (Prolongement par continuité)

Let f a function defined on interval $I - \{x_0\}$. If $\lim_{x \to x_0} f(x) = l$ (existe) then the function $\tilde{f}$ défined on I

$$\tilde{f} = \begin{cases} f(x) \text{ if } x \neq x_0 \\ l \text{ if } x = x_0 \end{cases}$$

is called the extension by continuity from f to x_0. The function f is then continuous in x_0.

Example 17 *Let* f *defined by:*

$$f(x) = \frac{\sin(x)}{x}$$

The domain $D_f = \mathbb{R} - \{0\}$ *and the function* f *is continuous in* D_f *but* $\lim\limits_{x \to 0} \frac{\sin(x)}{x} = 1$.

So we can extend f *to* $x_0 = 0$

$$\tilde{f} = \begin{cases} \frac{\sin(x)}{x} \text{ if } x \neq 0 \\ \\ 1 \text{ if } x = 0 \end{cases}.$$

1.4 Differentiability (Dérivabilité)

Definition 18 *Differentiability left and right (Dérivée sur à droite et à gauche): Let* f *a function defined on interval* I *in* R. *Let* a *a point of* I.

-The function f *is differentiable in left at a point* x_0 *if and only if*

$$\lim_{\substack{< \\ x \to x_0}} \frac{f(x) - f(x_0)}{x - x_0} = l.$$

-The function f is differentiable in right at a point x_0 *if and only if*

$$\lim_{\substack{> \\ x \to x_0}} \frac{f(x) - f(x_0)}{x - x_0} = l'$$

-The function f is differentiable at a point x_0 *if and only if*

$$\lim_{\substack{< \\ x \to x_0}} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{\substack{> \\ x \to a}} \frac{f(x) - f(x_0)}{x - x_0} = f'(x_0).$$

This number $f\prime(x_0)$ *is called derivative of* f *at* x_0.

Definition 19 *derivative on* I *(Dérivée sur* I*): We say that* f *is differentiable on* I *if, for all* x *of* I, f *is differentiable on* x. *This function is called the derivative of* f, *denoted* $f\prime$.

Example 20 *-Polynomial functions are differentiable at any point in* $\mathbb{R}$.

-Rational fractions are differentiable where they are defined.

-Let f a function definie by: $f(x) = \frac{x+2}{x}$ and $D_f = \mathbb{R} - \{0\}$. the function f is differentiable at $x_0 = 1$ because:

$$\lim_{\substack{< \\ x\to 1}} \frac{\frac{x+2}{x} - 3}{x-1} = \lim_{\substack{< \\ x\to 1}} \frac{\frac{-2x+2}{x}}{x-1} = \lim_{\substack{< \\ x\to 1}} \frac{-2x+2}{x(x-1)} = \frac{0}{0}(IF)$$

we use hospital rule:

$$\lim_{\substack{< \\ x\to 1}} \frac{-2x+2}{x(x-1)} = \lim_{\substack{< \\ x\to 1}} \frac{-2}{2x-1} = -2$$

also

$$\lim_{\substack{> \\ x\to 1}} \frac{-2x+2}{x(x-1)} = \lim \frac{-2}{\underset{\substack{> \\ x\to 1}}{2x-1}} = -2$$

then f is differentiable at $x_0 = 1$.

1.4.1 Properties (Propriétés)

Let f and g two fonctions differentiable on I.then we have:

1. for α and $\beta \in \mathbb{R}$: $(\alpha f + \beta g)' = \alpha f' + \beta g'$linearity(linéarité)

$$(2\cos(x) + 3\ln(x))' = -2\sin(x) + 3(\frac{1}{x}) = -2\sin(x) + \frac{3}{x}.$$

2. **Product derivative (Dérivé de produit)**:

$$(fg)' = f'g + fg'$$

as

$$(x\ln(x))' = \ln(x) + x(\frac{1}{x}) = \ln(x) + 1$$

3. **Derivative of the quotient** :

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

we take an example:

$$\left(\frac{x}{x^2+2}\right)' = \frac{1(x^2+2) - x(2x)}{(x^2+2)^2} = \frac{-x^2+2}{(x^2+2)^2}$$

4. **Derivative of composition functions**:

$$(f(g(x)))' = g'(x)f'(g(x))$$

for example:

$$\big(\cos(x^2+2x)\big)' = -(2x+2)\sin(x^2+2x)$$

Derivatives of usual functions: They are presented in the following table:

Function f	Derivative f'	Function f	Derivative f'
x^α	$\alpha x^{\alpha-1}$	$sin(x)$	$cos(x)$
$\ln(x)$	$\frac{1}{x}$	e^x	e^x
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	a^x	$ln(a)a^x$
$\frac{1}{x}$	$\frac{-1}{x^2}$	$\sqrt[n]{x}$	$\frac{1}{n\sqrt[n]{x^{n-1}}}$
$cos(x)$	$-sin(x)$	$u(x)^n$	$nu'(x)u^{n-1}$