Arch Analysis

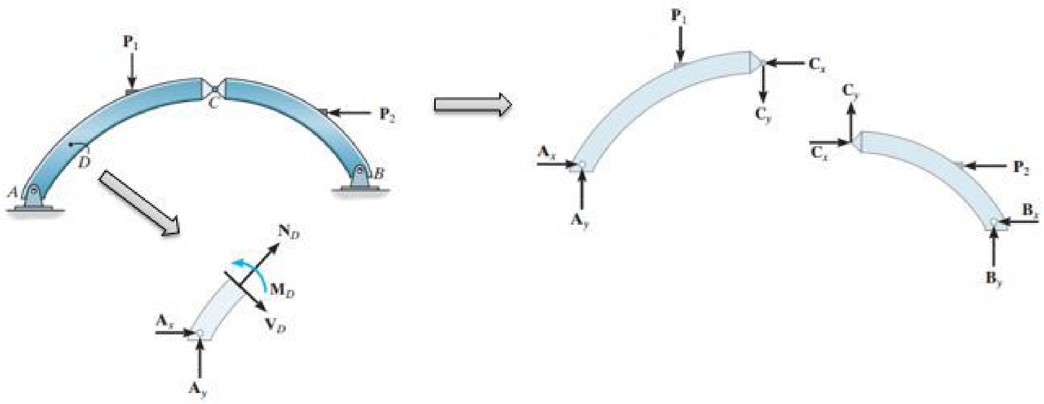
Table des matières

Three-Hinged Arch Analysis

I

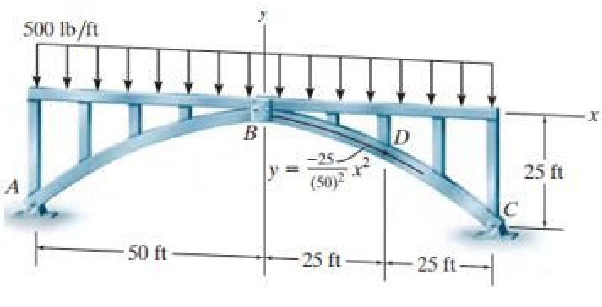
1. Three-Hinged Arch

Three-hinged arches are statically determinate and can be analyzed by separating the two members and applying the equations of equilibrium to each member.



Exemple

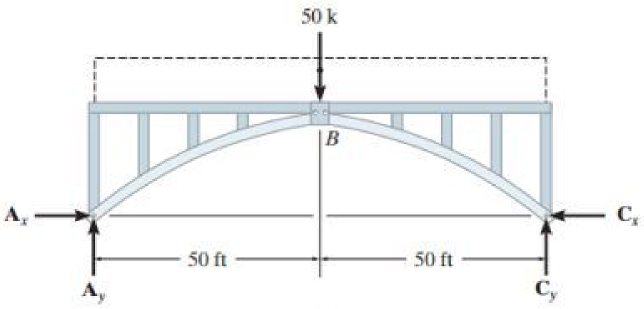
The three-hinged open-spandrel arch bridge like the one shown in the photo has a parabolic shape. If this arch were to support a uniform load and have the dimensions shown in Figure a, show that the arch is subjected only to axial compression at any intermediate point such as point D. Assume the load is uniformly transmitted to the arch ribs.



(a)

Solution:

Here the supports are at the same elevation. The free-body diagrams of the entire arch and part BC are shown in Figure b and Figure c. Applying the equations of equilibrium, we have:

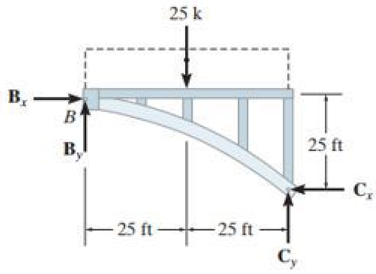


(b)

Entire arch:

$$\circlearrowleft+\Sigma M_A = 0; \qquad C_y(100\text{ ft}) - 50\text{ k}(50\text{ ft}) = 0$$
$$C_y = 25\text{ k}$$

Arch segment *BC*:



(c)

$$\circlearrowleft+\Sigma M_B = 0; \qquad -25\text{ k}(25\text{ ft}) + 25\text{ k}(50\text{ ft}) - C_x(25\text{ ft}) = 0$$
$$C_x = 25\text{ k}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 25\text{ k}$$

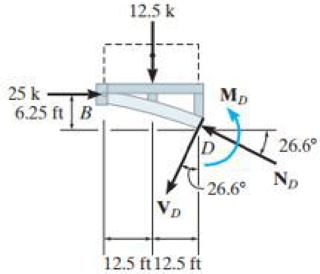
$$+\uparrow \Sigma F_y = 0; \qquad B_y - 25\text{ k} + 25\text{ k} = 0$$
$$B_y = 0$$

A section of the arch taken through point D, x = 25 ft, y = -25 (25)^2 / (50)^2 = -6.25 ft, is shown in Figure d. The slope of the segment at D is:

$$\tan\theta = \frac{dy}{dx} = \frac{-50}{(50)^2}x\Big|_{x=25\text{ ft}} = -0.5$$

$$\theta = -26.6^\circ$$

Applying the equations of equilibrium, Fig. 5–10d we have:



(d)

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 25\text{ k} - N_D\cos 26.6^\circ - V_D\sin 26.6^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad -12.5\text{ k} + N_D\sin 26.6^\circ - V_D\cos 26.6^\circ = 0$$

$$\circlearrowleft+\Sigma M_D = 0; \qquad M_D + 12.5\text{ k}(12.5\text{ ft}) - 25\text{ k}(6.25\text{ ft}) = 0$$

$$N_D = 28.0\text{ k}$$
$$V_D = 0$$
$$M_D = 0$$