

Exercise 1

A. Make truth table for the statements:

$$P \vee Q \Rightarrow P \wedge Q$$

$\overline{P \wedge Q}$ and $\overline{P} \vee \overline{Q}$ are logically.

B. Let P be a false assertion, Q a true assertion and R a false assertion. What are the assertions true?

- Q and (P or R)
- P or (Q and R)
- Non (P and Q and R)
- (P or Q) and (Q or R)

C. Give the negation of each statement:

$$\forall x \in R_+ \forall y \in R_+ \quad x + y > x.y$$

$$\forall x \in R \exists y \in R \quad x.y = 1$$

D. Which assertions are true?

- $\forall x \in R \quad x^2 - x \geq 0$
- $\forall n \in N \quad n^2 - n$
- $\exists x < 0 \quad \exp(x) < 0$
- $\exists x > 0 \quad \sqrt{x} = x$

Attention: $x^2 - x$ is negative for $x = \frac{1}{2}$ for example!

Exercise 2

Prove that:

1. if n positive integer with $n^2 > 100$ then $n > 10$.
2. $\sqrt{3}$ is irrational.
3. Prove using mathematical induction the following statement:

$$\forall n \in N^* \quad P(n) = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$