2014/2 ENGINEERING DEPARTMENTS PHYSICS 2

RECITATION 3

(Electric Potential)

1. **(a)** Find the work done by a displacement of +2q charge from A(0,6a) to B(3a,0).
   (b) Find the potential energy of the new system.

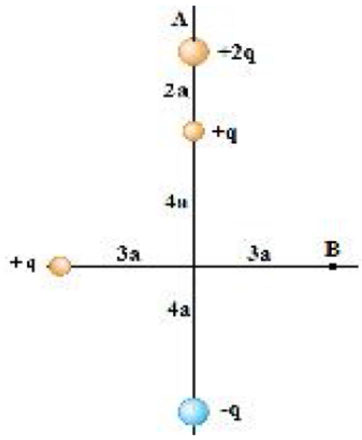
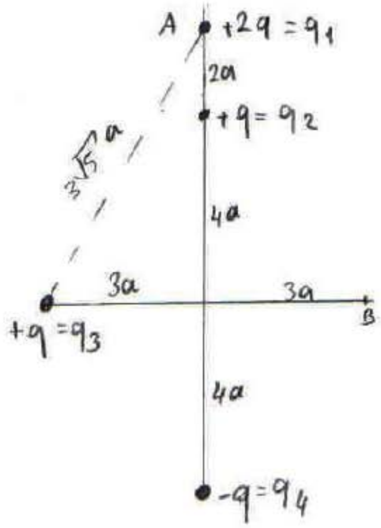


Figure 1



(a)

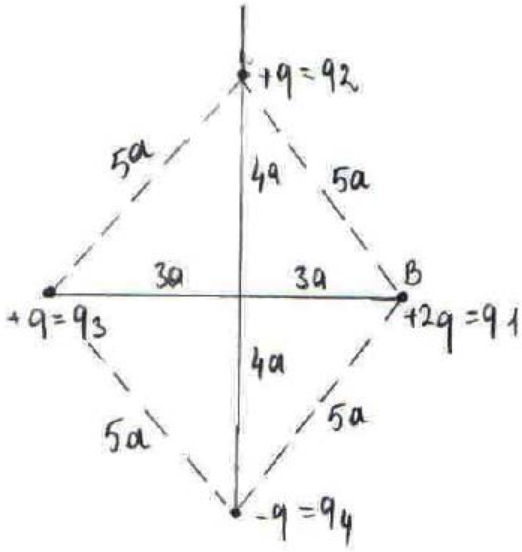
$$U_{12} = k\,\frac{q_1 q_2}{r_{12}}$$

$$U_A = k\,\frac{2q(q)}{2a} + k\,\frac{q(2q)}{3\sqrt{5}a} + k\,\frac{(-q)(2q)}{10a}$$

$$U_A = U_{21} + U_{31} + U_{41}$$

$$= k\,\frac{q^2}{a} + k\,\frac{2q^2}{3\sqrt{5}a} - k\,\frac{q^2}{5a}$$

$$= k\,\frac{q^2}{a}\left(\frac{4}{5} + \frac{2}{3\sqrt{5}}\right)$$



$$U_B = \underbrace{k\frac{2q(q)}{5a}}_{U_{21}} + \underbrace{k\frac{(2q)q}{6a}}_{U_{31}} + \underbrace{\frac{k(-q)(2q)}{5a}}_{U_{41}}$$

$$U_B = \frac{2kq^2}{5a} + k\frac{q^2}{3a} - \frac{2kq^2}{5a}$$

$$U_B = \frac{kq^2}{3a}$$

$$U_B - U_A = \frac{kq^2}{a}\left(\frac{1}{3} - \left(\frac{4}{5} + \frac{2}{3\sqrt{5}}\right)\right)$$

$$W_{A\to B} = U_B - U_A = k\frac{q^2}{a^2}\left(\frac{-2}{3\sqrt{5}} - \frac{7}{15}\right)$$

b)

Ulatest $= U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$

Ulatest $= \frac{k(2q)(q)}{5a} + \frac{k(2q)(q)}{6a} + k\frac{(-q)(2q)}{5a} + \frac{k(q)(q)}{5a} + k\frac{(q)(-q)}{8a} + k\frac{(q)(-q)}{5a}$

$\boxed{\text{Ulatest} = \frac{5}{24}k\frac{q^2}{a}}$

2. A uniform electric field of magnitude **325 V/m** is directed in the negative y direction in Figure 2. The coordinates of point **A** are **(-2,-3) m**, and those of point **B** are **(4, 5) m**. Calculate the electric potential difference **(V_B - V_A)** using **ACB** and **AB** paths.

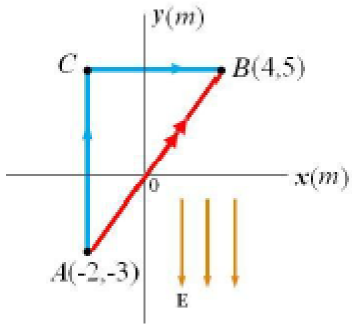
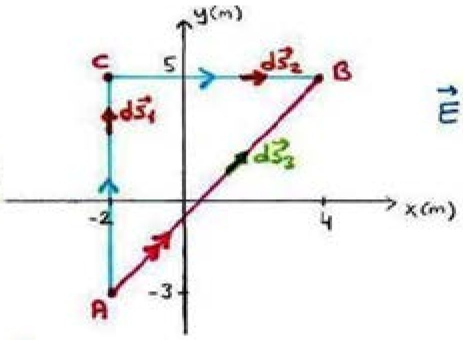


Figure 2

$d\vec{s}_1 = dy\,\hat{j}$

$d\vec{s}_2 = dx\,\hat{i}$

$d\vec{s}_3 = dx\,\hat{i} + dy\,\hat{j}$



$\vec{E} = -325\,\hat{j}$ (V/m)

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

ACB path

$$V_B - V_A = -\int_A^C \vec{E} \cdot d\vec{s}_1 - \int_C^B \vec{E} \cdot d\vec{s}_2$$

$$V_B - V_A = -\int_A^C (-325\,\hat{j}) \cdot dy\,\hat{j} - \int_C^B (-325\,\hat{j}) \cdot dx\,\hat{i} \qquad (\hat{j} \cdot \hat{i} = 0)$$

$$V_B - V_A = 325 \int_A^C dy$$

$$V_B - V_A = 325 \int_{-3}^{5} dy = 325 \left[y \right]_{-3}^{5} = 325 \left[5 - (-3) \right]$$

$$\boxed{V_B - V_A = 2600 \text{ (V)}}$$

AB path:

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}_3$$

$$V_B - V_A = -\int_A^B (-325\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$V_B - V_A = 325\int_A^B dy$$

$$V_B - V_A = 325\int_{-3}^{5} dy = 325\left[y\right]_{-3}^{5}$$

$$\boxed{V_B - V_A = 2600 (V)}$$

3. An electric dipole consists of two charges of **+5 μC** and **−5 μC** are placed at the points with coordinates **(-0.2; 0)m** and **(0.2; 0)m**, as shown in Figure 3. A test charge of **q_o=3 μC** is moved from the point **x=0.6m** to the point **x=-0.4m** with constant speed over a semicircle path intersecting y axis (radius of the path is **0.5m**).
How much work is done to move the test charge?

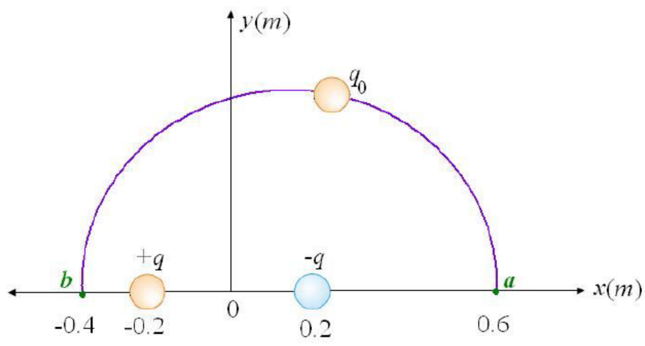
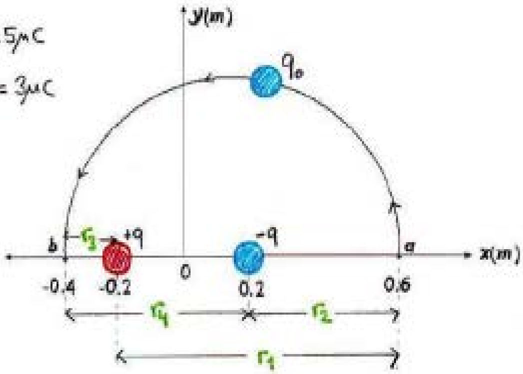


Figure 3

$q = 5\mu C$

$q_0 = 3\mu C$



$$W_{a\to b} = \Delta U = q_0 \, \Delta V = q_0 \, (V_b - V_a)$$

$$W_{a\to b} = 3.10^{-6}\left[150000 - (-56250)\right]$$

$$W_{a\to b} \cong 0,62 \ (J)$$

$$V = k\,\frac{q}{r}$$

$$V_a = k\,\frac{q}{r_1} - k\,\frac{q}{r_2}$$

$$V_a = 9.10^9\left(\frac{5.10^{-6}}{0,8} - \frac{5.10^{-6}}{0,4}\right)$$

$$V_a = -56250 \ (V)$$

$$V_b = k\,\frac{q}{r_3} - k\,\frac{q}{r_4}$$

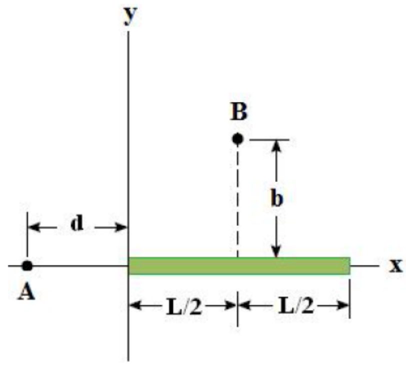
$$V_b = 9.10^9\left(\frac{5.10^{-6}}{0,2} - \frac{5.10^{-6}}{0,6}\right)$$

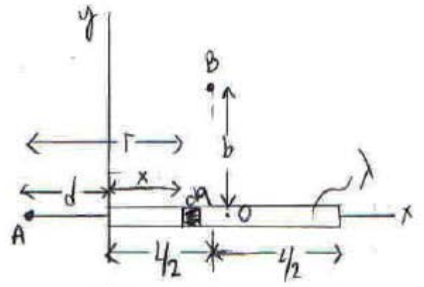
$$V_b = 150000 \ (V)$$

4. A thin rod of length **L** with charge per unit length **λ** lies along the x axis, as shown in Figure 4.

a) What are the electrical potentials of A and B points?

b) If the thin rod has a non-uniform charge density of **λ=αx** (**α**: constant), calculate the electrical potentials of A and B points.





(a)

$$V = k\int \frac{dq}{r}$$

$$dq = \lambda dx$$

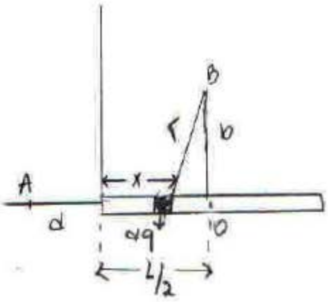
$$V_A = k\int_0^L \frac{dq}{r} = k\int_0^L \frac{\lambda dx}{r} \qquad \boxed{r = d + x}$$

$$V_A = k\lambda \int_0^L \frac{dx}{x+d} \qquad \left[\int \frac{dx}{(ax+b)} = \frac{1}{a}\ln(ax+b) \right]$$

$$V_A = k\lambda \left(\ln(x+d) \Big|_0^L \right)$$

$$V_A = k\lambda \ln\left(1 + \frac{L}{d}\right)$$

$$V = k\int \frac{dq}{r} \qquad r = \sqrt{b^2 + \left(\frac{L}{2} - x\right)^2}$$



$$V_B = \int \frac{k\lambda\, dx}{\left(b^2 + (L/2 - x)^2\right)^{1/2}} \qquad \boxed{\begin{array}{l} u = \frac{L}{2} - x \\ du = -dx \end{array}}$$

$$V_B = k\lambda \int \frac{-du}{\sqrt{b^2 + u^2}} \qquad \left[\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2}) \quad \text{veya} \quad \sinh^{-1}\frac{x}{a} \right]$$

$$V_B=-k\lambda\left(\ln\left(u+\sqrt{u^2+b^2}\right)\right)$$

$$V_B=-k\lambda\left(\ln\left(\frac{L}{2}-x\right)+\sqrt{\left(\frac{L}{2}-x\right)^2+b^2}\right)\Big|_0^L$$

$$V_B=-k\lambda\left[\ln\left(-\frac{L}{2}+\sqrt{\left(-\frac{L}{2}\right)^2+b^2}\right)-\ln\left(\frac{L}{2}+\sqrt{\left(\frac{L}{2}\right)^2+b^2}\right)\right]$$

$$V_B=k\lambda\left[\ln\left(\frac{\frac{L}{2}+\sqrt{L^2/4+b^2}}{-\frac{L}{2}+\sqrt{\frac{L^2}{4}+b^2}}\right)\right]$$

b) $\lambda=\alpha x$ ise

$$V_A=\int k\frac{dq}{r}=k\int\frac{\lambda dx}{r}=k\alpha\int_0^L\frac{x\,dx}{(x+d)}\quad,\left[\int\frac{x\,dx}{ax+b}=\frac{x}{a}-\frac{b}{a^2}\ln(ax+b)\right]$$

$$V_A=k\alpha\left(x-d.\ln(x+d)\right)\Big|_0^L$$

$$V_A=k\alpha\left(L-d.\ln\left(1+\frac{L}{d}\right)\right)$$

$$V_B=\int k\frac{dq}{r}=k\alpha\int\frac{x\,dx}{\sqrt{b^2+(L/2-x)^2}}\qquad\left[\begin{array}{l}u=L/2-x\\du=-dx\end{array}\right]$$

$$V_B=k\alpha\int\frac{(L/2-u)(-du)}{\sqrt{b^2+u^2}}$$

$$V_B=k\alpha\left[\int\frac{-L/2\,du}{\sqrt{b^2+u^2}}+\int\frac{u\,du}{\sqrt{b^2+u^2}}\right]\qquad\left[\int\frac{x\,dx}{\sqrt{x^2+a^2}}=x^2+a^2\right]$$

$$\left[\int\frac{dx}{\sqrt{x^2+a^2}}=\ln\left(x+\sqrt{x^2+a^2}\right)\right]$$

$$V_B=-k\alpha\left[-\frac{L}{2}\left(\ln\left(u+\sqrt{u^2+b^2}\right)\Big|_0^L+(u^2+b^2)\Big|_0^L\right)\right]$$

$u=L/2-x$

$$V_B=k\frac{\alpha L}{2}\ln\left(\frac{\sqrt{L^2/4+b^2}+L/2}{\sqrt{L^2/4+b^2}-L/2}\right)$$

5. **(a)** When an electron is accelerated from plate A to plate B by an electric field, it gains kinetic energy of 5.25x10^{-15} J. What is the potential difference between the plates? Which plate has the high potential?

(b) The electrical field for the region is $\vec{E} = 5x^2\hat{i} - 3\hat{j} + 2\hat{k} \quad kV/m$. If point A is located at origin and point B is at (4,3, 0)m. Determine the electrical potential between the points of A(0,0,0) and B(4,3,0).

(a) $W = \Delta K = q|\Delta V|$

$5{,}25.10^{-15} = 1{,}6.10^{-19}|\Delta V|$

$|\Delta V| = 32{,}8.10^{3}\,V$

Electrical field is formed from the high to low electrical potentials. Addition to this, electron moves reverse direction according to the electrical field that is why plate B has the high potential according to the potential of point A.

(b) $V_B - V_A = -\int_A^B \vec{E}.d\vec{r}$ A(0,0,0) $\rightarrow$ B(4,3,0)

$V_B - V_A = -\int_A^B E_x\,dx + \int_A^B E_y\,dy - \int_A^B E_z\,dz$

$V_B - V_A = -\int_0^4 5x^2\,dx + \int_0^3 3\,dy - \cancel{\int_0^0 2\,dz}$ 0

$V_B - V_A = -\frac{5x^3}{3}\Big|_0^4 + 3y\Big|_0^3$

$V_B - V_A = -97{,}6\,kV$

6. A thin rod of length **L** and charge **Q** has a charge per unit length **λ = αy** (α: cons.) lies along the y axis, as shown in Figure 5.

a) Calculate the electric potential at P point on x axis.

b) Obtain **x** component of the electric field at point **P** using the electric potential obtained in part (a).

c) If charge **q** is located at point **P**, calculate **x** component of the electric force exerted by the rod.

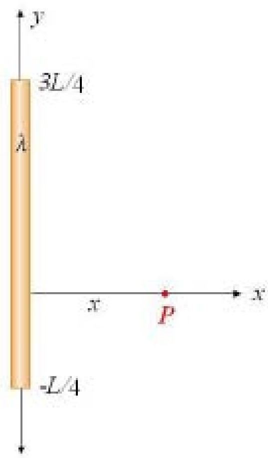
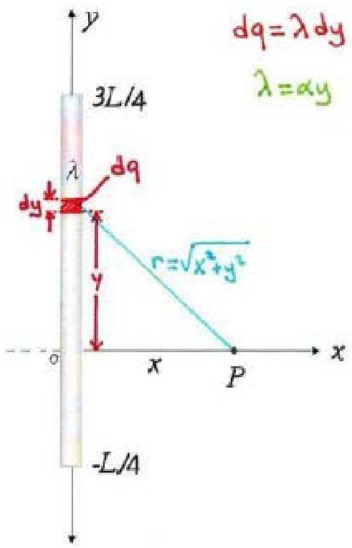


Figure 5



$dq=\lambda dy$

$\lambda=\alpha y$

$$V=k\int\frac{dq}{r}$$

a) $$V_P=k\int\frac{dq}{r}=k\int\frac{\lambda\,dy}{\sqrt{x^2+y^2}}$$

$$V_P=k\int_{-L/4}^{3L/4}\frac{\alpha y\,dy}{\sqrt{x^2+y^2}}$$

$x^2+y^2=u$

$2y\,dy=du$

$y=-\frac{L}{4}\ ;\ u=x^2+\frac{L^2}{16}$

$y=\frac{3L}{4}\ ;\ u=x^2+\frac{9L^2}{16}$

$$V_P=\frac{k\alpha}{2}\int_{x^2+\frac{L^2}{16}}^{x^2+\frac{9L^2}{16}}\frac{du}{u^{1/2}}$$

$$V_P=k\alpha\left[u^{1/2}\right]_{x^2+\frac{L^2}{16}}^{x^2+\frac{9L^2}{16}}$$

$$\boxed{V_P=k\alpha\left(\sqrt{x^2+\frac{9L^2}{16}}-\sqrt{x^2+\frac{L^2}{16}}\right)}$$

b)

$$\vec{E}=-\vec{\nabla}V=-\left(\frac{\partial V}{\partial x}\hat{i}+\frac{\partial V}{\partial y}\hat{j}+\frac{\partial V}{\partial z}\hat{k}\right)$$

$$E_x=-\frac{\partial V}{\partial x}\ \rightarrow\ E_x=-\frac{dV}{dx}$$

$$E_x=-\frac{d}{dx}\left[k\alpha\left(\sqrt{x^2+\frac{9L^2}{16}}-\sqrt{x^2+\frac{L^2}{16}}\right)\right]$$

$$E_x = -k\alpha \left[\frac{1}{2}\left(x^2 + \frac{9L^2}{16}\right)^{-1/2} 2x - \frac{1}{2}\left(x^2 + \frac{L^2}{16}\right)^{-1/2} 2x \right]$$

$$\boxed{E_x = k\alpha \left(\frac{x}{\sqrt{x^2 + \frac{L^2}{16}}} - \frac{x}{\sqrt{x^2 + \frac{9L^2}{16}}} \right)}$$

c) $\vec{F}_e = q\vec{E} = q\left(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}\right)$

$F_{e_x} = qE_x$

$$\boxed{F_{e_x} = k\alpha q \left(\frac{x}{\sqrt{x^2 + \frac{L^2}{16}}} - \frac{x}{\sqrt{x^2 + \frac{9L^2}{16}}} \right)}$$

7. Consider two thin, conducting, spherical shells as shown in Figure 6. The inner shell has a radius **r_1=15cm** and a charge of **10 nC**. The outer shell has a radius **r_2=30cm** and a charge of **−15 nC**. Find

a) the electric field E and

b) the electric potential V in regions **A**, **B**, and **C**, with V=0 at $r=\infty$.

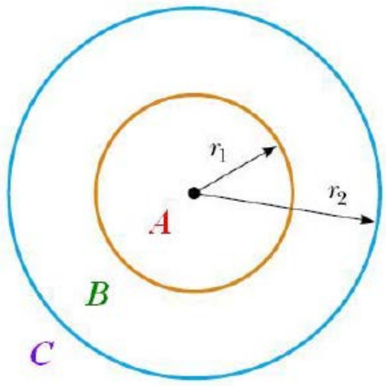


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Figure 6

a) $\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q_i}{\varepsilon_0}$

A region;

$q_{iç}=0$; $\boxed{E_A=0}$ $(r<r_1)$

B region;

$E_B.(4\pi r^2) = \frac{q_1}{\varepsilon_0}$

$E_B = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r^2} = k\frac{q_1}{r^2}$

$E_B = 9.10^9 . \frac{10.10^{-9}}{r^2}$

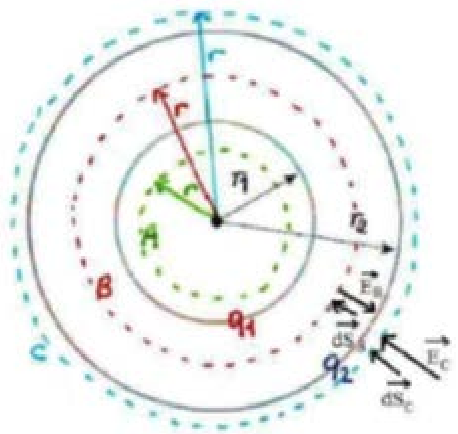
$\boxed{E_B = \frac{90}{r^2} \ (V/m)}$ $(r_1<r<r_2)$

C region;

$E_C.(4\pi r^2) = \frac{q_1+q_2}{\varepsilon_0}$

$E_C = \frac{1}{4\pi\varepsilon_0} \frac{q_1+q_2}{r^2} = k\frac{q_1+q_2}{r^2} = 9.10^9 . \frac{(10-15).10^{-9}}{r^2}$

$\boxed{E_C = -\frac{45}{r^2} \ (V/m)}$ $(r>r_2)$



b) $V_C = k\,\dfrac{(q_1+q_2)}{r}$; $V_C = 9.10^9 \cdot \dfrac{(10-15).10^{-9}}{r}$; $\boxed{V_C = -\dfrac{45}{r}\ (V)}$

$V_B = V_{r_2} + \int_{r_2}^{r} k\,\dfrac{q_1}{r^2}\,dr$ $\qquad r \to r_2 \qquad V_{r_2} = -\dfrac{45}{0,3} = -150\ (V)$

$V_B = -150 + k q_1 \left(\dfrac{1}{r} - \dfrac{1}{r_2}\right)$

$V_B = -150 + 9.10^9 \cdot 10.10^{-9} \left(\dfrac{1}{r} - \dfrac{1}{0,3}\right)$

$\boxed{V_B = -450 + \dfrac{90}{r}\ (V)}$

$r \to r_1 \qquad V_{r_1} = -450 + \dfrac{90}{0,15} = +150\ (V)$

$\boxed{V_A = +150\ V}$

2nd Method:

$$V_C - V_\infty = -\int_\infty^C \vec{E}_C . d\vec{S}_C$$

$$[dS_C = -dr]$$

$$V_C = -\int_\infty^C \vec{E}_C . d\vec{S}_C = -\int_\infty^C E_C dS_C Cos0 = -\int_\infty^r \frac{45}{r^2}(-dr) = +\int_\infty^r \frac{45}{r^2}dr = 45\left|-\frac{1}{r}\right|_\infty^r = -\frac{45}{r}(V)$$

$r = r_2 \qquad V_{r2} = -\dfrac{45}{0,3} = -150(V)$ $\qquad [dS_B = -dr]$

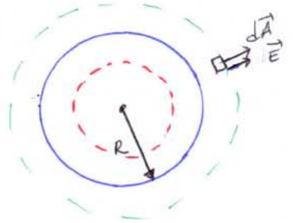
$$V_B = -\int_\infty^{r2} \vec{E}_C . d\vec{S}_C - \int_{r2}^{B} \vec{E}_B d\vec{S}_B = -\int_\infty^{r2} E_C dS_C Cos0 - \int_{r2}^{B} E_B dS_B Cos180 = -\int_\infty^{r2}\frac{45}{r^2}(-dr) - \int_{r2}^{r}\frac{90}{r^2}(-dr)(-1) =$$

$$= +\int_\infty^{r2}\frac{45}{r^2}dr - \int_{r2}^{r}\frac{90}{r^2}dr = 45\left|-\frac{1}{r}\right|_\infty^{r2} - 90\left|-\frac{1}{r}\right|_{r2}^{r} = -\frac{45}{r_2} + 90\left(\frac{1}{r} - \frac{1}{r_2}\right) = -\frac{45}{0,3} + 90\left(\frac{1}{r} - \frac{1}{0,3}\right) = -450 + \frac{90}{r}(V)$$

$r = r_1 \qquad V_{r1} = -450 + \dfrac{90}{0,15} = 150(V)$

$V_A = 150(V)$

8. A solid insulating sphere of radius **R** and charge **Q** has a non-uniform charge density that varies with r according to the expression $\rho = Ar^2$, where A is a constant and **r**, **R** is measured from the center of the sphere. Use Gauss's law to
   a) Determine the magnitudes of the electric fields outside and inside the sphere.
   b) Determine the electric potential of a point inside the sphere.
   c) Draw **E=f(r)** and **V=f(r)** graphs.



$r > R$

a) $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \dfrac{q_i}{\epsilon_0}$

$$q_i = Q = \int_0^r \rho \, dV = \int_0^R Ar^2 4\pi r^2 dr = A 4\pi \frac{R^5}{5}$$

$$E_{out} 4\pi r^2 = 4\pi A \frac{r^5}{\epsilon_0} \Rightarrow \vec{E}_{out} = \frac{AR^5}{5\epsilon_0 r^2}\hat{r}$$

$r < R$

$$q_i = \int_0^r \rho \, dV = \int_0^r Ar^2 4\pi r^2 dr = A 4\pi \frac{r^5}{5}$$

$$E_i \, 4\pi r^2 = 4\pi A \frac{r^5}{5\epsilon_0}$$

$$\vec{E}_i = A \frac{r^3}{5\epsilon_0}\hat{r}$$

b) $V_A - V_\infty = V_A = -\int_\infty^R \vec{E}_{out} \cdot d\vec{s} - \int_R^r \vec{E}_i \cdot d\vec{s}$ $\qquad ds = dr$

$$V_A = -\int_\infty^R \frac{AR^5}{5\epsilon_0 r^2} dr - \int_R^r A \frac{r^3}{5\epsilon_0} dr$$

$$V_A = \frac{AR^5}{5\epsilon_0}\left(\frac{1}{R} - 0\right) - \frac{A}{5\epsilon_0}\left(\frac{r^4}{4} - \frac{R^4}{4}\right)$$

$$V_A = \frac{A}{20\epsilon_0}(5R^4 - r^4)$$

c)