## Tutorial ${ }^{\circ} 4$ : Exercises on plane and spherical mirrors and the reduced eye.

## Exercise 4.1:

We consider a convex spherical mirror, with center C , vertex S , radius of curvature $R=\overline{S C}=$ +30 cm and an object of height 1 cm .

1) Give the position of the focus $F$.
2) Determine the image $\overline{A^{\prime} B^{\prime}}$ of the object $\overline{A B}$ by specifying its position, its magnification, its size and its nature in the case where $\overline{S A}=-30 \mathrm{~cm}$.
3) Construct the image (Diagram).

## Solution

1- The position of focus $F$.
The focus F of the convex spherical mirror is in the middle of the segment $[S C]$ and $\overline{S F^{\prime}}=$ $\overline{S F}=15 \mathrm{~cm}$.

2- The position of $A^{\prime}$ :

$$
\frac{1}{\overline{S A^{\prime}}}+\frac{1}{\overline{S A}}=\frac{1}{\overline{S F}} \Rightarrow \overline{S A^{\prime}}=\frac{\overline{S A} \cdot \overline{S F}}{\overline{S A}-\overline{S F}}=\frac{(-30) .(15)}{-30-15}=+10 \mathrm{~cm}
$$

The magnification $\gamma: \gamma=\frac{\overline{A^{\prime} B^{\prime}}}{\overline{A B}}=-\frac{\overline{S A^{\prime}}}{\overline{S A}}=-\frac{10}{-30}=0.33 \Rightarrow \overline{A^{\prime} B^{\prime}}=0.33 .1 \mathrm{~cm}=0.33 \mathrm{~cm}$
Nature of mage: The image is virtual, vertical and smaller than the object
3-


## Exercise 4.2:

1. Construct the image from the object:
a- Concave mirror

b- Convex mirror


## Solution

https://www.physagreg.fr/optique-12-generalites-systemes-miroirs.php

## Miroir concave



Miroir concave : objet et image réels


Miroir concave : objet réel, image virtuelle


## Miroir convexe



## Exercise 4.3:

A- A myopic eye is comparable, when it does not accommodate, to a 15 mm lens focal distance.
The retina is then located 1 mm beyond the image focus $F_{o}^{\prime}$.
Determine:

1. The distance from the eye to the Remotum point.
2. The number of the corrective lens to use

B- Same questions as part A for a hyperopic eye whose focal length is 15 mm when it does not accommodate and the retina is then located 1 mm below the focus image ${F^{\prime}}_{H}$.

## Solution

1- the distance $O R: \frac{1}{\overline{O R^{\prime}}}-\frac{1}{\overline{O R}}=\frac{1}{\overline{O F_{o}^{\prime}}}$

with $\overline{O F_{o}^{\prime}}=15 \mathrm{~mm}$ and $\overline{O R^{\prime}}=\overline{O F_{o}^{\prime}}+\overline{F_{o}^{\prime} R^{\prime}}=15+1=16 \mathrm{~mm}$

$$
\overline{O R}=\frac{\overline{O F_{o}^{\prime}} \cdot \overline{O R^{\prime}}}{\overline{O F_{o}^{\prime}}-\overline{O R^{\prime}}}=\frac{15.16}{15-16}=-240 \mathrm{~mm}=-0.24 \mathrm{~m}
$$

2-Corrective lens: The corrective lens is a divergent lens whose image focus is in $R$.
Its focal length is therefore $F_{C}^{\prime}=-0.24 m$ and its vergence is: $C=-\frac{1}{0.24}=-4.16 \delta$ this is the requested number.
B- the distance $O R: \frac{1}{O R^{\prime}}-\frac{1}{O R}=\frac{1}{O F_{H}^{\prime}}$

with $\overline{O F_{H}^{\prime}}=15 \mathrm{~mm}$ and $\overline{O R^{\prime}}=\overline{O F_{H}^{\prime}}-\overline{F_{H}^{\prime} R^{\prime}}=15-1=14 \mathrm{~mm}$

$$
\overline{O R}=\frac{\overline{O F_{H}^{\prime}} \cdot \overline{O R^{\prime}}}{\overline{O F_{H}^{\prime}}-\overline{O R^{\prime}}}=\frac{15.14}{15-14}=+210 \mathrm{~mm}=+0.21 \mathrm{~m}
$$

2-Corrective lens: The corrective lens is a convergent lens whose image focus is in R .
Its focal length is therefore $F_{C}^{\prime}=+0.21 \mathrm{~m}$ and its vergence is: $C=+\frac{1}{0.21}=+4.76 \delta$ this is the requested number.

## Exercise 4.4:

A myopic person cannot see objects clearly further than $2 m$ and their minimum distinct vision distance is 10 cm .

1. What is its amplitude of accommodation?
2. What must be the vergence $V_{1}$ of the contact lens $L_{1}$ of the corrective lenses so that he can see objects at infinity? Deduce the type of lens of these corrective lenses.
3. What happens to the Punctum Proximum of these eyes when wearing these corrective lenses? 4. Instead of the contact lens, a lens $L_{2}$ is placed 2 cm on the cornea. What should be the vergence $V_{2}$ of this lens to maximize the visual field of these eyes at infinity.

## Solution

1. The amplitude of accommodation $A$ is defined by:
$A=\mathrm{V}_{\text {max }}-\mathrm{V}_{\text {min }}=\frac{1}{\overline{P R}}-\frac{1}{\overline{P P}}$
$\overline{P R}=-2 \mathrm{~cm}$ Punctum Remotum .
$\overline{P P}=-10 \mathrm{~cm}=0.1 \mathrm{~m}$ Punctum Proximum
$A=\frac{1}{(-2)}-\frac{1}{(-0.1)}=9.5 \delta$
2- The contact lens $L_{1}$ of optical center $O_{1}$ (confused with the optical center of the lens of the crystalline lens) and of vergence $V_{1}$ shape of the object located at infinity
( $\overline{O_{1} A}=-\infty$ ) form an image $A^{\prime}$ that must be confused with PR: $\overline{O_{1} A^{\prime}}=\overline{P R}=-2 m$
The conjugation relation for the contact lens $L_{1}$ :
$\frac{1}{\overline{O_{1} A^{\prime}}}-\frac{1}{\overline{O_{1} A}}=V_{1}$
The vergence $V_{1}$ of corrective lenses:
$V_{1}=\frac{1}{(-2)}-\frac{1}{(-\infty)}=-0.5 \delta$
$V_{1}<0$ The lens is divergent.
3- The new punctum proximum $P P_{N}$ is the object through which the lens $L_{1}$ gives an image to the "natural" punctum proximum $P P$ of the uncorrected eye.

From the conjugation relation of $L_{1}$, we have:

$$
\begin{aligned}
& \frac{1}{\overline{P P}}-\frac{1}{\overline{P P_{N}}}= V_{1} \\
& \Rightarrow \frac{1}{\overline{P P_{N}}}=\frac{1}{\overline{P P}}-V_{1} \Rightarrow \overline{P P_{N}}=\frac{\overline{P P}}{1-V_{1} \cdot \overline{P P}}=\frac{(-0.1)}{1-(-0.5) \cdot(-0.1)} \\
&=-0.105 m
\end{aligned}
$$

4- Eye-lens distance $L_{1}: \overline{O O_{2}}=-0.02 \mathrm{~m}$
$\overline{O_{2} A^{\prime}}=\overline{O_{2} O}+\overline{O A^{\prime}}=+0.02-2=-1.98 m$ the distance $O_{2} A^{\prime}=-1.98 \mathrm{~m}$ for an object A placed at infinity $\overline{O_{2} A}=-\infty$

$$
\frac{1}{\overline{O_{2} A^{\prime}}}-\frac{1}{\overline{O_{2} A}}=V_{2} \Rightarrow V_{2}=\frac{1}{(-1.98)}=-0.505 \delta
$$

