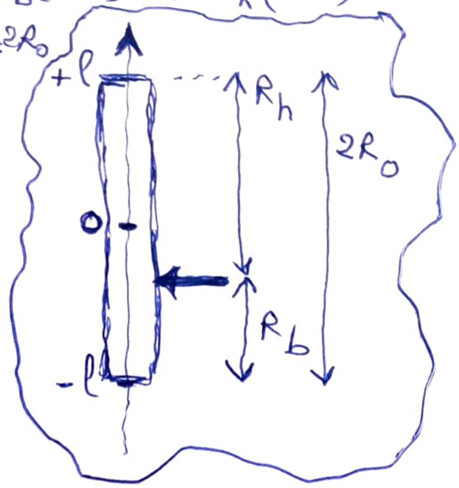
Ex 01

# TD N° 1

1) Expression des résistances $R_b(\Delta x)$ et $R_h(\Delta x)$ :

$R_b(-l) = 0$ , $R_b(0) = R_o$ , $R_b(+l) = 2R_o$

$R_h(-l) = 2R_o$ , $R_h(0) = R_o$ , $R_h(+l) = 0$




$R_b(\Delta x) = a_1 \Delta x + b_1$

$R_b(0) = R_o = b_1 \Rightarrow b_1 = R_o$

$R_b(-l) = 0 = -a_1 l + R_o$

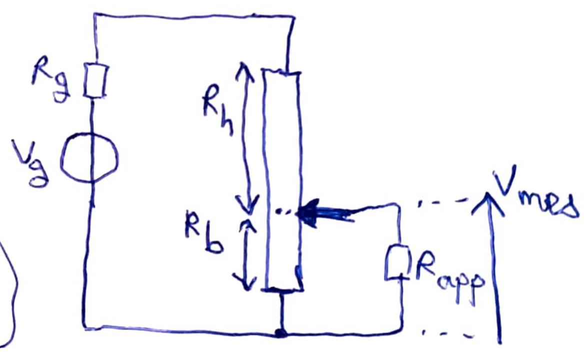
$\Rightarrow a_1 = \frac{R_o}{l}$

$\Rightarrow R_b(\Delta x) = \frac{R_o}{l} \Delta x + R_o$

$\Rightarrow$ $R_b(\Delta x) = R_o\left(1 + \frac{\Delta x}{l}\right)$

$R_h(\Delta x) = R_o\left(1 - \frac{\Delta x}{l}\right)$

2) Expression de $V_{mes}$ :




$$V_{mes} = \frac{R_b \parallel R_{app}}{(R_b \parallel R_{app}) + R_h + R_g} . V_g$$

3) Expression de $V_{mes}$ pour $R_{app} \gg R_o \Rightarrow R_{app} \gg R_b$ :

$$R_{eq} = R_b \parallel R_{app} = \frac{R_b R_{app}}{R_b + R_{app}} = \begin{cases} R_b \text{ pour } R_b \to 0 \ \forall R_{app} \\ R_b \text{ pour } R_{app} \gg R_b \end{cases}$$

$\Rightarrow R_{eq} = R_b$

$\Rightarrow V_{mes} = \frac{R_b}{R_b + R_h + R_g} . V_g \Rightarrow$ $V_{mes} = \frac{R_b}{2R_o + R_g} . V_g$

4) La sensibilité de la mesure $S_{mes}$ :

$$S_{mes} = \frac{\Delta V_{mes}}{\Delta x}$$

$V_{mes}(0) = \frac{R_o}{2R_o + R_g} . V_g$ , $V_{mes}(\Delta x) = \frac{R_o\left(1 + \frac{\Delta x}{l}\right)}{2R_o + R_g} . V_g$

$\Rightarrow \Delta V_{mes} = V_{mes}(\Delta x) - V_{mes}(0) = \frac{\frac{R_o \Delta x}{l}}{2R_o + R_g} . V_g$

$$\Delta V_{mes} = \frac{\frac{R_0 \Delta x}{l}}{2R_0 + R_g} \cdot V_g \Rightarrow \boxed{S'_{mes} = \frac{\Delta V_{mes}}{\Delta x} = \frac{\frac{R_0}{l}}{2R_0 + R_g} \cdot V_g}$$

5) Pour que $S_{mes}$ ~~soit~~ devient maximale : $R_g \rightarrow 0$

$$\Rightarrow \boxed{S_{mes,max} = \frac{V_g}{2l}}$$

$$\Rightarrow V_{mes} = \frac{R_b}{2R_0} \cdot V_g \Rightarrow \boxed{V_{mes} = \left(1 + \frac{\Delta x}{l}\right) \cdot \frac{V_g}{2}}$$

* La sensibilité réduite : $S_r = \frac{S_{mes}}{E_{Alimentation}} = \frac{S'_{mes}}{V_g}$

$$\Rightarrow \boxed{S'_r = \frac{1}{2l}}$$ ~~unité : V/cm~~

unité V/cm.V

6)

vitesse de déplacement du curseur : $v = \frac{d\Delta x}{dt}$

mouvement sinusoïdal : $\Delta x = l \sin(2\pi f t) + x_0$ , $x_0 = 0$

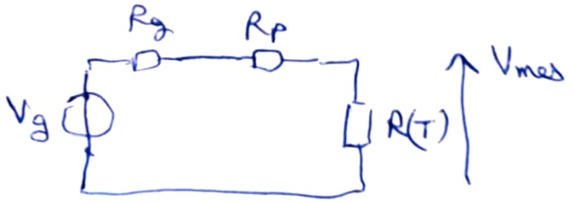
$$v = \frac{d\Delta x}{dt} = 2\pi f l \cos(2\pi f t)$$

$$V_{max} = 2\pi f_{max} l \quad , \quad \cos(2\pi f t) = 1$$

$$\Rightarrow \boxed{f_{max} = \frac{V_{max}}{2\pi l}}$$

## Exo 2 :

1) $$\boxed{V_{mes} = \frac{R(T)}{R(T) + R_p + R_g} \cdot V_g}$$




2) $R(T) = R_0(1 + AT + BT^2)$ , $T \in [-10°C, +10°C]$

$R(0) = R_0$

$$\Rightarrow \boxed{\Delta R(T) = R(T) - R_0 = R_0(AT + BT^2)}$$

3)
$$V_{mes}(T=0) = \frac{R_0}{R_0 + R_p + R_g} \cdot V_g$$

$$V_{mes}(T) = \frac{R(T)}{R(T) + R_p + R_g} \cdot V_g$$

$$\Delta V_{mes} = V_{mes}(T) - V_{mes}(0)$$

$$= \left( \frac{R(T)}{R(T) + R_p + R_g} - \frac{R_0}{R_0 + R_p + R_g} \right) \cdot V_g$$

$$= \frac{(R_p + R_g)(R(T) - R_0)}{(R_0 + \Delta R(T) + R_p + R_g)(R_0 + R_p + R_g)} \cdot V_g$$

$$\Rightarrow \boxed{\Delta V_{mes} = \frac{(R_p + R_g)\,\Delta R(T)}{(R_0 + R_p + R_g)^2 \left(1 + \frac{\Delta R(T)}{R_0 + R_p + R_g}\right)} \cdot V_g}$$

4) Pour avoir un maximum de sensibilité :

$$\frac{d S_{mes}}{d R_p} = 0 \quad , \quad S_{mes} = \frac{\Delta V_{mes}}{\Delta R(T)}$$

On considère la partie linéaire de $\Delta V_{mes}$ :

$$\Rightarrow \Delta R(T) \ll R_0 + R_p + R_g$$

$$\Rightarrow \Delta V_{mes,lin} = \frac{(R_p + R_g)}{(R_0 + R_p + R_g)^2} \cdot V_g \cdot \Delta R(T)$$

$$\Rightarrow S_{mes,lin} = \frac{\Delta V_{mes,lin}}{\Delta R(T)} = \frac{(R_p + R_g)}{(R_0 + R_p + R_g)^2} \cdot V_g$$

$$\frac{d S_{mes}}{d R_p} = \frac{(R_0 + R_p + R_g)^2 - (R_p + R_g) \cdot 2 \cdot (R_0 + R_p + R_g)}{(R_0 + R_p + R_g)^4} \cdot V_g$$

$$= \frac{R_0^2 - R_g^2 - 2 R_g R_p - R_p^2}{(R_0 + R_p + R_g)^4} \cdot V_g = 0$$

$$\Rightarrow R_0^2 - R_g^2 - 2 R_g R_p - R_p^2 = 0 \Rightarrow R_0^2 - (R_g + R_p)^2 = 0$$

$$\Rightarrow R_g + R_p = \pm R_0 \Rightarrow \boxed{R_p = R_0 - R_g}$$

5) On prend le cas où : $R_p = R_0 - R_g$

$$\Delta V_{mes} = \frac{(R_p+R_g)\,\Delta R(T)}{(R_0+R_p+R_g)^2\left(1+\frac{\Delta R(T)}{R_0+R_p+R_g}\right)} \cdot V_g$$

$$\Delta V_{mes} = \frac{\Delta R(T)}{4R_0\left(1+\frac{\Delta R(T)}{2R_0}\right)} \cdot V_g$$

$$\Rightarrow \boxed{\Delta V_{mes} = \frac{R_0(AT+BT^2)}{4R_0\left(1+\frac{R_0(AT+BT^2)}{2R_0}\right)} \cdot V_g}$$

6) $\Delta V_{mes,lin} = ?$

on a : $\Delta V_{mes} = \frac{1+\frac{BT}{A}}{1+\frac{AT+BT^2}{2}} \cdot \frac{AV_g}{4} \cdot T$

on a : $\frac{B \cdot T_{max}}{A} = 0{,}01 \ll 1$

$\frac{AT_{max}+BT_{max}^2}{2} = 0{,}028 \ll 1$

$\Rightarrow \boxed{\Delta V_{mes,lin} = \frac{AV_g}{4} \cdot T}$ , $\Delta T = T - 0 = T$

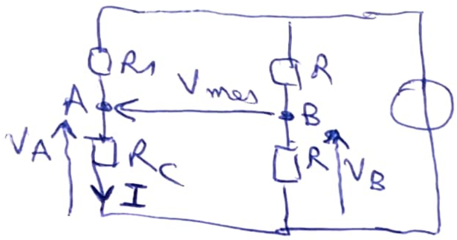
$\Rightarrow S_{mes,lin} = \frac{\Delta V_{mes,lin}}{\Delta T} = \frac{AV_g}{4} = 13{,}73 \text{ mV/°C}$

# TD N° 2

## Ex 2 :

1) La valeur de $R_1$ pour équilibrer le pont à $T_0 = 50°C$ :

$V_{mes} = V_A - V_B$




$$V_A = \frac{R_C}{R_C + R_1} \cdot V_g$$

$$V_B = \frac{R}{R+R} \cdot V_g = \frac{V_g}{2}$$

$$V_{mes} = \frac{R_C}{R_C + R_1} V_g - \frac{V_g}{2} \Rightarrow V_{mes} = \frac{R_c - R_1}{R_c + R_1} \cdot \frac{V_g}{2}$$

à $T_0 = 50°C$ on a : $R_{C_0} = R_0(1 + \alpha \cdot 50) = 119{,}25\ \Omega$

À l'équilibre : $V_{mes} = 0 \Rightarrow R_1 = R_C = R_{C_0}$

$\Rightarrow R_1 = R_{C_0} = 119{,}25\ \Omega$

2) $$I = \frac{V_{g\,max}}{R_{C_{min}} + R_1} < I_{max}$$

$R_{C_{min}} = R_c(T = 0°C) = R_0$

$\Rightarrow V_{g\,max} < (R_{C_{min}} + R_{C_0})\, I_{max}$

$\Rightarrow V_{g\,max} < 1{,}096\ V$ , on prend : $V_g = 1{,}096\ V$

3) $$V_{mes} = \frac{R_c - R_1}{R_c + R_1} \cdot \frac{V_g}{2} = \frac{R_0(1+\alpha T) - R_{C_0}}{R_0(1+\alpha T) + R_{C_0}} \frac{V_g}{2}$$

$$\Rightarrow \Delta V_{mes} = V_{mes} = \frac{R_0(1+\alpha T) - R_0(1+\alpha T_0)}{R_0(1+\alpha T) + R_0(1+\alpha T_0)} \cdot \frac{V_g}{2}$$

$(V_{mes}(T_0) = 0$ et $\Delta V_{mes} = V_{mes}(T) - V_{mes}(T_0))$

$$\Rightarrow \Delta V_{mes} = \frac{\alpha R_0 \Delta T}{2R_0(1+\alpha T_0) + \alpha R_0 \Delta T} \cdot \frac{V_g}{2} \quad , \ \Delta T = T - T_0$$

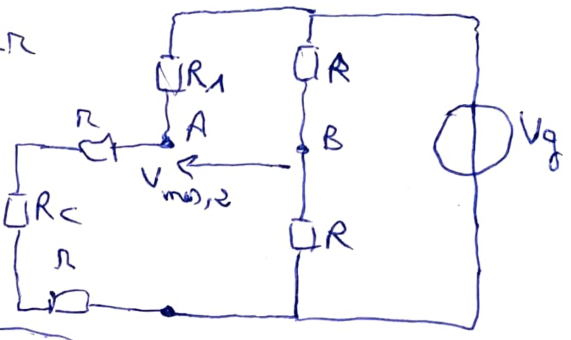
$$\Rightarrow \Delta V_{mes} = \frac{\alpha}{(1+\alpha T_0)\left(1 + \frac{\alpha \Delta T}{2(1+\alpha T_0)}\right)} \cdot \frac{V_g}{4} \cdot \Delta T$$

$$\Delta V_{mes,lin} = \frac{\alpha}{1+\alpha T_0} \cdot \frac{V_g}{4} \cdot \Delta T$$

$$\left( \frac{\alpha \, \Delta T_{max}}{2(1+\alpha T_0)} = 0{,}08 \ll 1 \right)$$

$$\Rightarrow S_{mes,lin} = \frac{\Delta V_{mes,lin}}{\Delta T} = \frac{\alpha}{1+\alpha T_0} \frac{V_g}{4} = 88{,}48 \times 10^{-5} \, V/°C$$

4) • on remplace $R_c$ par $R_c + 2r$




$$\Rightarrow \boxed{V_{mes,2} = \frac{R_c + 2r - R_{c0}}{R_c + 2r + R_{c0}} \cdot \frac{V_g}{2}}$$

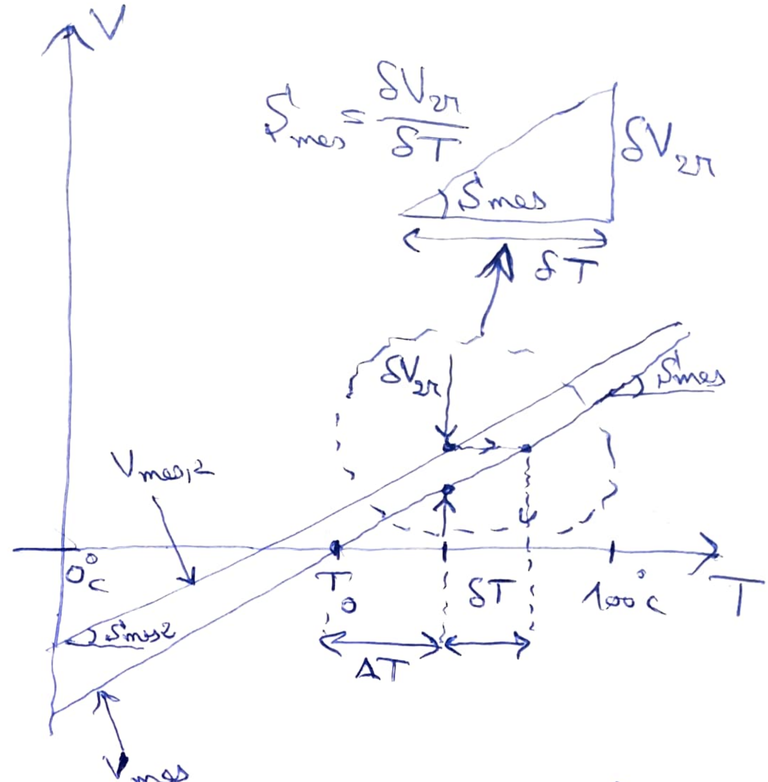
• $$\delta V_{2r} = V_{mes,2} - V_{mes} = \left( \frac{R_c + 2r - R_{c0}}{R_c + 2r + R_{c0}} - \frac{R_c - R_{c0}}{R_c + R_{c0}} \right) \frac{V_g}{2}$$

$$\Rightarrow \delta V_{2r} = \frac{4 r R_{c0}}{(2R_{c0} + 2r + \Delta R_c)(2R_{c0} + \Delta R_c)} \cdot \frac{V_g}{2}$$

Cette erreur est ~~minimale~~ maximale pour $\Delta R_c = \Delta R_{c\,min} = R_0 - R_{c0}$

$$\Rightarrow \delta V_{2r,max} = \frac{4 r R_{c0}}{\left(1 + \frac{2r}{R_{c0} + R_0}\right)(R_{c0} + R_0)^2} \cdot \frac{V_g}{2} \quad (I)$$

$$\delta V_{2r} = S_{mes} \cdot \delta T$$




$\delta T \simeq 0{,}2\,°C$

$$(I) \Rightarrow 4 r R_{C_0} \frac{V_g}{2} = S_{mes} \cdot \delta T \left(1 + \frac{2r}{R_{C_0} + R_0}\right)\left(R_{C_0} + R_0\right)^2$$

$$\Rightarrow \boxed{r = \frac{(R_{C_0} + R_0)^2 S_{mes}\, \delta T}{2\left(R_{C_0} V_g - (R_{C_0} + R_0) S_{mes}\, \delta T\right)}} \Rightarrow r = 32{,}5\ m\Omega$$

La longueur de fils :

$$R = \frac{\rho \cdot l}{S} \Rightarrow \boxed{l = \frac{R \cdot S}{\rho}}$$

$$S = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4} \quad , \quad \rho = 1{,}72 \cdot 10^{-8}\ \Omega \cdot m$$

$$\Rightarrow l = 37{,}15\ cm$$

<u>Ex 2 :</u> Voir correction TP 01 : moodle