Chapter I : Logical and Proof Mathematics

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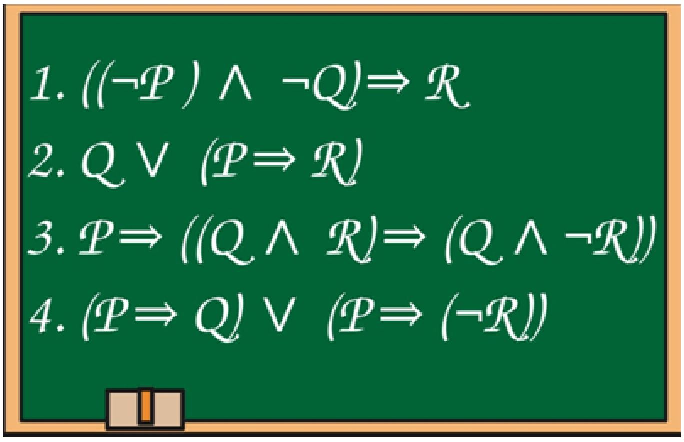
1. Proposition logic



1.1. Notions of Logic

Definition: A logical statement[*] is a sentence that has a single value of truth: true or false (not at the same time).

A logical statement is generally denoted by a capital letter: P, Q,R...

When the proposition is true, it is assigned the letter “T”

When the proposition is false, it is assigned the letter “F” (1)[*].

Example

1. (Every prime number is even), this proposition is false.
2. (√2 is an irrational number), this statement is true.

1.2. Negation: p

Definition: Given a logical proposition P, we call the negation of P the logical proposition P, which is false when P is true and which is true when P is false, so we can represent it as follows:

P	$\overline{P}$
T	F
F	T

Example

P: the function f is positive, then $\overline{P}$: the function f is not positive.

2. Logical connectors and Quantifiers

2.1. Logical connectors

Definition : A Logical connector* is a symbol which is used to connect two or more propositional or predicate logic's in such a manner that resultant logic depends only on the input logic's and the meaning of the connective used. Generally, there are five connectives which are (2)*:

2.1.1. The conjunction (and); (^)

Let P, Q be two propositions, the proposition (P and Q) or $(P \wedge Q)$ is the conjunction of the two propositions P, Q. $(P \wedge Q)$ is true if P and Q are both true, $(P \wedge Q)$ is false in all other cases.

This is summarized in the following truth table:

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

2.1.2. The disjunction (or), (∨)

The dis junction between P; Q is denoted by (P or Q), $(P \vee Q)$.

$(P \vee Q)$ is false if P and Q are both false, if not $(P \vee Q)$ is true.

This is summarized in the following truth table:

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

2.1.3. The implication (⇒)

"$P \Rightarrow Q$" is the logical proposition that is false if “P” is true, and “Q”is false. So, its truth table is as follows:

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	F

2.1.4. Equivalence (⇔)

We say that the two logical propositions P and Q are logically equivalent, if they are simultaneously true or simultaneously false, and we note “P, Q”, its truth table is:

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

2.2. Quantifiers

Definition:To formulate more complex mathematical statements, we use the quantifiers such as "there exists at least and for all" is a predicate.

See ""

2.2.1. Universal quantifier

The expression " For all" denoted $"\forall"$ is called a universal quantifier, the latter allows us to rewrite the assertion "for all x in E, x verifies A" in the form " $\forall x \in E, A(x)$"

2.2.2. Existential quantifier

The expression "there exists at least" denoted $"\exists"$ is called an existential quantifier, the latter allows us to rewrite the assertion "there exists at least one x in E, x verifies A" in the form: $\exists x \in E, A(x)$".

Example

- The logical proposition $\forall a \in \Re, a^2 < 0$ is false
- The logical proposition $\exists a \in \Re, a^2 < 0$ is true

3. Proof methods

Definition: A proof* of a theorem is a written verification that shows that the theorem is and unequivocally true. There are many ways to go about proving something, we'll discuss 4 methods: direct proof, Contrapositive proof, proof by contradiction, proof by induction (3)*.

3.1. Direct proof

To show that $(P \Rightarrow Q)$ is true, we assume that P is true and demonstrate that Q is also true.

3.2. Contrapositive proof

Contrapositive proof is based on the following equivalence:

$(P \Rightarrow Q) \Leftrightarrow (\overline{Q} \Rightarrow \overline{P})$, So if we want to show the assertion"$P \Rightarrow Q$."it is sufficient to show that is true.

3.3. Proof by contradiction

To show that $(P \Rightarrow Q)$ we assume both that P is true, and that Q is false, and look for a contradiction.Thus, if P is true then Q must be true and so $P \Rightarrow Q$ is true.

3.4. Proof by induction

To show that $\forall n \in N, n \geq n_0, P(n)$ is true we follow the following steps:

we show that$P(n_0)$ is true, (initial value).

Assume that$P(n)$ is true and show that $P(n+1)$ is true.

4. Exercice : Test

[solution n°1 p.9*]*

*Let P be a true assertion and Q a false assertion. Which assertions are true?

- ☐ P or Q
- ☐ P and Q
- ☐ Not (P) or Q
- ☐ Not (P and Q)

5. Exercice : Test

[solution n°2 p.9]

*I want to show by induction the assertion H_n, for any integer n large enough. Which initialization step is valid?

- start at n = 0
- start at n = 1
- I start at n = 2
- I start at n = 3

Exercises solution

> **Solution** n°1

Exercice p. 7

*Let P be a true assertion and Q a false assertion. Which assertions are true?

- [x] P or Q
- [ ] P and Q
- [ ] Not (P) or Q
- [x] Not (P and Q)

> **Solution** n°2

Exercice p. 8

*I want to show by induction the assertion H_n, for any integer n large enough. Which initialization step is valid?

- ○ start at n = 0
- ○ start at n = 1
- ○ I start at n = 2
- ◉ I start at n = 3

Glossary

Logical connector

is a constant used to join two ideas or formulas that have a particular relation.

proof

A proof is a structured argument that follows a set of logical steps, to prove if a mathematical statement or conjecture is true.

Statement

is a declarative sentence that is either true or false called a proposition

References

1 C. Degrave (2003)

2 S. Balac et F. Sturm (2003)

3 Laurent, A. (2009)