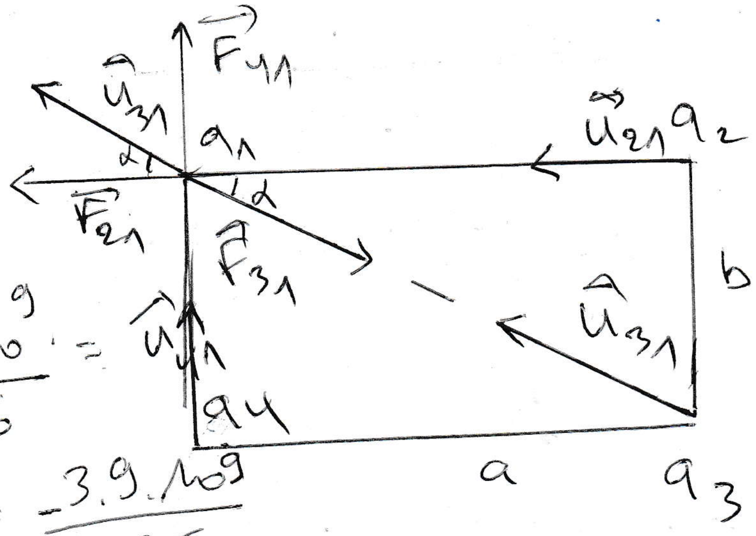
Exercise 01

Finding the direction and magnitude of the force exerted on charge q_1 by the 3 charges:

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41} \quad \text{---} (1)$$



Where:

$$\begin{cases} F_{21} = \dfrac{k q_1 q_2}{a^2} = \dfrac{k q^2}{a^2} = \dfrac{9.10^9}{16} \\ F_{31} = \dfrac{k q_1 q_3}{a^2+b^2} = \dfrac{k(-3q^2)}{a^2+b^2} = \dfrac{-3.9.10^9}{25} \\ F_{41} = \dfrac{k q_1 q_4}{b^2} = \dfrac{k(4q^2)}{b^2} = \dfrac{4.9.10^9}{9} \end{cases}$$

and:

$$\begin{cases} \vec{u}_{21} = -\vec{i} \\ \vec{u}_{31} = -\cos\alpha\,\vec{i} + \sin\alpha\,\vec{j} \;/\; \cos\alpha = \dfrac{4}{5},\ \sin\alpha = \dfrac{3}{5} \\ \vec{u}_{41} = \vec{j} \end{cases}$$

By substituting into the equation (1)

$$\vec{F}_1 = \frac{9.10^9}{16}(-\vec{i}) + \left(-\frac{27}{25}.10^9\right)\left(-\cos\alpha\,\vec{i} + \sin\alpha\,\vec{j}\right) + 4.10^9\,\vec{j}$$

$$\vec{F}_1 = -\frac{9}{16}.10^9\,\vec{i} + (0{,}864\,\vec{i} - 0{,}648\,\vec{j}).10^9 + 4.10^9\,\vec{j}$$

$$\vec{F}_1 = \underbrace{0{,}302.10^9}_{F_x}\,\vec{i} + \underbrace{3{,}352.10^9}_{F_y}\,\vec{j}$$

$$\|\vec{F_1}\| = \sqrt{F_x^2+F_y^2} = \sqrt{(0,302)^2+(3,352)^2}\cdot 10^9$$

$$\boxed{F_1 = 3,36\cdot 10^9\ N}$$

$$tg\alpha = \frac{F_y}{F_x} = \frac{3,35}{0,301} = 11,12$$

$$\alpha = arctg(11,12) = 84,86^\circ$$

Exercice 02:

1/ The potential V_0:

$$V_0 = V_A + V_B + V_C + V_D$$

$$= \frac{kq_A}{a} + \frac{kq_B}{a} + \frac{kq_C}{a} + \frac{kq_D}{a}$$

$$V_0 = \frac{k}{a}(-q+2q+3q-2q) = \frac{k}{a}(2q)$$

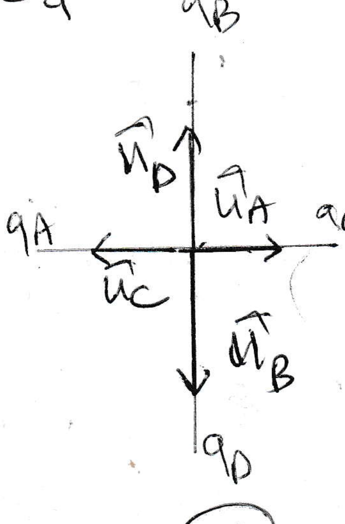
$$V_0 = \frac{9\cdot 10^9\cdot 2\cdot 10^{-9}}{5\cdot 10^{-2}} = 3,6\cdot 10^{-2}\ \text{volt.}$$

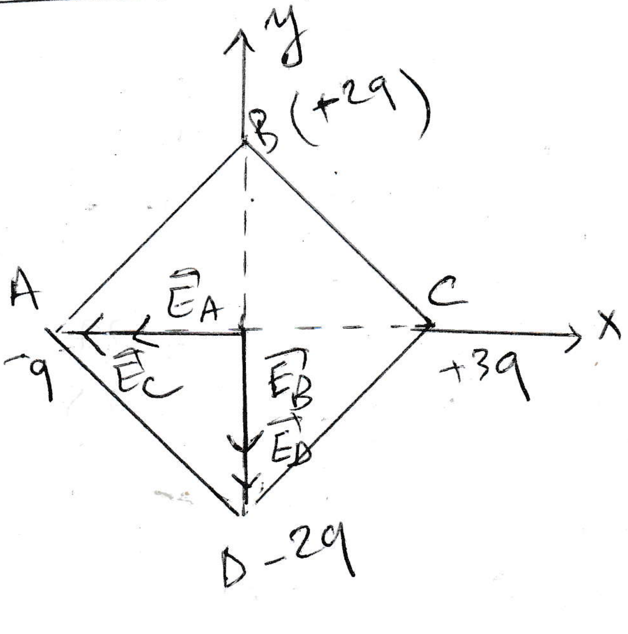
2/ The total electric field vector $\vec{E_0}$ at point (O)

$$\vec{E_0} = \vec{E_1} + \vec{E_2} + \vec{E_3} + \vec{E_4}.$$

where:

$$\begin{cases} \vec{E_A} = \dfrac{kq_A}{a^2}\vec{u_A} \\ \vec{E_B} = \dfrac{kq_B}{a^2}\vec{u_B} \\ \vec{E_C} = \dfrac{kq_C}{a^2}\vec{u_C} \end{cases}$$





$$\vec{E}_D' = \frac{kq_D}{a^2}\vec{u}_D$$

Such as : $\vec{u}_A = \vec{i}$, $\vec{u}_B = -\vec{j}$, $\vec{u}_C = -\vec{i}$, $\vec{u}_D = \vec{j}$

So: $\vec{E}_0 = \frac{k(-q)}{a^2}\vec{i} + k\frac{(2q)}{a^2}(-\vec{j}) + \frac{k(3q)}{a^2}\vec{i} + \frac{k(-2q)}{a^2}\vec{j}$

$$= \frac{kq}{a^2}\left(-\vec{i} - 2\vec{j} + 3\vec{i} - 2\vec{j}\right)$$

$$\vec{E}_0 = \frac{kq}{a^2}\left(-4\vec{i} - 4\vec{j}\right) = -\frac{4kq}{a^2}(\vec{i}+\vec{j})$$

$$\|\vec{E}_0\| = 4\sqrt{2}\frac{kq}{a^2} = \frac{4\sqrt{2}.9.10^9.10^{-9}}{25.10^{-4}} = 2.10^4\ V/m.$$

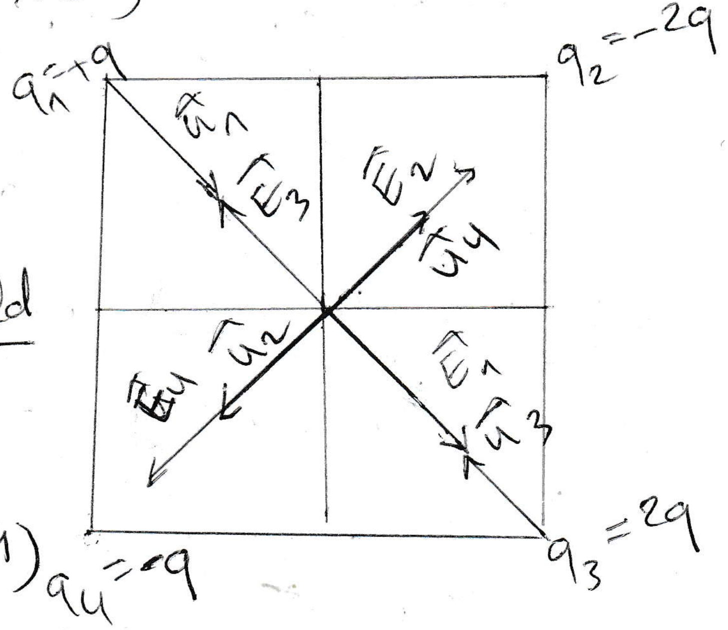
3/ The force $\vec{F}$ at point (o) if $q' = -q/2$

$$\vec{F}_0 = q'\vec{E}_0 = q'\left[-4\frac{kq}{a^2}(\vec{i}+\vec{j})\right] = -\frac{q}{2}\left[-4\frac{kq}{a^2}(\vec{i}+\vec{j})\right]$$

$$\vec{F}_0 = \frac{2kq^2}{a^2}(\vec{i}+\vec{j}).$$

So: $\|\vec{F}_0\| = \frac{2kq^2}{a^2} = \frac{2\sqrt{2}.9.10^9(10^{-9})^2}{(5.10^{-2})^2} =$

$$\|\vec{F}_0\| = 10^{-5}\ N$$



Exercise 03

Determine the electric field at point (o) :

We have:

$$\vec{E}_0 = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 \longrightarrow (1)$$

$$\begin{cases} \vec{E}_1 = \dfrac{kq_1}{OA^2}\vec{u}_1 \\ \vec{E}_2 = \dfrac{kq_2}{OB^2}\vec{u}_2 \\ \vec{E}_3 = \dfrac{kq_3}{OC^2}\vec{u}_3 \\ \vec{E}_4 = \dfrac{kq_4}{OD^2}\vec{u}_4 \end{cases} \Bigg/ \begin{cases} \vec{u}_1 = \dfrac{\sqrt{2}}{2}\vec{i} - \dfrac{\sqrt{2}}{2}\vec{j} \\ \vec{u}_2 = -\dfrac{\sqrt{2}}{2}\vec{i} - \dfrac{\sqrt{2}}{2}\vec{j} \\ \vec{u}_3 = -\dfrac{\sqrt{2}}{2}\vec{i} + \dfrac{\sqrt{2}}{2}\vec{j} \\ \vec{u}_4 = \dfrac{\sqrt{2}}{2}\vec{i} + \dfrac{\sqrt{2}}{2}\vec{j} \end{cases}$$

By substituting into the equation (1):

$$\vec{E}_0 = \frac{kq}{OA^2}\left(\frac{\sqrt{2}}{2}\vec{i} - \frac{\sqrt{2}}{2}\vec{j}\right) + \frac{k(-2q)}{OB^2}\left(-\frac{\sqrt{2}}{2}\vec{i} - \frac{\sqrt{2}}{2}\vec{j}\right) + \frac{k(2q)}{OC^2}\left(-\frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j}\right) + \frac{k(-q)}{OD^2}\left(\frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j}\right).$$

Where as: $OA = OB = OC = OD = a/\sqrt{2}$

$$\vec{E}_0 = \frac{kq}{\frac{a^2}{2}}\left(\cancel{\frac{\sqrt{2}}{2}\vec{i}} + \cancel{\frac{2\sqrt{2}}{2}\vec{i}} - \cancel{\frac{2\sqrt{2}}{2}\vec{i}} - \cancel{\frac{\sqrt{2}}{2}\vec{i}}\right) + \frac{kq}{OA^2}\left(-\frac{\sqrt{2}}{2}\vec{j} + 2\frac{\sqrt{2}}{2}\vec{j} + 2\frac{\sqrt{2}}{2}\vec{j} - \frac{\sqrt{2}}{2}\vec{j}\right)$$

$$\vec{E}_0 = \frac{2kq}{a^2}\left(-\frac{\cancel{2}\sqrt{2}}{\cancel{2}}\vec{j} + \frac{4\sqrt{2}}{2}\vec{j}\right) = 2\sqrt{2}\,\frac{kq}{a^2}\vec{j}$$

$$\|\vec{E}_0\| = \frac{2\sqrt{2}\,kq}{a^2} \text{ V/m or N/C.}$$

2/ The potential V_0

$$V_0 = V_1 + V_2 + V_3 + V_4 = \frac{kq_1}{OA} + \frac{kq_2}{OB} + \frac{kq_3}{OC} + \frac{kq_4}{OD}$$

$$= \frac{k}{OA}(q - 2q + 2q - q) = 0 \text{ volt}$$