

University of Mohamed Khider-Biskra



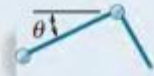
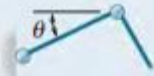










Architecture Department

**Module: Structure 2**  
**“TD”**

Semester 2  
(2023/2024)

3<sup>rd</sup> year Bachelor  
(Architecture)

# 1. Supports of structure:

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1)  light cable  weightless link	 		One unknown. The reaction is a force that acts in the direction of the cable or link.
(2)  rollers  rocker	  		One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(3)  smooth contacting surface			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.

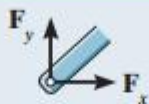
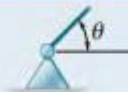
(4)



smooth pin-connected collar

One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.

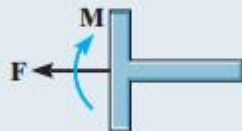
(5)



smooth pin or hinge

Two unknowns. The reactions are two force components.

(6)



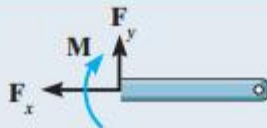
slider

Two unknowns. The reactions are a force and a moment.



fixed-connected collar

(7)



fixed support

Three unknowns. The reactions are the moment and the two force components.

## 2. Equilibrium:

The equations of equilibrium to each member are:

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_O = 0$$

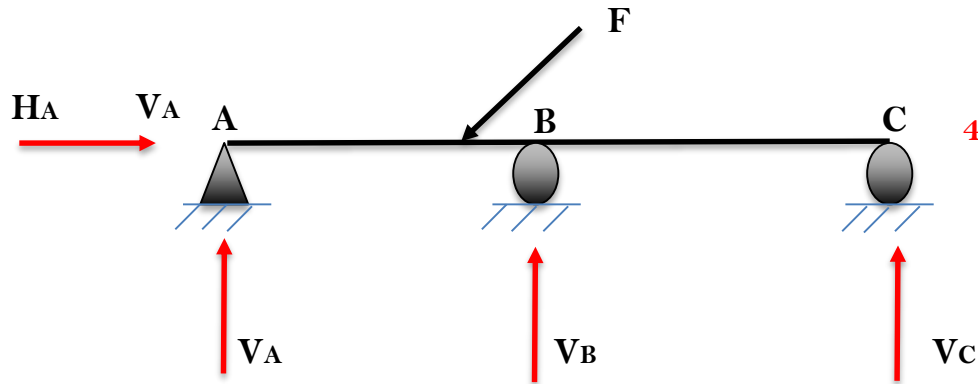
# TD N°01:

## Beam Analysis

### ”Analysis of Statically Determinate Beames”

#### 1) Determinacy:

- The structure is *statically determinate* when all force in a structure can be determined strictly from equilibrium equations.
- The structure is *statically indeterminate* when structures having more unknown forces than available equilibrium equations.



$r = 3n$ , statically determinate  
 $r > 3n$ , statically indeterminate

Where:

**r** is the number of force and moment reaction components

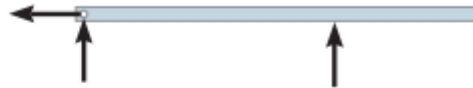
**n** is the number of parts

Example:



(a)

$$r = 3, n = 1, 3 = 3(1)$$

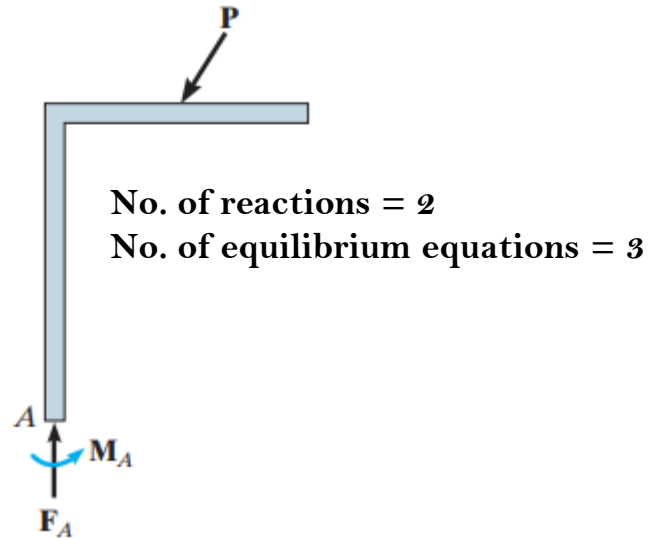
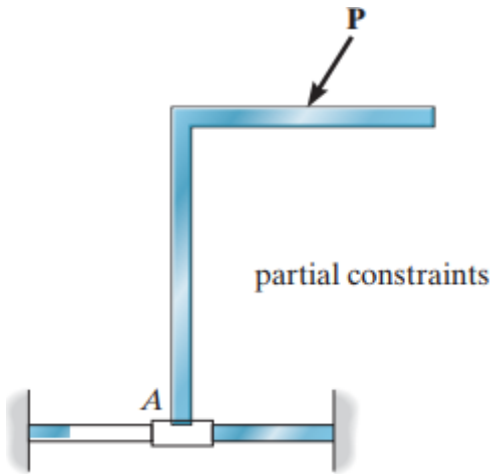


Statically determinate

## 2) Stability:

### a) Partial constraints

A structure or one of its members may have **fewer** reactive forces than equations of equilibrium that must be satisfied.



## b) Improper constraints

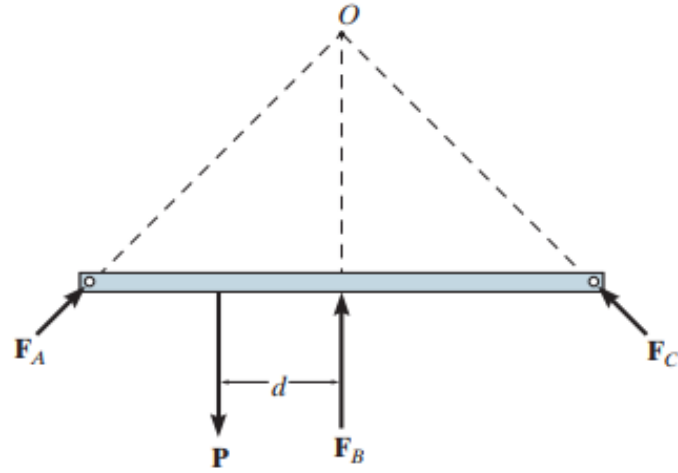
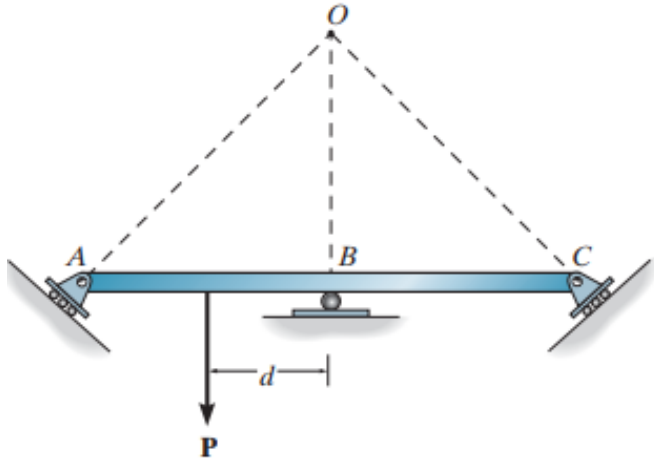
- Improper constraining by the supports causing instability although there may be as many unknown forces as there are equations of equilibrium.
- This can occur if:
  - All the supports reactions are concurrent at a point.
  - The reactive forces are all parallel

### **NOTE:**

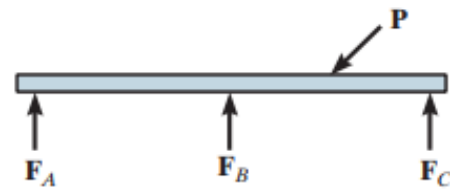
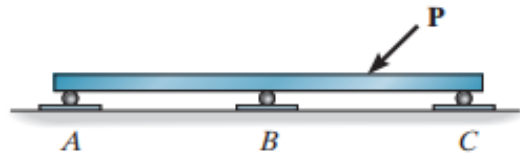
If the structure is unstable, it does not matter if it is statically determinate or indeterminate. In all cases such types of structures must be avoided in practice.



## Cases of Improper constraints



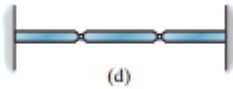
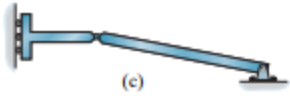
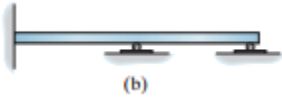
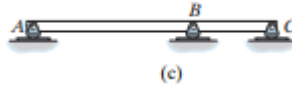
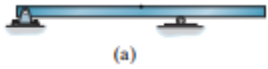
concurrent reactions



parallel reactions

Exercise:

Classify each of the beams in bellow Figures as unstable, statically determinate, or statically indeterminate

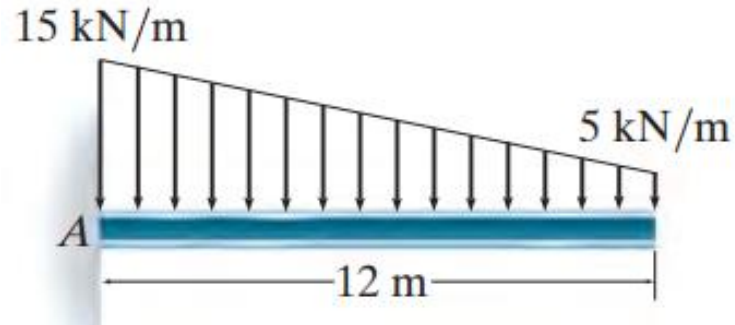


### 3. Internal loading developed in a beam (Section Method)

#### a- Determine the member's support reactions

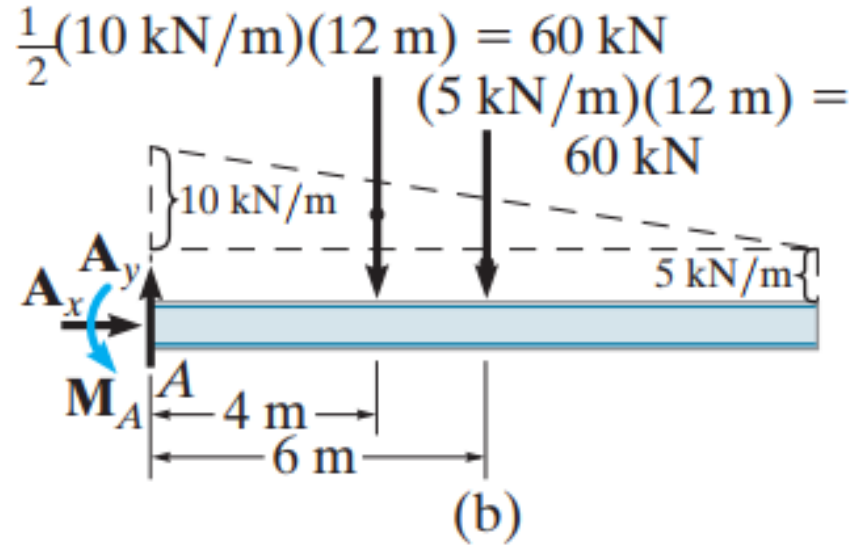
Example :

Determine the reactions on the beam in the Figure (a)



(a)

Solution:



Equations of equilibrium:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 60 - 60 = 0 \quad A_y = 120 \text{ kN}$$

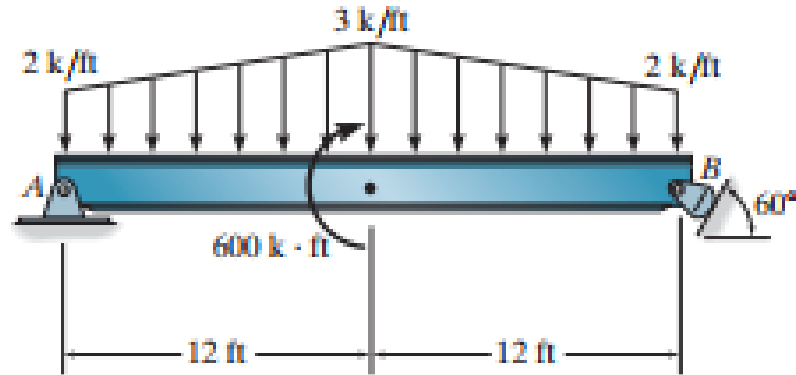
$$\curvearrowleft + \Sigma M_A = 0; \quad -60(4) - 60(6) + M_A = 0 \quad M_A = 600 \text{ kN} \cdot \text{m}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 60 - 60 = 0 \quad A_y = 120 \text{ kN}$$

$$\curvearrowleft + \Sigma M_A = 0; \quad -60(4) - 60(6) + M_A = 0 \quad M_A = 600 \text{ kN} \cdot \text{m}$$

2-19. Determine the reactions on the beam.



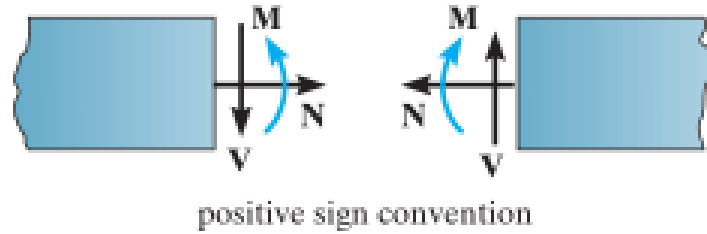
Solution:

$$F_B = 110 \text{ k}$$

$$A_x = 95.3 \text{ k}$$

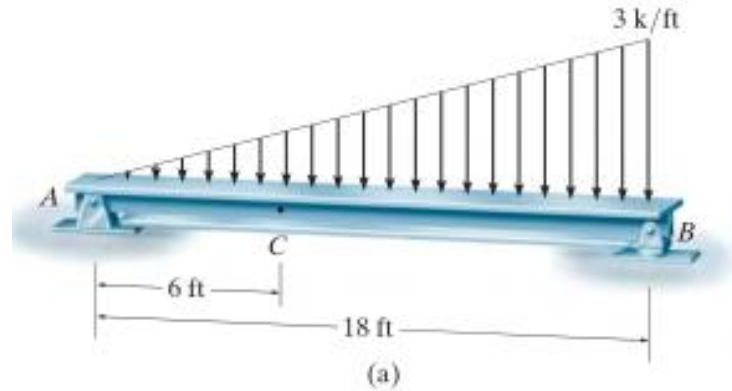
$$A_y = 5.00 \text{ k}$$

b- Section Method to determine the internal forces:



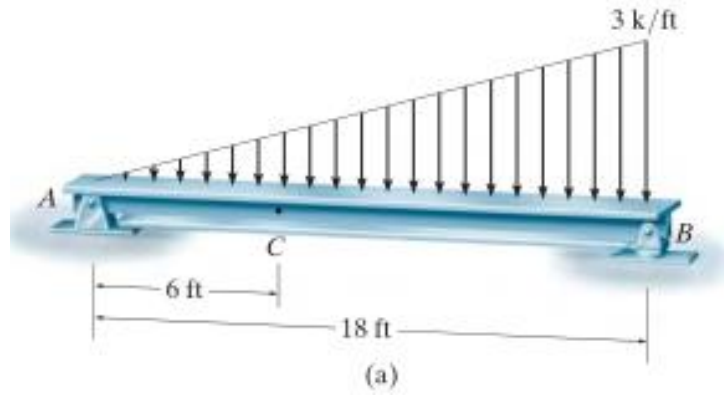
**Example N°01 :**

**Determine the internal shear ( $V$ ) and moment ( $M$ ) acting at a section passing through point C in the beam shown in Figure (a)**



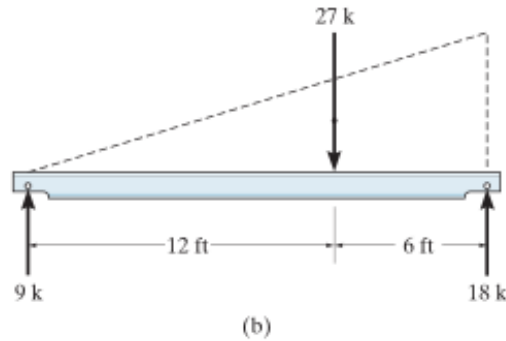


**Solution:**

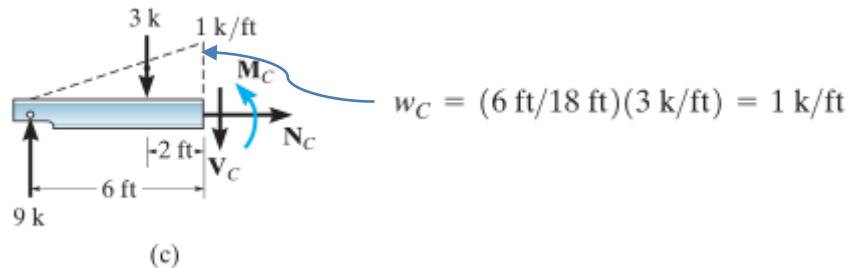


## Solution:

1. **Support Reactions:** replacing the distributed load by its resultant force and computing the reactions yields the results shown in Figure (b)



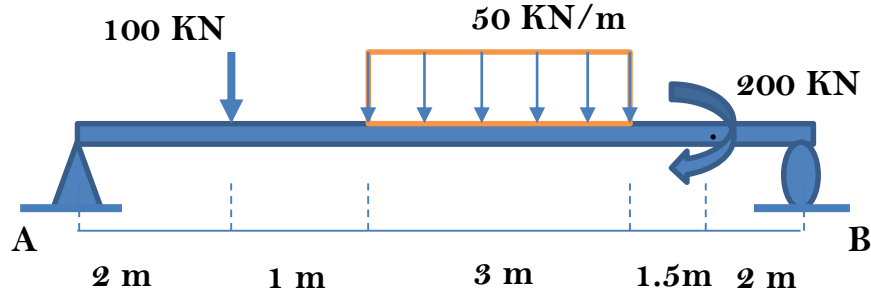
2. **Section Method:** segment AC will be considered since it yields the simplest solution, Figure (c). The distributed load intensity at C is computed by proportion, that is:



$$\begin{aligned} +\uparrow \Sigma F_y = 0; & \quad 9 - 3 - V_C = 0 & \quad V_C = 6 \text{ k} \\ \curvearrowright +\Sigma M_C = 0; & \quad -9(6) + 3(2) + M_C = 0 & \quad M_C = 48 \text{ k} \cdot \text{ft} \end{aligned}$$

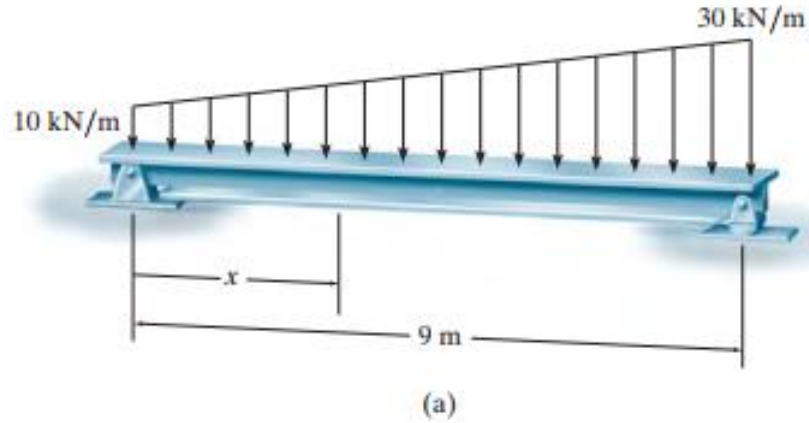
**Exercise:**

Find the internal forces in the beam shown in Fig (a) at 7m from the point A.



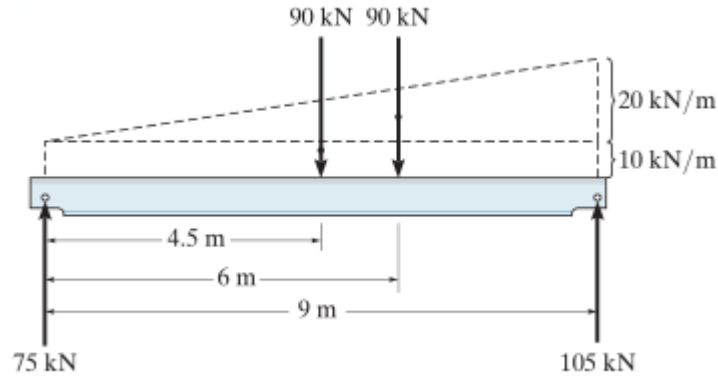
**Example N°03:**

Determine the internal shear and moment in the beam shown in Fig (a) as a function of  $x$ .

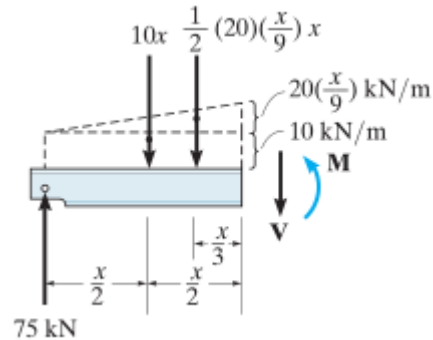


## Solution:

### 1. Support Reactions:



### 2. Section Method :



$$+\uparrow \Sigma F_y = 0; \quad 75 - 10x - \left[ \frac{1}{2}(20)\left(\frac{x}{9}\right)x \right] - V = 0$$

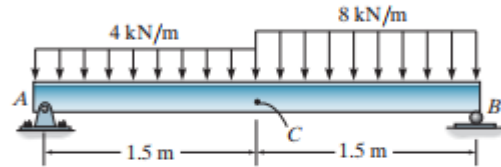
$$V = 75 - 10x - 1.11x^2$$

$$\curvearrowleft + \Sigma M_S = 0; \quad -75x + (10x)\left(\frac{x}{2}\right) + \left[ \frac{1}{2}(20)\left(\frac{x}{9}\right)x \right] \frac{x}{3} + M = 0$$

$$M = 75x - 5x^2 - 0.370x^3$$

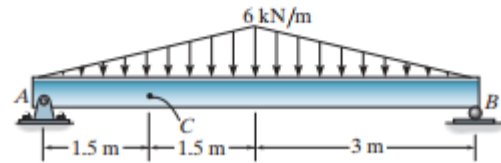
Exercise:

**F4-2.** Determine the internal normal force, shear force, and bending moment acting at point  $C$  in the beam.



**F4-2**

**F4-3.** Determine the internal normal force, shear force, and bending moment acting at point  $C$  in the beam.



**F4-3**



## 2. Shear Force and Moment Diagramme (S.F.D & B.M.D)

The following procedure provides a method for constructing the shear and moment diagrams for a beam:

### Support Reactions

- Determine the support reactions and resolve the forces acting on the beam into components which are perpendicular and parallel to the beam's axis.

### Shear Diagram

- Establish the  $V$  and  $x$  axes and plot the values of the shear at the two ends of the beam.
- Since the slope of the shear diagram at any point is equal to the intensity of the distributed loading  $w$  at the point.(Note that  $w$  is positive when it acts upward.)
- If a numerical value of the shear is to be determined at the point,

one can find this value either by using the method discussed before or by using this Eq:

$$\Delta V = \int w(x) dx$$

$$\left. \begin{array}{l} \text{Change in} \\ \text{Shear} \end{array} \right\} = \left\{ \begin{array}{l} \text{Area under} \\ \text{Distributed Loading} \\ \text{Diagram} \end{array} \right.$$

, which states that the change in the shear force is equal to the area under the distributed loading diagram.

- Since  $w(x)$  is integrated to obtain  $V$ , if  $w(x)$  is a curve of degree  $n$ , then  $V(x)$  will be a curve of degree  $n+1$ . For example, if  $w(x)$  is uniform,  $V(x)$  will be linear.

### Moment Diagram

- Establish the **M** and  $x$  axes and plot the values of the moment at the ends of the beam.
- Since the slope of the moment diagram at any point is equal to the intensity of the shear at the point.
- At the point where the shear is zero, and therefore this may be a point of maximum or minimum moment.
- If the numerical value of the moment is to be determined at a point, one can find this value either by using the method of sections as discussed before or by using Eq:

$$\Delta M = \int V(x) dx$$

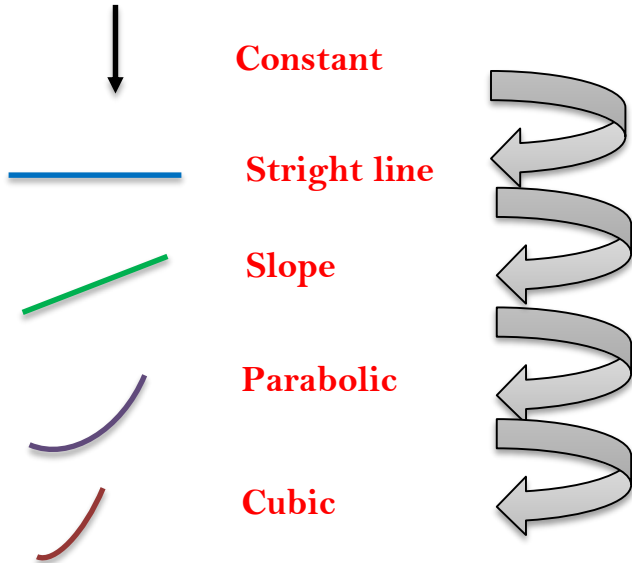
$$\left. \begin{array}{l} \text{Change in} \\ \text{Moment} \end{array} \right\} = \left\{ \begin{array}{l} \text{Area under} \\ \text{Shear Diagram} \end{array} \right.$$

which states that the change in the moment is equal to the area under the shear diagram.

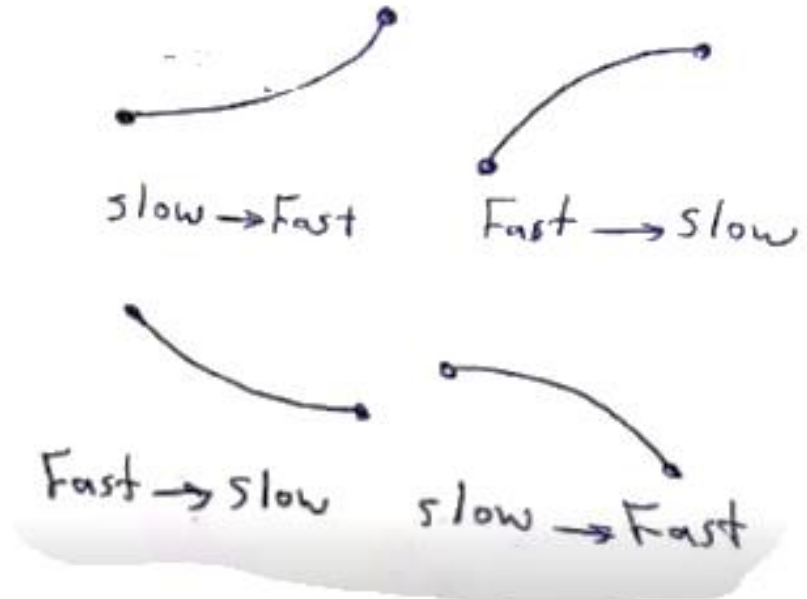
- Since  $V(x)$  is integrated to obtain  $M$ , if  $V(x)$  is a curve of degree  $n$ , then  $M(x)$  will be a curve of degree  $n+1$ . For example, if  $V(x)$  is linear,  $M(x)$  will be parabolic.

### Graphic Method

Order of the Lines:

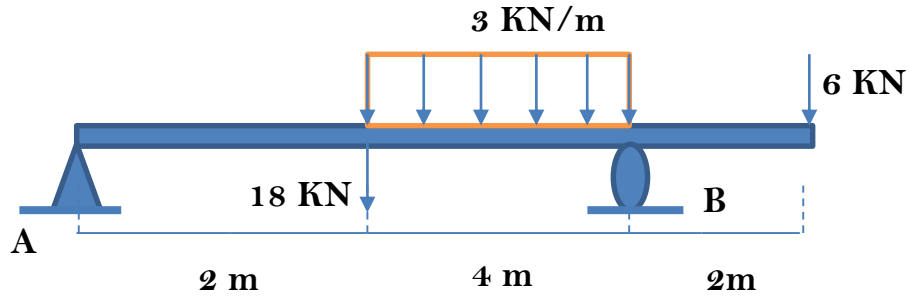


For the of bending moment:



**Example N°01:**

**Draw the shear and moment diagrams for the beam in Figure (a)**



## Solution:

1) Support reactions:

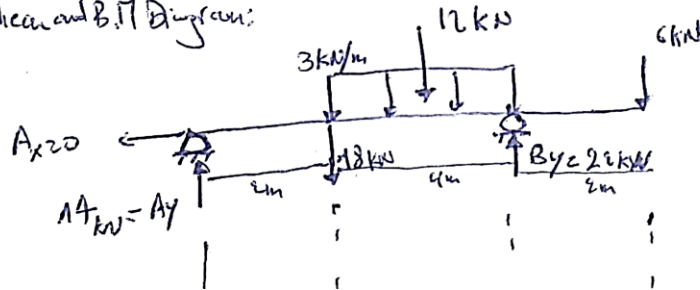
$$\sum F_y = 0 : A_y + B_y = 18 + 12 + 6 = 36$$

$$\sum M_A = 0 : -B(2) - 12(4) + B_y(6) - 6(8)$$

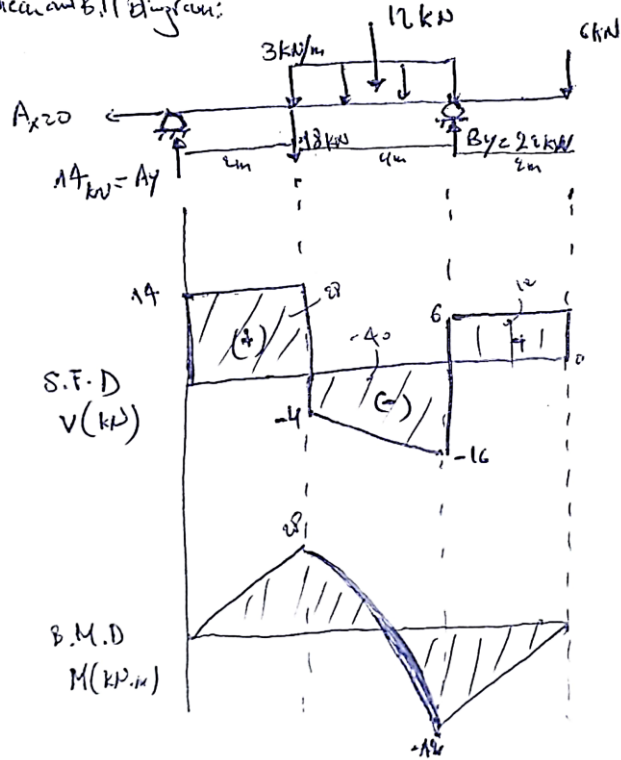
$$B_y = 22 \text{ kN}$$

$$\Rightarrow A_y = 14 \text{ kN}$$

2) Shear and B.M Diagrams:

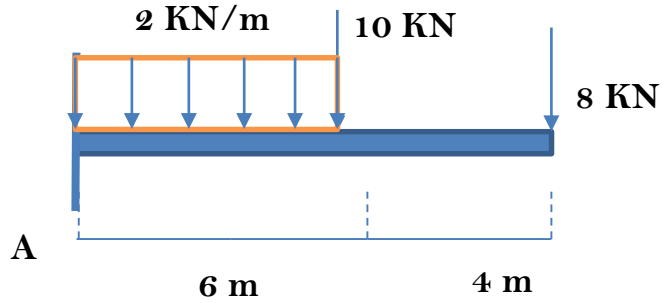


2) Shear and B.M Diagrams:



**Example N°02:**

**Draw the shear and moment diagrams for the beam in Figure (b)**



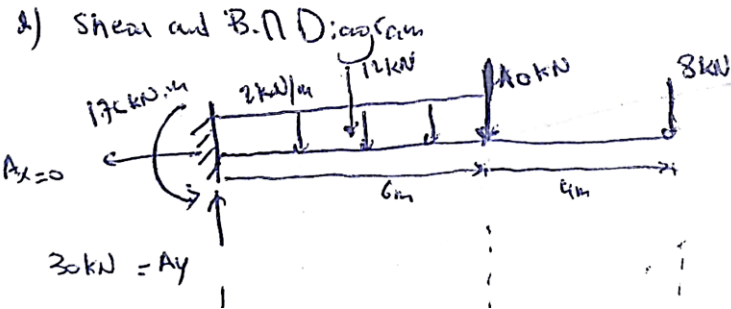
## Solution:

1) Support reactions.

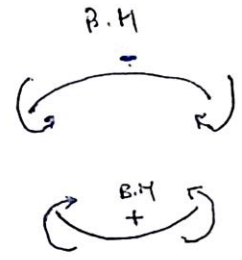
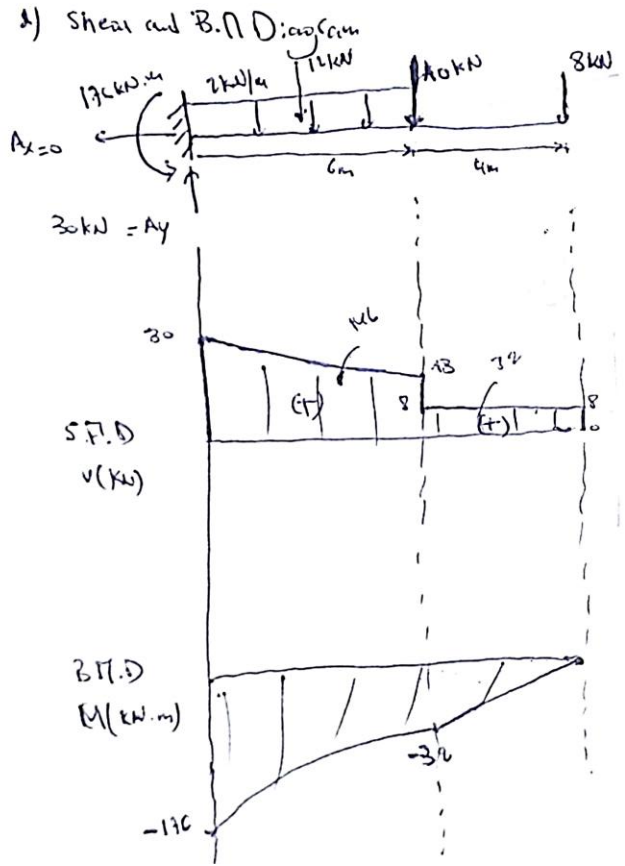
$$\sum F_y = 0 : A_y = 12 + 10 + 8 = 30 \text{ kN}$$

$$\sum M_A = 0 = \uparrow R_A - 12(3) - 10(6) - 8(10)$$

$$R_A = 176 \text{ kN.m}$$

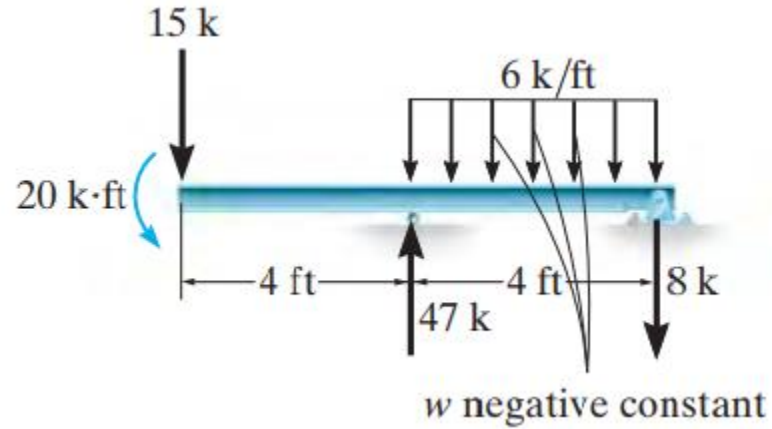




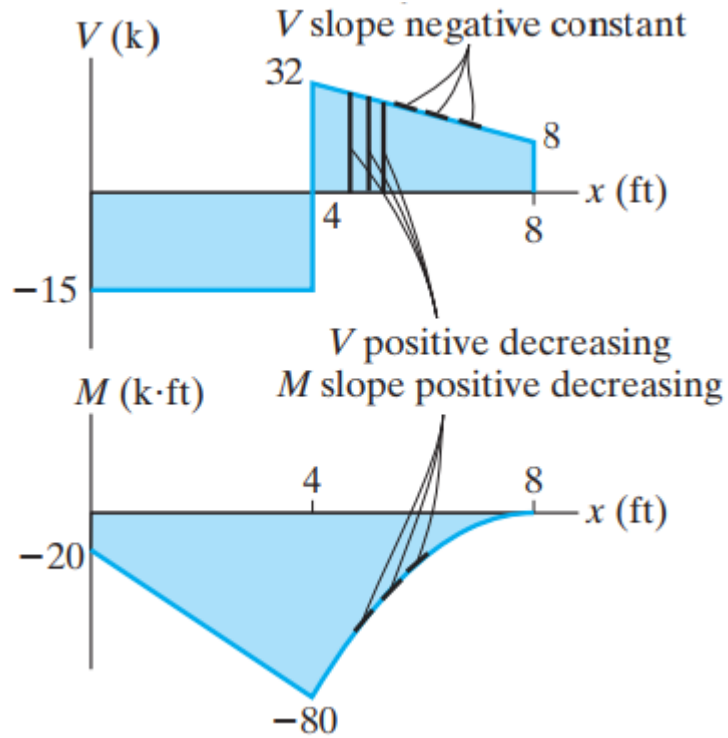


**Example N°03:**

**Draw the shear and moment diagrams for the beam in Figure (c)**

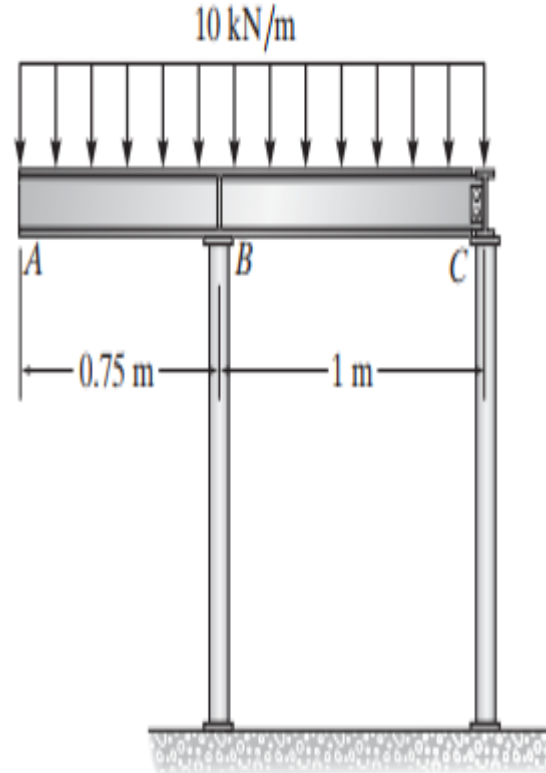


Solution:

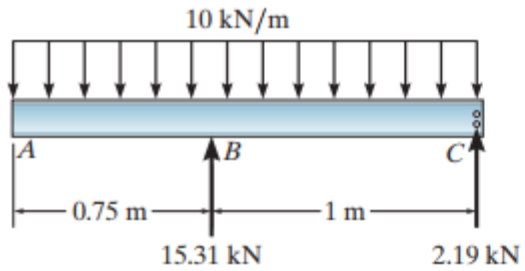


**Problem:**

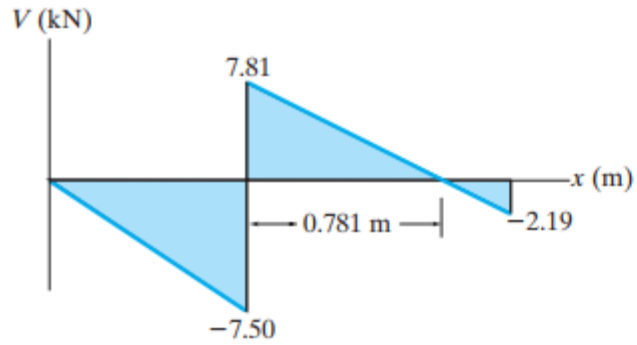
The beam shown in the photo is used to support a portion of the overhang for the entranceway of the building. The idealized model for the beam with the load acting on it is shown in Fig d. Assume B is a roller and C is pinned. Draw the shear and moment diagrams for the beam.



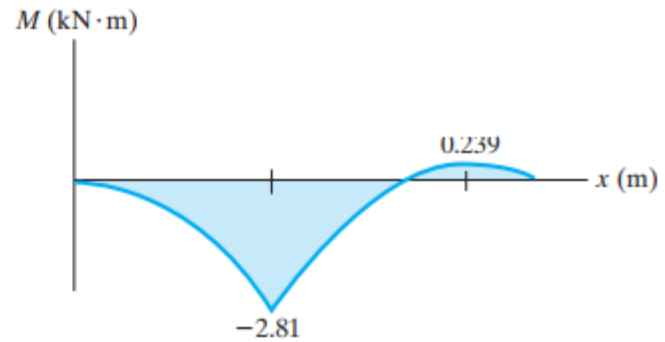
**Solution:**



(b)



(c)



(d)

# References

1. Hibbeler, R. C. 2012. **Structural Analysis. Eighth Edition, Pearson Prentice Hall, New Jersey.**